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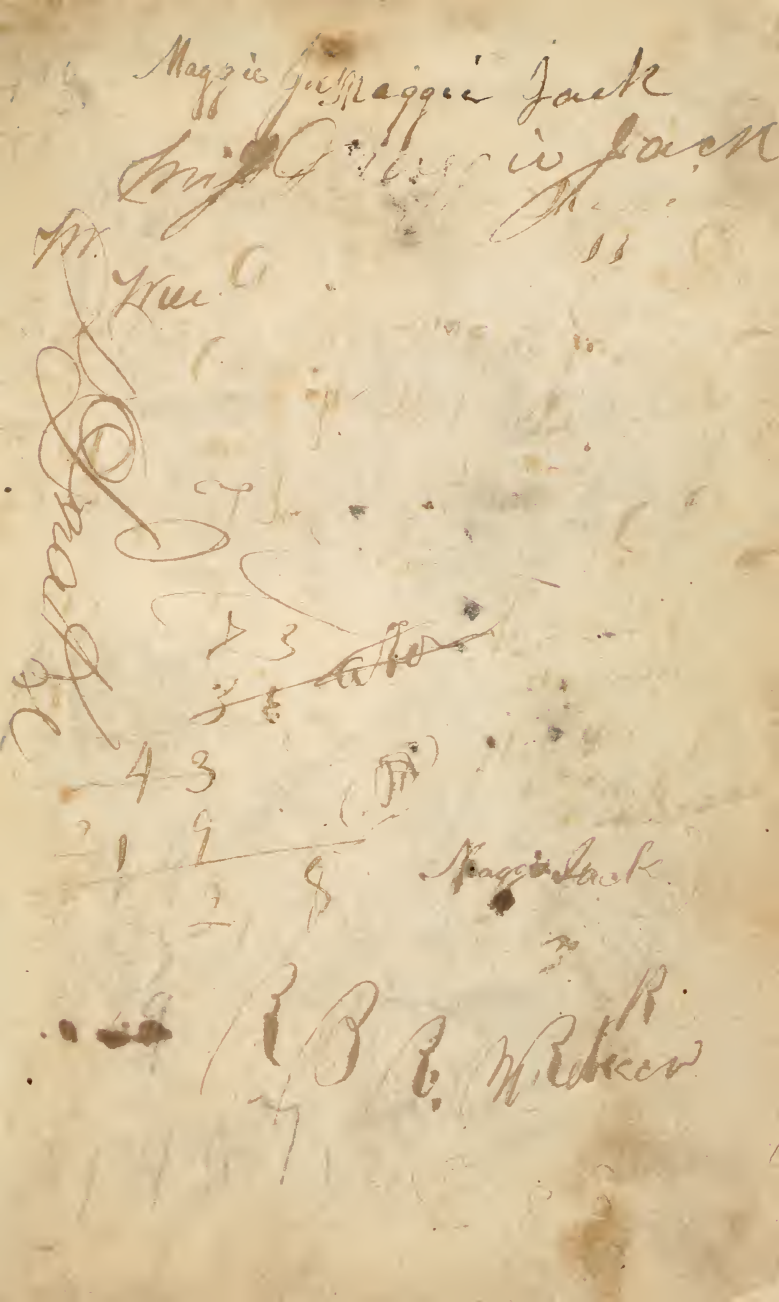

16.

THE

PUBLIC SCHOOL

ARITHMETIC,

IN WHICH

THE ELEMENTS OF THE SCIENCE ARE DEFINED, ITS PRINCIPLES DEMONSTRATED, ITS METHODS EXPLAINED, AND ITS COMMON APPLICATIONS ABUNDANTLY EXEMPLIFIED.

Designed for Public Schools and Academies.

BY

PHILOTUS DEAN, A.M.

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PREFACE.

THIS text-book on Arithmetic has been prepared to meet the requirements both of the individual learner, and of class recitations in large schools. To the former, the written and mental processes, explanations, and models, will be an embodiment of all the instruction that he will need in those cases in which they are given. For schools and classes, the same features of the book will be a well-defined track, by following which the progress of the pupil can be made rapid and satisfactory. The Mental Processes are approximate models of those explanations which every teacher should require from his pupils in giving their solutions. Experience has demonstrated that a well-studied *formula* of reasoning and recitation is the best means of giving both to thought and its expression the requisite degree of brevity, clearness, and force. The teacher is therefore advised to require his pupils to explain their solutions, either in the methods furnished in the book, or in such as he may construct for his own use. The author is convinced from long experience that, when the exercises of the learner are thus conducted, no harm can result from having before the eye the answers to the questions. It is the teacher's duty to see that the *mind* of the pupil performs its work. He who accepts a copied answer, or a defective process because it produces the answer, is in no sense a *teacher*. The results of the author's computations are, therefore, given to the learner merely as a check against mistake,

suggesting to him, who would otherwise be satisfied with an incorrect process, that he should review his work. The book thus becomes *an assistant* to teachers of large schools, saving much time which is needed for other things.

The author trusts that this work will be found superior to most works of its grade, in amount and variety of matter, accuracy and completeness of definitions, scope and thoroughness of commercial rules, and in the omission of their theoretical errors. The arrangement is such as to lead the learner naturally and logically from topic to topic, whilst he has a sufficient number of examples for practice, as he goes, to impart familiarity and skill. Yet in this, as in other works, the student may sometimes find it advisable, in his first advance through the book, to defer for review such portions as can be omitted without detriment to his understanding of any following matter, and can be better comprehended when he has attained to a greater maturity of thought.

In conclusion, the author takes this occasion to express to the distinguished teachers who have assisted him, his high appreciation of their talents, and his heartfelt gratitude.

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ARITHMETIC.

CHAPTER I.

DEFINITIONS. NOTATION. NUMERATION.

DEFINITIONS.

Art. 1. ARITHMETIC is the science of numbers and the art of computing by them.

A *number* is an expression of quantity.

Quantity is that which is capable of measurement.

Art. 2. A *unit* of any thing is a single thing of that kind, and is used as a *measure* of the *quantity* of that thing. It is sometimes called *unity*.

Unity, properly, is *singleness*, or the state of being *single or one*.

Number, properly, is the state of being *one or more*.

Art. 3. In reference to its nature, a unit is either *arbitrary* or *natural*.

An *arbitrary* unit is a portion of a quantity, assumed for the purpose of estimating and expressing that quantity; as, a *pound* for weight, a *minute* for time, a *yard* for length, &c. Such units can be assumed of any extent at our pleasure.

Natural unity is *individuality*, and a natural unit of a certain

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QUESTIONS.—1. What is Arithmetic? What is a number? What is quantity?—2. What is a unit of any thing? For what is it used? What else is it called? What is unity? What is number?—3. What is a unit in reference to its nature? What is an arbitrary unit? Is its extent limited by nature? What is natural unity? What is a natural unit? Is its extent limited by nature?

kind is an *individual* of that kind; as, a *tree*, a *cloud*, a *house*, a *man*, &c. Such units cannot be varied in their extent at our pleasure.

Art. 4. In reference to their *signification*, numbers are *concrete* or *abstract*.

A *concrete* number is an expression of a particular *kind* of quantity; as *two men*, *nine dollars*, *seventy-five pounds*, &c.

An *abstract* number is an expression of quantity without reference to its kind; as *two*, *nine*, *seventy-five*, &c.

Art. 5. In reference to the *mode* of expressing numbers, Arithmetic is either *oral* or *written*.

Oral Arithmetic treats of numbers as *spoken*. In it our numerical perceptions and reasonings are expressed by speech.

Written Arithmetic treats of numbers as *written*. In it our numerical perceptions and processes are expressed by written *words* and *symbols*.

Art. 6. A *rule* in Arithmetic is a statement of the mode of computing.

There are six primary processes in written Arithmetic, viz.: Notation, Numeration, Addition, Subtraction, Multiplication, and Division. The last four are called *fundamental rules*, because every other arithmetical process depends upon one or more of them.

NOTATION.

Art. 7. NOTATION is the art of expressing numbers by symbols. There are two methods of notation in common use, viz.: the *Roman*, and the *Arabic* or *Decimal*.

ROMAN NOTATION.

Art. 8. The symbols of the Roman notation are seven capital letters, viz.: I, which signifies *one;* V, *five;* X, *ten;* L, *fifty;* C, *one hundred;* D, *five hundred;* and M, *one thousand.*

QUESTIONS.—4. What are numbers in reference to their signification? What is a concrete number? What is an abstract number?—5. What is Arithmetic in reference to its mode of expression? What is oral Arithmetic? What is written Arithmetic?—6. What is a rule? What are the primary processes of written Arithmetic? What are its fundamental rules? Why?—7. What is notation? What are the common methods?—8. What are the symbols of the Roman notation?

To express other numbers, these letters are combined. Thus II signifies *two;* VI, *six;* XV, *fifteen;* &c.

TABLE OF ROMAN NOTATION.

I	signifies	one.	XXI	signifies	twenty-one.
II	"	two.	XXX	"	thirty.
III	"	three.	XL	"	forty.
IV	"	four.	L	"	fifty.
V	"	five.	LX	"	sixty.
VI	"	six.	LXX	"	seventy.
VII	"	seven.	LXXX	"	eighty.
VIII	"	eight.	XC	"	ninety.
IX	"	nine.	C	"	one hundred.
X	"	ten.	CC	"	two hundred.
XI	"	eleven.	CCC	"	three hundred.
XII	"	twelve.	CCCC	"	four hundred.
XIII	"	thirteen.	D or IↃ	"	five hundred.
XIV	"	fourteen.	DC	"	six hundred.
XV	"	fifteen.	DCC	"	seven hundred.
XVI	"	sixteen.	DCCC	"	eight hundred.
XVII	"	seventeen.	DCCCC	"	nine hundred.
XVIII	"	eighteen.	M or CIↃ	"	one thousand.
XIX	"	nineteen.	MM	"	two thousand.
XX	"	twenty.	MMM	"	three thousand.

The letter I before V signifies *one* less than *five;* before X, *one* less than *ten;* after V, *one* more than *five;* after X, *one* more than *ten*. The letter X before L signifies *ten less than fifty;* before C, *ten less than one hundred;* after L, *ten more than fifty;* after C, *ten more than one hundred.*

Every repetition of a letter signifies a repetition of its value. A line over a letter signifies a thousand times the value of the letter: thus, $\overline{\text{X}}$, *ten thousand;* $\overline{\text{D}}$, *five hundred thousand.*

Art. 9. To write numbers in the Roman notation.

RULE. *Write the highest part of the number first, the next less at its right hand, and so on to the lowest part.*

QUESTIONS.—How are other numbers expressed? Repeat the table of Roman notation. What does I before a letter signify? What does I after a letter signify? What does X before a letter signify? What does X after a letter signify? What does a repetition of a letter signify? What does a line over a letter signify?

Example. Write, in the Roman notation, one hundred and forty-three.

Here, we first write C for one hundred, then XL for forty, then III for three. Answer, CXLIII.

EXERCISES.

Write, in the Roman Notation,

1. Twenty-two.
2. One hundred thirty-nine.
3. One hundred and one.
4. Fifty-five.
5. Sixty-seven.
6. Seventy-three.
7. Two hundred eighty-four.
8. Four hundred ninety-nine.
9. Five hundred and five.
10. Six hundred and sixteen.
11. One thousand seven hundred and fourteen.
12. Eighteen hundred and fifty-eight.

Art. 10. Read the following numbers.

XCIX.	$\overline{\text{X}}$M.	MDCCLXXVII.
CI.	MCCXLVI.	MDCCCLX.
DLXI.	DCCCCIX.	$\overline{\text{D}}$V.
CCCXXXIII.	MMDCXIV.	$\overline{\text{M}}$.

Art. 11. The foregoing is called the Roman notation, because it was used by the ancient Romans. Its chief use is in numbering divisions of books, dial faces, &c.

ARABIC NOTATION.

ART. **12.** The symbols of the Arabic or decimal notation are ten, viz.:—

1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

one, two, three, four, five, six, seven, eight, nine, cipher.

The first nine of these figures are called *digits*, from the process of counting on the fingers; and *significant* figures, because they always signify some value. The *cipher* is called *naught*, or *zero*, because *when alone* it signifies no value. When used

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QUESTIONS.—9. How are numbers written in Roman notation?—11. Why is it called *Roman*? What is its chief use?—12. What are the symbols of the Arabic notation? What are they called, and why? What is the use of 0 with other figures?

with the significant figures, it assists them in expressing number. Thus *one* and *naught* (10) do not signify *one*, but *ten;* eight and two naughts (800) do not signify *eight*, but *eight hundred*, &c.

Art. 13. When figures stand *alone*, they signify the number of *units* expressed by their names; thus 3 signifies three units, 7, seven units, &c. When a figure is written at the left of another, it signifies the number of *tens* expressed by its name. Thus, 49 signifies *four tens*, or forty, and nine units, both expressing *forty-nine;* 50 signifies *five tens* no units, both expressing *fifty;* and 99 signifies *nine tens*, or ninety, and nine units, both expressing *ninety-nine.*

TABLE.

10 is ten, or one ten.
11 is eleven, or one ten one unit.
12 is twelve, or one ten two units.
13 is thirteen, or one ten three units.
14 is fourteen, or one ten four units.
15 is fifteen, or one ten five units.
16 is sixteen, or one ten six units.
17 is seventeen, or one ten seven units.
18 is eighteen, or one ten eight units.
19 is nineteen, or one ten nine units.
20 is twenty, or two tens no units.
21 is twenty-one, or two tens one unit.
30 is thirty, or three tens.
40 is forty, or four tens.
50 is fifty, or five tens.
60 is sixty, or six tens.
70 is seventy, or seven tens
80 is eighty, or eight tens.
90 is ninety, or nine tens.
100 is one hundred.

Art. 14. Figures at the left of tens, signify *hundreds;* at the left of hundreds, *thousands;* at the left of thousands, *ten thousands;* at the left of ten thousands, *hundred thousands;* at the left of hundred thousands, *millions.*

QUESTIONS.—13. What does a figure express when alone? What does a gure express at the left of another?—14. What are the successive significations of a figure at the left of two, three, four, five, and six figures?

Illustration.

Numbers.	Signification.
2	Two.
23	Twenty-three.
234	Two hundred thirty-four.
2345	Two thousand three hundred forty-five.
23456	Twenty-three thousand four hundred fifty-six.
234567	Two hundred thirty-four thousand five hundred sixty-seven.
2345678	Two millions three hundred forty-five thousand six hundred seventy-eight.

Thus a figure expresses a ten times greater quantity, every time it is placed at the left of one more figure.

The *place* of a figure is its position, as determined by the number of figures it is removed from units.

Art. 15. The *value* of a figure is its power of signifying or expressing. Figures have two *kinds* of value, viz.: *simple* and *local*.

The *simple value* of a figure is its significance when it is alone.

The *local value* of a figure is its significance as modified by occupying a place in a number.

A *particular local value* of a figure is its significance in a given place in a number. In the place of units the simple and local values are the same.

Art. 16. The foregoing is called the *Arabic Notation*, because the Europeans obtained it from the Arabs. It is called the *Decimal Notation*, because it uses *ten* symbols so combined as to signify *tenfold* at every occupation of one more place toward the left.

NUMERATION.

Art. 17. Numeration is the art of reading numbers expressed by figures.

To assist in reading and writing numbers in the Arabic Notation, the figures are arranged in groups, called *periods*, beginning at the place of units. There are two methods of grouping in use, viz.: the *French* and the *English*.

Questions.—When does a figure express ten times more? What is the place of a figure?—15. What is the value of a figure? What are its *kinds* of value? What is its simple value? What is its local value? What is its particular local value? Where do the two values agree?—16. Why is this called the Arabic notation? Why decimal notation?—17. What is numeration? How are figures arranged in numeration? What are the two methods?

FRENCH METHOD OF NUMERATION.

Art. 18. In the French method three figures form a period. Proceeding from right to left, the names of the periods are Units, Thousands, Millions, Billions, Trillions, Quadrillions, Quintillions, Sextillions, Septillions, Octillions, Nonillions, Decillions, Undecillions, Duodecillions, Tredecillions, Quatuordecillions, Quindecillions, Sexdecillions, Septendecillions, Octodecillions, Novendecillions, Vigintillions, &c.

The right hand figure of each period is the *units* of that period; the middle figure, its *tens;* and the left hand figure its *hundreds.* This is illustrated by the following

French Numeration Table.

	Hundreds of Octillions.	Hundreds of Septillions.	Hundreds of Sextillions.	Hundreds of Quintillions.	Hundreds of Quadrillions.	Hundreds of Trillions.	Hundreds of Billions.	Hundreds of Millions.	Hundreds of Thousands.	Hundreds.
	Tens of Octillions.	Tens of Septillions.	Tens of Sextillions.	Tens of Quintillions.	Tens of Quadrillions.	Tens of Trillions.	Tens of Billions.	Tens of Millions.	Tens of Thousands.	Tens.
	Octillions.	Septillions.	Sextillions.	Quintillions.	Quadrillions.	Trillions.	Billions.	Millions.	Thousands.	Units.
10th Period. Octillions.	1	8	7							
9th Period. Septillions.	4	2	9							
8th Period. Sextillions.	6	1	8							
7th Period. Quintillions.	3	9	7							
6th Period. Quadrillions.	4	5	6							
5th Period. Trillions.	2	7	3							
4th Period. Billions.	1	6	9							
3d Period. Millions.	9	1	2							
2d Period. Thousands.	6	8	7							
1st Period. Units.	4	3	5							

Art. 19. To read numbers.

Rule.—*Beginning at units, mark off the numbers into as many periods as possible, of three figures each, naming the periods as you proceed.*

Then, beginning at the left hand, read and name each period in order by itself, omitting the name of the last.

Example 1.—The number in the numeration table is read thus:—One hundred eighty-seven octillions, four hundred twenty-nine septillions, six hundred eighteen sextillions, three hundred ninety-seven quintillions, four hundred fifty-six quad-

Questions.—18. What are the names of the periods? What do the right, middle, and left figures of a period signify? Give in order the name of each place in the table.—19. What is the rule for reading numbers? Read the example in the table.

rillions, two hundred seventy-three trillions, one hundred sixty-nine billions, nine hundred twelve millions, six hundred eighty-seven thousand, four hundred thirty-five.

Ex. 2. Pronounce 70140091001. This number, marked off, is 70,140,091,001, and is read thus:—seventy billions, one hundred forty millions, ninety-one thousand, and one.

Pronounce or write in words the following numbers:—

8	1001	80100	210987654321
27	2800	78002	50000400400020
36	3080	24440	76088900404033
459	4701	66666	10111100010001
302	8750	100001	22033044055066
101	5075	802021	55606707808909
940	9866	707070	30004000005000
700	9999	990880	80008080008008
850	70333	404404	10000000010000
110	20010	878934	123003003003003

98017263545005328907600210000

10056750000000006000000070000

60000090810120230344400055560

86007000809014007302000601010

Art. 20. To write numbers.

RULE.—*First write the number of the highest period, then that of each period successively toward the right, filling with ciphers the places and periods not expressed.*

Write in figures the following numbers:—

1. Eighty-four.
2. Five hundred seventy-two.
3. Six hundred and three.
4. Nine hundred and ten.
5. Three thousand thirty-one.
6. Seventeen thousand and six.
7. Eleven thousand nineteen.
8. Eighty thousand and eight.
9. Four hundred thousand forty.
10. Two hundred and one thousand and five.
11. Six millions and sixty-three.
12. Forty-two millions forty-two.

QUESTION.—20. What is the rule for writing numbers?

13. Fourteen millions, fifty thousand.
14. Fifty billions, ten thousand.
15. One hundred billions, twenty thousand and fifty-six.
16. Thirty trillions, three millions, three.
17. Seven quadrillions, nine billions, sixty-five thousand.
18. Ten septillions, one thousand.
19. Four octillions, twenty quintillions.
20. Five vigintillions, six quindecillions, eighty one decillions, ten septillions, and twenty-three.

Art. 21. The French method of numeration is generally used on the continent of Europe and in the United States.

ENGLISH METHOD OF NUMERATION.

Art. 22. In the English method, six figures form a period. Proceeding from right to left, the names of the periods are, Units, Millions, Billions, Trillions, &c., as in the French method. This is illustrated in the following

English Numeration Table.

of Quadrillions.	of Trillions.	of Billions.	of Millions.	of Units.
Hundreds of Thousands	Hundreds of Thousands	Hundreds of Thousands	Hundreds of Thousands	Hundreds of Thousands
Tens of Thousands	Tens of Thousands	Tens of Thousands	Tens of Thousands	Tens of Thousands
Thousands	Thousands	Thousands	Thousands	Thousands
Hundreds	Hundreds	Hundreds	Hundreds	Hundreds
Tens	Tens	Tens	Tens	Tens
Units	Units	Units	Units	Units
1 8 7 4 2 9	6 1 8 3 9 7	4 5 6 2 7 3	1 6 9 9 1 2	6 8 7 4 3 5

This number, thus grouped, is read

One hundred eighty-seven thousand four hundred and twenty-nine quadrillions; six hundred eighteen thousand three hundred and ninety-seven trillions; four hundred fifty-six thousand two hundred and seventy-three billions; one hundred sixty-nine thousand nine hundred and twelve millions; six hundred eighty-seven thousand four hundred and thirty-five.

Art. 23. To read numbers in English numeration.

Questions.—21. Where is the French numeration generally used?—22. In the English method how many figures form a period?

RULE.—*Beginning at units, mark off the number into as many periods as possible of six figures each, naming the periods as you proceed.*

Then, beginning at the left hand, read and name each period in order by itself, omitting the name of the last.

Pronounce or write in words the following numbers.

106425	354010217230496
987401002	80700650400300210
7456003120	8544342491200710397568
1024061018	407800620003450000092125650789
3406084645	132578904500638007204001750003
7650430219	876543210123456789012345678900

Art. 24. To write numbers expressed in the English method.

RULE.—*First write the number of the highest period, then that of each period successively toward the right, filling with ciphers the places and periods not expressed.*

Write in figures the following numbers.

1. Twelve thousand four hundred and three billions; one hundred and sixteen thousand millions; ninety thousand and nine.

2. Seven hundred fifty-six thousand eight hundred and four trillions; one thousand and one billions; two hundred thousand and two millions; eighty-five thousand and eighty-five.

3. Eight thousand eight hundred and one quadrillions; forty thousand and sixty-two trillions; five thousand and twenty-eight millions; and ten thousand and one.

4. Sixty-one thousand quintillions; one hundred twenty-nine trillions; and four hundred seventeen thousand.

5. Eighteen quintillions; eighteen thousand quadrillions; one hundred eighty thousand trillions; one thousand eight hundred billions; one million; and eight hundred thousand.

6. Fifty thousand octillions; three hundred thousand septillions; nine thousand sextillions; seven hundred quintillions; forty-five quadrillions; three trillions; one hundred forty-eight thousand two hundred six billions; four hundred thousand and twelve millions; ten thousand and ten.

QUESTION.—23. What is the rule for reading in English numeration?

CHAPTER II.

ADDITION.

Art. 25. ADDITION is the process of uniting two or more numbers, so as to form one number, which is called their *sum*, or *amount*.

Learn perfectly the following

ADDITION TABLE.

1 and 1 are 2	1 and 2 are 3	1 and 3 are 4	1 and 4 are 5
2 and 1 are 3	2 and 2 are 4	2 and 3 are 5	2 and 4 are 6
3 and 1 are 4	3 and 2 are 5	3 and 3 are 6	3 and 4 are 7
4 and 1 are 5	4 and 2 are 6	4 and 3 are 7	4 and 4 are 8
5 and 1 are 6	5 and 2 are 7	5 and 3 are 8	5 and 4 are 9
6 and 1 are 7	6 and 2 are 8	6 and 3 are 9	6 and 4 are 10
7 and 1 are 8	7 and 2 are 9	7 and 3 are 10	7 and 4 are 11
8 and 1 are 9	8 and 2 are 10	8 and 3 are 11	8 and 4 are 12
9 and 1 are 10	9 and 2 are 11	9 and 3 are 12	9 and 4 are 13
10 and 1 are 11	10 and 2 are 12	10 and 3 are 13	10 and 4 are 14
11 and 1 are 12	11 and 2 are 13	11 and 3 are 14	11 and 4 are 15
12 and 1 are 13	12 and 2 are 14	12 and 3 are 15	12 and 4 are 16
1 and 5 are 6	1 and 6 are 7	1 and 7 are 8	1 and 8 are 9
2 and 5 are 7	2 and 6 are 8	2 and 7 are 9	2 and 8 are 10
3 and 5 are 8	3 and 6 are 9	3 and 7 are 10	3 and 8 are 11
4 and 5 are 9	4 and 6 are 10	4 and 7 are 11	4 and 8 are 12
5 and 5 are 10	5 and 6 are 11	5 and 7 are 12	5 and 8 are 13
6 and 5 are 11	6 and 6 are 12	6 and 7 are 13	6 and 8 are 14
7 and 5 are 12	7 and 6 are 13	7 and 7 are 14	7 and 8 are 15
8 and 5 are 13	8 and 6 are 14	8 and 7 are 15	8 and 8 are 16
9 and 5 are 14	9 and 6 are 15	9 and 7 are 16	9 and 8 are 17
10 and 5 are 15	10 and 6 are 16	10 and 7 are 17	10 and 8 are 18
11 and 5 are 16	11 and 6 are 17	11 and 7 are 18	11 and 8 are 19
12 and 5 are 17	12 and 6 are 18	12 and 7 are 19	12 and 8 are 20
1 and 9 are 10	1 and 10 are 11	1 and 11 are 12	1 and 12 are 13
2 and 9 are 11	2 and 10 are 12	2 and 11 are 13	2 and 12 are 14
3 and 9 are 12	3 and 10 are 13	3 and 11 are 14	3 and 12 are 15
4 and 9 are 13	4 and 10 are 14	4 and 11 are 15	4 and 12 are 16
5 and 9 are 14	5 and 10 are 15	5 and 11 are 16	5 and 12 are 17
6 and 9 are 15	6 and 10 are 16	6 and 11 are 17	6 and 12 are 18
7 and 9 are 16	7 and 10 are 17	7 and 11 are 18	7 and 12 are 19
8 and 9 are 17	8 and 10 are 18	8 and 11 are 19	8 and 12 are 20
9 and 9 are 18	9 and 10 are 19	9 and 11 are 20	9 and 12 are 21
10 and 9 are 19	10 and 10 are 20	10 and 11 are 21	10 and 12 are 22
11 and 9 are 20	11 and 10 are 21	11 and 11 are 22	11 and 12 are 23
12 and 9 are 21	12 and 10 are 22	12 and 11 are 23	12 and 12 are 24

QUESTION.—25. What is addition?

Let the pupil vary and extend the following

Mental Exercises.

1. How many are 1 and 0? 5 and 0? 9 and 0? 7 and 0? &c.
2. How many are 1 and 9? 1 and 19? 1 and 29? 1 and 39? &c.
3. How many are 2 and 9? 2 and 19? 2 and 29? 2 and 39? &c.
4. How many are 3 and 6? 3 and 16? 3 and 26? 3 and 36? &c.
5. How many are 3 and 7? 3 and 17? 3 and 27? 3 and 37? &c.
6. How many are 4 and 7? 4 and 17? 4 and 27? 4 and 37? &c.
7. How many are 5 and 8? 5 and 18? 5 and 28? 5 and 38? &c.
8. How many are 5 and 4? 5 and 14? 5 and 24? 5 and 34? &c.
9. How many are 6 and 10? 6 and 20? 6 and 30? 6 and 40? &c.
10. How many are 7 and 7? 7 and 17? 7 and 27? 7 and 37? &c.
11. How many are 8 and 8? 8 and 18? 8 and 28? 8 and 38? &c.
12. How many are 9 and 8? 9 and 18? 9 and 28? 9 and 38? &c.
13. How many are 8 and 7? 8 and 17? 8 and 27? 8 and 37? &c.
14. How many are 5 and 9? 5 and 19? 5 and 29? 5 and 39? &c.
15. How many are 8 and 4? 8 and 14? 8 and 24? 8 and 34? &c.
16. How many are 4 and 9? 4 and 19? 4 and 29? 4 and 39? &c.
17. How many are 9 and 5? 9 and 15? 9 and 25? 9 and 35? &c.
18. How many are 7 and 6? 7 and 16? 7 and 26? 7 and 36? &c.
19. How many are 6 and 5? 6 and 15? 6 and 25? 6 and 35? &c.
20. How many are 10 and 8? 10 and 18? 10 and 28? 10 and 38? &c.

21. How many are 11 and 9? 11 and 19? 11 and 29? 11 and 39? &c.

22. How many are 11 and 4? 11 and 14? 11 and 24? 11 and 34? &c.

23. How many are 12 and 9? 12 and 19? 12 and 29? 12 and 39? &c.

24. How many are 12 and 5? 12 and 15? 12 and 25? 12 and 35? &c.

25. How many are 12 and 2? 12 and 12? 12 and 22? 12 and 32? &c.

Art. 26. The sign of addition is a cross formed by a perpendicular and a horizontal line; thus, +. Its name is *plus*, a Latin word signifying *more*.

The sign of equality is two parallel horizontal lines; thus, =. The expression $9 + 4 = 13$ is read, *nine plus four is equal to thirteen*.

The things signified by the numbers to be added must be alike. Thus, 9 units + 4 units = 13 units; but 9 units + 4 tens do not equal either 13 units or 13 tens. So 5 apple-trees + 3 peach-trees = 8 *trees*, but not 8 *apple-trees*, nor 8 *peach-trees*.

WRITTEN EXERCISES IN ADDITION.

Case I.

Art. 27. When the sum of each column is less than 10.

Ex. 1. What is the sum of 235, 421, and 22?

Ans. 678.

Written Process.	Mental Process.
235 421 22 678	Because the things signified by the figures to be added must be alike, we form one separate column of the units, another of the tens, and another of the hundreds. Then say, 2 units and one more are 3 units and 5 more are 8 units. Write 8 under the column of units. Then 2 tens and 2 more are 4 tens and 3 more are 7 tens. Write 7 under the column of tens. Then 4 hundreds and two more are 6 hundreds. Write 6 under the column of hundreds.

Questions.—26. What is the sign of addition? What is its name? What i the sign of equality? What numbers only can be added?—27. What is Case I in addition?

RULE.— *Write the numbers so that their units may form one column, their tens another, &c.; and draw a line under them. Beginning with the units, find successively the sum of each column, and write it under the column added.*

PROOF.—To detect error, begin with the units and find the sums of the columns by reckoning the figures in each in a direction different from that of the first process: if the two results are equal, the work is probably correct.

REMARK.—This is not *certain*, because mistakes may be made in either trial or both; but we are not likely to make the same mistake in both trials.

EXAMPLES FOR PRACTICE.

(2.)	(3.)	(4.)	(5.)	(6.)
21	123	52	2070	12013
13	30	424	406	400
735	35	10	5121	50172
769	188	486	7597	62585

7. What is the sum of 11, 12, 13, 20, and 33? Ans. 89.

8. What is the sum of 3, 30, 121, and 4040? Ans. 4194.

9. What is the sum of ten, fifteen, one hundred fifty, two thousand and one, and four thousand three hundred twenty? Ans. 6496.

10. A boy sold from his garden some potatoes for twenty-five cents, tomatoes for thirty-one cents, corn for twenty cents, and beans for twenty-two cents. How many cents did he get? Ans. 98 cents.

11. Gave twenty-three cents for a slate, ten cents for a spelling-book, forty-four cents for an Arithmetic, one cent for a slate-pencil, and eleven cents for an inkstand. How many cents did I give for all? Ans. 89 cents.

12. What is the sum of ten thousand two hundred three, four thousand fifty, six hundred forty-five, and one hundred one? Ans. 14999.

QUESTIONS.—27. What is the rule? How do we detect error in addition?

Case II.

Art. 28. When the sum of any column equals or exceeds 10.

Ex. 1. One book has 345 pages, another 567 pages, and another 719 pages; how many pages are there in the three books? Ans. 1631.

Written Process.

345
567
719
1631

Explanation.

The sum of the units is 21 or 2 tens and 1 unit. Write the 1 under the units, and reckon the 2 tens with the column of tens, which will make it 13 tens. This equals 1 hundred and 3 tens. Write 3 under the tens, and reckon the 1 hundred with the column of hundreds, which will make it 16 hundreds. This equals 1 thousand and 6 hundred. Write the 6 under the hundreds, and put the 1 thousand in the place of thousands, since there is no column with which to reckon it. These operations are called *carrying to the next column.*

Mental and Recitation Model.

16, 21. Write the 1 and carry the 2:—3, 9, 13. Write the 3 and carry the 1:—8, 13, 16. Write the whole.

Rule.—*Write the numbers so that their units may form one column, their tens another, &c.; and draw a line under them.*

First find the sum of the units. If it is less than ten, write it under the column; if it is ten or more, write its right-hand figure under the column, and add its other figure or figures to the next column.

Do this with each column successively to the last, of which write the whole sum.

Proof.—To detect error, review the work, as directed in Case I.

Examples for Practice.

(2.)	(3.)	(4.)	(5.)	(6.)	(7.)
13	20	307	987	123	5043
84	37	560	654	456	697
92	89	77	321	789	8210
78	99	808	12	12	888
267	245	1752	1974	1380	14838

Question.—28. What is the rule for addition?

(8.)	(9.)	(10.)	(11.)	(12.)
357	470	75	1020	13009
864	98	557	304	20780
139	689	755	5060	4056
505	777	575	708	1066
90	63	757	9	12005
1955	2097	2719	7101	50916

(13.)	(14.)	(15.)	(16.)
88070	10200	98765	80706
76054	11302	43210	50403
32091	12403	12345	20100
64010	13504	67890	19187
87011	10000	89670	15143
20020	30001	88888	12299
367256		400768	

(17.)	(18.)	(19.)
123456789	750461	9
12345678	2301	89
1234567	27040	789
123456	163040	6789
12345	8010	56789
1234	90082	456789
123	41	3456789
12	880880	23456789
1	3003	123456789

20. Add 54, 1008, 10008, 990, and 9990. Ans. 22050.

21. Add 245, 87, 9989, 8898, and 91. Ans. 19310.

22. What is the sum of 999 + 898 + 988 + 789 + 698 + 589 + 497 + 679 + 59 + 69 + 78 + 88? Ans. 6431.

23. Add 191 + 292 + 393 + 494 + 595 + 696 + 797 + 898 + 999 + 888 + 777 + 666 + 555 + 444.

Ans. 8685.

24. How many are 8 + 98 + 1005 + 80001 + 90805 + 19 + 142063 + 3487005 + 75075 + 63?

Ans. 3876142.

25. How many are 37063 + 5087 + 6391 + 122436 + 98117612 + 84 + 16000 + 879? Ans. 98305552.

26. Find the sum of forty-nine, nine hundred fifty-eight,

seven thousand three hundred thirty-seven, one million and one, four hundred fifty thousand fifty-four, two hundred sixty-eight, and sixty-five. Ans. 1458732.

27. B. Felton's farm is valued at eighteen thousand five hundred fifty dollars, his house at three thousand four hundred seventy-five, his live-stock at four thousand eight hundred twenty-five, and his share in a mill at ten thousand one hundred eighty-eight dollars. What is the amount of this valuation?

Ans. 37038 dollars.

28. A coal-dealer sold in one day to A 437 bushels, to B 550 bushels, to C 235 bushels, to D 88 bushels, and to F 675 bushels. How many bushels did he sell? Ans. 1985.

29. A man willed to his wife 25000 dollars, to his two sons 10375 dollars apiece, to his three daughters 7750 dollars apiece, and to a charitable institution 3469 dollars. What was the sum of the legacies? Ans. 72469 dollars.

30. Before getting into a stage, nine men weighed themselves and their baggage. Their weights were 145, 162, 133, 157, 181, 174, 149, 125, and 138 pounds. Their trunks weighed 52, 48, 37, 63, 46, 31, 35, 44, and 39 pounds. The driver weighed 146 pounds. What was the whole weight of men? of baggage? of both?

Ans. Men 1510 pounds; baggage 395 pounds; both 1905 pounds.

31. In the year 1775 the white population of the colonies was estimated as follows:—New Hampshire, 102000; Massachusetts, 352000; Rhode Island, 58000; Connecticut, 262000; New York, 238000; New Jersey, 138000; Pennsylvania, 341000; Delaware, 37000; Maryland, 174000; Virginia, 300000; North Carolina, 181000; South Carolina, 93000; Georgia, 27000. Slaves, 500000. What was the estimated population of the thirteen colonies in that year, excluding the free colored?

Ans. 2803000.

32. In the year 1850, the white population of the same States was estimated as follows:—New Hampshire, 317456; Massachusetts, 985450; Rhode Island, 143875; Connecticut, 363099; New York, 3048325; New Jersey, 465509; Pennsylvania, 2258160; Delaware, 71169; Maryland, 417943; Virginia, 894800; North Carolina, 553028; South Carolina, 274563; Georgia, 521572. Slaves, 1620636; free colored, 333935. What was their estimated population in 1850? Ans. 12269520.

33. In 1850, the estimated numbers of the free who could not read and write were as follows:—Maine, 6282; New Hampshire,

3009; Vermont, 6240; Massachusetts, 28345; Rhode Island, 3607; Connecticut, 5306; New York, 98722; New Jersey, 18665; Pennsylvania, 76272; Delaware, 10181; Maryland, 41877; Virginia, 88520; North Carolina, 80423; South Carolina, 16564; Georgia, 41667. How many free persons in these states could not read and write in 1850? Ans. 525680.

34. The same in other states numbered in 1850 as follows:—Florida, 4129; Alabama, 33992; Mississippi, 13528; Louisiana, 24610; Texas, 10583; Arkansas, 16935; Tennessee, 78619; Kentucky, 69706; Missouri, 36778; Illinois, 41283; Indiana, 72710; Ohio, 66020; Michigan, 8281; Wisconsin, 6453; Iowa, 8153; California, 5235. How many free persons in these states could not read and write in 1850? Ans. 497015.

35. The numbers of copies of newspapers and periodicals issued in 1850 were estimated as follows:—New England, 81683724; New York, New Jersey, Pennsylvania, Delaware, District of Columbia, and Maryland, 235543963; Virginia, North Carolina, South Carolina, Georgia, Florida, 22780228; Alabama, Mississippi, Louisiana, Arkansas, Texas, Tennessee, 25446143; Kentucky, Missouri, Illinois, Indiana, Ohio, Michigan, Wisconsin, Iowa, California, 60858132. What was the total number of copies issued in these states in 1850?

Ans. 426312190.

36. A republic was established at Athens in the year 510 B.C. How long was it to the Declaration of American Independence, in A.D. 1776? Ans. 2286 years.

37. Babylon is thought to have been built in the year 2100 B.C. How long was it to the battle of Waterloo, A.D. 1815?

Ans. 3915 years.

38. The length of the Danube is estimated at 1750 miles; of the Dnieper, 1260 miles; of the Don, 1120 miles; of the Rhine, 700 miles; of the Elbe, 800 miles; and of the Rhone, 650 miles. What is the aggregate length of these European rivers?

Ans. 6280 miles.

39. The length of the Amazon river is estimated at 3600 miles; of the Mississippi, 4100 miles; of the Rio de La Plata, 2240 miles; of Nelson's river, 2000 miles; of the St. Lawrence, 2100 miles; of the Orinoco, 1600 miles; and of the Rio Grande, 2150 miles. What is the aggregate length of these American rivers? Ans. 17790 miles.

Counting-Room Columns.

Art. 29. In long columns, such as often occur in business, it is advisable,

1st. To acquire the habit of adding *rapidly*, as well as accurately.

2d. To write down in a convenient place the figure to be carried, for reference in reviewing the work, and to enable us to resume adding after interruption.

3d. Sometimes to add portions only at a time, putting the sums in the margin, and then adding these sums, as in Ex. (2) and (4).

4th. To acquire the power of adding more than one column at a time.

For these purposes, practice upon examples like the following:—

(1.)	(2.)		(3.)	(4.)
1357	428		6981	5678
962	1352		7543	3214
2578	7960		8209	4769
4030	288		9765	3588
89	705		5432	275
909	3030		874	3899
5753	18		25	4790
1056	1716	15497	157	49
357	154		4396	9468
9864	1314		844	3255
2102	25		3322	8899
708	6070		7788	3889
9006	8531		1000	999
55	2663		100	8098
881	949		900	888
3111	12	19718	8066	341
2525	7		742	9872
73	401		4523	4683
6642	4333		5009	6678
8405	2806		557	3455
6061	5055		2264	209
857	9800		3113	566
2354	622		877	78
19	909	23933		
301		59148		

CHAPTER III.

SUBTRACTION.

Art. 30. SUBTRACTION is the process of finding the difference between two numbers.

Learn perfectly the following

SUBTRACTION TABLE.

1 from 1 leaves 0	2 from 2 leaves 0	3 from 3 leaves 0
1 from 2 leaves 1	2 from 3 leaves 1	3 from 4 leaves 1
1 from 3 leaves 2	2 from 4 leaves 2	3 from 5 leaves 2
1 from 4 leaves 3	2 from 5 leaves 3	3 from 6 leaves 3
1 from 5 leaves 4	2 from 6 leaves 4	3 from 7 leaves 4
1 from 6 leaves 5	2 from 7 leaves 5	3 from 8 leaves 5
1 from 7 leaves 6	2 from 8 leaves 6	3 from 9 leaves 6
1 from 8 leaves 7	2 from 9 leaves 7	3 from 10 leaves 7
1 from 9 leaves 8	2 from 10 leaves 8	3 from 11 leaves 8
1 from 10 leaves 9	2 from 11 leaves 9	3 from 12 leaves 9
4 from 4 leaves 0	5 from 5 leaves 0	6 from 6 leaves 0
4 from 5 leaves 1	5 from 6 leaves 1	6 from 7 leaves 1
4 from 6 leaves 2	5 from 7 leaves 2	6 from 8 leaves 2
4 from 7 leaves 3	5 from 8 leaves 3	6 from 9 leaves 3
4 from 8 leaves 4	5 from 9 leaves 4	6 from 10 leaves 4
4 from 9 leaves 5	5 from 10 leaves 5	6 from 11 leaves 5
4 from 10 leaves 6	5 from 11 leaves 6	6 from 12 leaves 6
4 from 11 leaves 7	5 from 12 leaves 7	6 from 13 leaves 7
4 from 12 leaves 8	5 from 13 leaves 8	6 from 14 leaves 8
4 from 13 leaves 9	5 from 14 leaves 9	6 from 15 leaves 9
7 from 7 leaves 0	8 from 8 leaves 0	9 from 9 leaves 0
7 from 8 leaves 1	8 from 9 leaves 1	9 from 10 leaves 1
7 from 9 leaves 2	8 from 10 leaves 2	9 from 11 leaves 2
7 from 10 leaves 3	8 from 11 leaves 3	9 from 12 leaves 3
7 from 11 leaves 4	8 from 12 leaves 4	9 from 13 leaves 4
7 from 12 leaves 5	8 from 13 leaves 5	9 from 14 leaves 5
7 from 13 leaves 6	8 from 14 leaves 6	9 from 15 leaves 6
7 from 14 leaves 7	8 from 15 leaves 7	9 from 16 leaves 7
7 from 15 leaves 8	8 from 16 leaves 8	9 from 17 leaves 8
7 from 16 leaves 9	8 from 17 leaves 9	9 from 18 leaves 9

Let the pupil vary and extend the following

MENTAL EXERCISES.

1. How many are 0 less 0? 1 less 0? 2 less 0? 3 less 0? &c.

QUESTION.—30. What is subtraction?

2. How many are 1 less 1? 11 less 1? 21 less 1? 31 less 1? &c.

3. How many are 2 less 2? 12 less 2? 22 less 2? 32 less 2? &c.

4. How many are 9 less 3? 19 less 3? 29 less 3? 39 less 3? &c.

5. How many are 0 less 9? 10 less 9? 20 less 9? 30 less 9? &c.

6. How many are 4 less 7? 14 less 7? 24 less 7? 34 less 7? &c.

7. How many are 9 less 7? 19 less 7? 29 less 7? 39 less 7? &c.

8. How many are 6 less 8? 16 less 8? 26 less 8? 36 less 8? &c.

9. How many are 4 less 4? 14 less 4? 24 less 4? 34 less 4? &c.

10. How many are 6 less 5? 16 less 5? 26 less 5? 36 less 5? &c.

11. How many are 4 less 5? 14 less 5? 24 less 5? 34 less 5? &c.

12. How many are 7 less 4? 17 less 4? 27 less 4? 37 less 4? &c.

13. How many are 0 less 7? 10 less 7? 20 less 7? 30 less 7? &c.

14. How many are 3 less 5? 23 less 5? 33 less 5? 43 less 5? &c.

15. How many are 5 less 3? 15 less 3? 25 less 3? 35 less 3? &c.

Art. 31. The word *subtraction* signifies the *taking from* one number that part of itself which is equal to another number. This is called simply *taking one number from another.*

The *minuend* is the number to be diminished.

The *subtrahend* is the number which is taken from the *minuend.*

The *difference*, or *remainder*, is the number which is left after subtraction.

The sign of subtraction is a short horizontal line, thus,—. Its name is *minus*, a Latin word signifying *less.* It indicates that the number following it is to be taken from the number

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Questions.—31. What does the word subtraction signify? What is the minuend? The subtrahend? The remainder? What is the sign of subtraction? What is its name? What does it indicate?

before it. The expression $9 - 4 = 5$ is read, *nine minus four equals five.*

The things signified by the numbers in subtraction must be alike. Thus 9 units — 4 units = 5 units; but 9 tens — 4 units do not equal either 5 tens or 5 units. Also 9 — 4 cannot express the difference between 9 apple-trees and 4 peach-trees, it being neither 5 apple-trees, nor 5 peach-trees.

WRITTEN EXERCISES IN SUBTRACTION.

Case I.

Art. 32. When each figure of the subtrahend does not exceed the corresponding figure of the minuend.

Ex. 1. A man paid 570 dollars for a piece of land, and sold it for 895 dollars. How much did he gain?

Ans. 325 dollars.

Written Process.	Mental Process.
895 570 325	A less number is here to be taken from a greater to find their difference. Since, in subtracting, minuend and subtrahend must signify like things, units must be taken from units, tens from tens, &c. Hence we write the subtrahend under the minuend, units under units, tens under tens, &c. Then beginning with the units, say, no units from 5 units leave 5 units; write 5 under the units; 7 tens from 9 tens leave 2 tens; write 2 under the tens; 5 hundreds from 8 hundreds leave 3 hundreds; write 3 under the hundreds. The difference is 325 dollars.

Rule.— *Write the less number under the greater, units under units, tens under tens, &c., and draw a line under them. Take each lower figure from that above it, and write the difference below.*

Ex. 2. What is 9568, less 357? Ans. 9211.

Ex. 3. What is 67059, less 5027? Ans. 62032.

Ex. 4. What is the difference between 184, and 2896?

Ans. 2712.

Ex. 5. A man was born A.D. 1745, and died A.D. 1798, on the same day of the year. How many years did he live?

Ans. 53 years.

Question.—31. What must the numbers and figures, in subtracting, signify?

Art. 33. If the same number be added to the minuend and subtrahend, the difference will still be the same.

ILLUSTRATION.

The minuend 578 + 111 = 689 a new minuend.
The subtrahend 234 + 111 = 345 a new subtrahend.

The difference 344 = 344 the new difference.

CASE II.

Art. 34. When any figure of the subtrahend is greater than the corresponding figure of the minuend.

Ex. 1. In one lot are 413 sheep, in another 265. How many more are in one than in the other? Ans. 148.

WRITTEN PROCESS.

413
265
148

EXPLANATION.

Since 5 units are more than 3 units, we cannot subtract 5 from 3. Add 10 units to 3 units, making it 13 units. Then 5 from 13 leaves 8 units. Write 8 under the units. Now, since we have added 10 units to the minuend, we add the same number, as 1 ten, to the 6 tens of the subtrahend, *so as not to alter the difference.* This makes 7 tens in the subtrahend. Now, to take 7 tens from 1 ten is impossible. Add 10 tens to 1 ten, making it 11 tens. Then 7 tens from 11 tens leave 4 tens. Write 4 under the tens. Now, since we added 10 tens to the minuend, we add the same number, as 1 hundred, to the 2 hundreds of the subtrahend, so as not to alter the difference. This makes it 3 hundreds; then 3 hundreds from 4 hundreds leave 1 hundred. Write 1 under the hundreds. The difference is 148, which must be correct, because we have added 110 to both minuend and subtrahend. The above process is sometimes called *borrowing ten and carrying one.* Some, after adding the 10, *take* 1 *from the next upper figure,* thus diminishing the upper number as much as increasing it.

MENTAL AND RECITATION MODEL.

5 from 3, impossible; 5 from 13 leaves 8. Write 8 and carry 1. 1 to 6 is 7; 7 from 1, impossible; 7 from 11 leaves 4. Write 4 and carry 1. 1 to 2 is 3; 3 from 4 leaves 1. Write 1.

RULE.— *Write the subtrahend under the minuend, units under units, tens under tens, &c., and draw a line under them.*

Then, beginning with units, subtract successively each figure from the figure over it, if possible, and write the difference below.

QUESTION.—33. How does increasing equally two numbers affect their difference?

When a figure exceeds the figure over it, add 10 *to the upper, from this sum subtract the lower, add* 1 *to the next lower figure or place, and proceed with this sum as usual.*

If the upper number has more figures than the lower, suppose ciphers on the left of the lower, and subtract to the last. If the last left-hand figure of the remainder is a cipher, omit it.

PROOF.—To detect error, add the remainder to the less number: the result should equal the greater.

EXAMPLES FOR PRACTICE.

	(2.)	(3.)	(4.)	(5.)
Minuend	469	805	7063	24002
Subtrahend	89	96	6927	9008
Difference	380	709	136	14994

	(6.)	(7.)	(8.)	(9.)
From	100101	87560	100000	750750
Take	90199	6080	80808	75075
Ans.	9902	81480	19192	675675

	(10.)	(11.)
From	101202303404505	100000200000300
Take	90208030704065	99999870304256
Ans.		

	(12.)	(13.)	(14.)
From	9876543210	8888888888	11111111100
Take	1234567890	7945878889	1234567890
Ans			

15. From 805326089004 take 796047188505.
Ans. 9278900499.

16. From 202404606 take 50570709.
Ans. 151833897.

17. Take 16171819 from 1213141516.
Ans. 1196969697.

18. Take 4990994 from 10200201. Ans. 5209207.

QUESTIONS.—34. How are numbers written for subtraction? Why? Where do we begin to subtract? What is the process when the lower figure exceeds the figure over it? Why? How do we detect error?

19. How many are 98877665 — 59933881?

Ans. 38943784.

20. How many are 120110100 — 119202122?

Ans. 907978.

21. From 1111111111 take 987654321.

Ans. 123456790.

22. What is the difference between one thousand nineteen, and nineteen thousand? Ans. 17981.

23. Find the difference between one hundred thousand, and nine thousand nine hundred ninety-nine. Ans. 90001.

24. Find the difference between two thousand five hundred seventy-five, and twenty-five thousand twenty-five.

Ans. 22450.

25. From nine millions eight thousand and seven, take ninety-nine thousand and eighty-eight. Ans. 8908919.

26. From a cask of molasses containing 83 gallons, 45 gallons were drawn, and afterwards 37 gallons were put in. How many gallons were there finally in the cask? Ans. 75 gallons.

27. What number added to 6789 will make 10000?

Ans. 3211.

28. Starting on a journey with 650 dollars, I returned with 388 dollars. How many dollars did I spend? Ans. 262.

29. From my farm of 473 acres I sold 95 acres. How many acres had I left? Ans. 378.

30. From a piece of cloth containing 63 yards, were sold at different times 5 yards, 13 yards, 9 yards, and 6 yards. How many yards remained? Ans. 30.

31. From 17256 take the sum of 9821 and 2587.

Ans. 4848.

Short Written Process.	Mental Process.
17256 9821 2587 4848	7 and 1 are 8, which from 16 leaves 8. Carrying 1 to 8 is 9 and 2 is 11, of which the right figure 1 from 5 leaves 4. Carrying the left 1 to 5 is 6 and 8 is 14, of which the right figure 4 from 12 leaves 8. Carrying the left 1, *also the borrowed* 1, to 2 is 4, and 9 is 13, of which the 3 from 7 leaves 4. The left 1 from 1 leaves 0.

32. Samuel Reed owns property valued at 28150 dollars; he owes J. Brown 6245 dollars, William Abel 462 dollars, and H. Clark 1328 dollars. What is Reed worth?

Ans. 20115 dollars.

33. A goes 275 miles, and B 168 in the same direction. How

far are they apart, and how *much farther* would they have been apart if they had gone in opposite directions?

Ans. 1st 107 miles: 2d 336 miles.

34. From 7543 — 2658 take 9125 — 5989.

Ans. 1749.

35. From 9784 — 3995 take 2147 + 908.

Ans. 2734.

36. An army of 15750 before battle, only numbered 12863 after it. What was the loss? Ans. 2887.

37. A farmer, having gathered 1033 bushels of potatoes, threw out the diseased ones, and then found that the good measured 758 bushels. How many were diseased?

Ans. 275 bushels.

38. A man willed an estate of 55200 dollars to his nephew; but other persons keeping it in a long law-suit, he only got 27425 dollars. What was the cost of defending his case?

Ans. 27775 dollars.

39. America was discovered by Columbus in A.D. 1492. How long was it to the Declaration of Independence in 1776?

Ans. 284 years.

40. Napoleon Bonaparte died in the year 1821, at the age of 52. When was he born? Ans. 1769.

41. St. Petersburg was founded by Peter the Great in 1703. How long was it to the burning of Moscow in 1812?

Ans. 109 years.

42. Of two candidates who together received 18923 votes, the successful one received 9984 votes. How many did the other receive, and what was the majority of the first?

Ans. 1st 8939; 2d 1045.

43. In 1800 the population of the United States was 5305925. In 1850 it was 23191876. What was the increase in that time?

Ans. 17885951.

44. If a certain star is distant from the earth sixty-two trillions four hundred eighty-one billions five hundred millions of miles, and the sun is distant ninety-five millions one hundred seventy-three thousand one hundred twenty-seven miles, how much farther from us is the star than the sun?

Ans. 62481404826873 miles.

45. From 831 octillions, 20 septillions, 405 quintillions, 60 quadrillions, 700 millions, 8 thousand, and 9, take 8 octillions, 600 sextillions, 504 quintillions, 80 billions, 621 millions, 8 thousand, and 301

Ans 823019399901059999920078999708

CHAPTER IV.

MULTIPLICATION.

Art. 35. MULTIPLICATION is a short process of producing the sum of one number used in addition as many times as there are units in another.

ILLUSTRATION.—To obtain the sum of 8 taken 9 times, we need not write 8 nine times and add the column. We can remember that 9 times 8 are 72: and if we commit to memory such facts in regard to small numbers, we can use them in operating with larger numbers. These facts arranged make a

MULTIPLICATION TABLE.

Once 0 is 0	2 times 0 are 0	3 times 0 are 0	4 times 0 are 0
Once 1 is 1	2 times 1 are 2	3 times 1 are 3	4 times 1 are 4
Once 2 is 2	2 times 2 are 4	3 times 2 are 6	4 times 2 are 8
Once 3 is 3	2 times 3 are 6	3 times 3 are 9	4 times 3 are 12
Once 4 is 4	2 times 4 are 8	3 times 4 are 12	4 times 4 are 16
Once 5 is 5	2 times 5 are 10	3 times 5 are 15	4 times 5 are 20
Once 6 is 6	2 times 6 are 12	3 times 6 are 18	4 times 6 are 24
Once 7 is 7	2 times 7 are 14	3 times 7 are 21	4 times 7 are 28
Once 8 is 8	2 times 8 are 16	3 times 8 are 24	4 times 8 are 32
Once 9 is 9	2 times 9 are 18	3 times 9 are 27	4 times 9 are 36
Once 10 is 10	2 times 10 are 20	3 times 10 are 30	4 times 10 are 40
Once 11 is 11	2 times 11 are 22	3 times 11 are 33	4 times 11 are 44
Once 12 is 12	2 times 12 are 24	3 times 12 are 36	4 times 12 are 48

5 times 0 are 0	6 times 0 are 0	7 times 0 are 0	8 times 0 are 0
5 times 1 are 5	6 times 1 are 6	7 times 1 are 7	8 times 1 are 8
5 times 2 are 10	6 times 2 are 12	7 times 2 are 14	8 times 2 are 16
5 times 3 are 15	6 times 3 are 18	7 times 3 are 21	8 times 3 are 24
5 times 4 are 20	6 times 4 are 24	7 times 4 are 28	8 times 4 are 32
5 times 5 are 25	6 times 5 are 30	7 times 5 are 35	8 times 5 are 40
5 times 6 are 30	6 times 6 are 36	7 times 6 are 42	8 times 6 are 48
5 times 7 are 35	6 times 7 are 42	7 times 7 are 49	8 times 7 are 56
5 times 8 are 40	6 times 8 are 48	7 times 8 are 56	8 times 8 are 64
5 times 9 are 45	6 times 9 are 54	7 times 9 are 63	8 times 9 are 72
5 times 10 are 50	6 times 10 are 60	7 times 10 are 70	8 times 10 are 80
5 times 11 are 55	6 times 11 are 66	7 times 11 are 77	8 times 11 are 88
5 times 12 are 60	6 times 12 are 72	7 times 12 are 84	8 times 12 are 96

9 times 0 are 0	10 times 0 are 0	11 times 0 are 0	12 times 0 are 0
9 times 1 are 9	10 times 1 are 10	11 times 1 are 11	12 times 1 are 12
9 times 2 are 18	10 times 2 are 20	11 times 2 are 22	12 times 2 are 24
9 times 3 are 27	10 times 3 are 30	11 times 3 are 33	12 times 3 are 36
9 times 4 are 36	10 times 4 are 40	11 times 4 are 44	12 times 4 are 48
9 times 5 are 45	10 times 5 are 50	11 times 5 are 55	12 times 5 are 60
9 times 6 are 54	10 times 6 are 60	11 times 6 are 66	12 times 6 are 72
9 times 7 are 63	10 times 7 are 70	11 times 7 are 77	12 times 7 are 84
9 times 8 are 72	10 times 8 are 80	11 times 8 are 88	12 times 8 are 96
9 times 9 are 81	10 times 9 are 90	11 times 9 are 99	12 times 9 are 108
9 times 10 are 90	10 times 10 are 100	11 times 10 are 110	12 times 10 are 120
9 times 11 are 99	10 times 11 are 110	11 times 11 are 121	12 times 11 are 132
9 times 12 are 108	10 times 12 are 120	11 times 12 are 132	12 times 12 are 144

Let the pupil vary and extend the following

Mental Exercises.

1. How many are 3 times 4? 4 times 3? 2 times 8? 8 times 2?

2. How many are 7 times 9? 9 times 7? 11 times 6? 6 times 11?

3. James wrote 5 nine times and added them, and Mary wrote 9 five times and added them. How much did their answers differ?

4. Harry and Jane counted the trees in their orchard: one said there were four rows of 12 trees in a row, the other said there were twelve rows of 4 trees in a row. Could both have been right?

5. What is the difference of value between *nothing* expressed three times and 3 expressed three times?

6. What is the difference of value between 0 expressed eight times and 0 expressed four times?

7. What is the difference of value between 0 expressed 7 times and 0 expressed 9 times?

8. What cost 6 oranges at 3 cents apiece?

Ans. As many *times* 3 cents as there are oranges, that is, 6 times 3 cents, or 18 cents.

9. What cost 10 melons at 12 cents apiece?

10. What is the difference *of cost* between 11 lemons at 2 cents apiece, and 2 pineapples at 11 cents apiece?

11. Sarah bought 12 spools of thread at 4 cents apiece. Not knowing the Multiplication Table, she added to find the cost. Should she have taken 12 four times, or 4 twelve times?

12. One man traveled 9 days 12 miles a day, and another 12 days 9 miles a day? Which made the longest journey?

13. How much more is 12 times 12 than 6 times 9?

14. A boy bought 8 peaches at 2 cents apiece, and offered a twenty-five cent piece. How much change should he have received?

15. A man bought 10 pounds of sugar at 8 cents a pound, and 4 quarts of molasses at 9 cents a quart. How much did both cost?

16. A boy bought 10 pounds of flour at 5 cents a pound, but spilled 3 pounds on his way home. How many cents' worth were left?

Questions.—35. What is multiplication? Show the use of a table.

Art. 36. To multiply one number by another is to seek the sum of the first taken as many times as there are units in the second.

The *multiplicand* is the number of whose repeated expression we seek the sum; or, simply, the number to be multiplied.

The *multiplier* shows the number of expressions whose sum we seek; or, simply, it is the number by which we multiply.

The *product* is the result of multiplication.

The *multiplicand* and *multiplier* are called the *factors* of the product, because they *make* or produce it.

The *sign* of multiplication is an oblique cross; thus, $\times$. The expression $3 \times 8 = 24$ may be briefly read thus. 3 *into* 8 equals 24. A parenthesis ( ), inclosing a numerical expression, signifies that the value of that expression is taken as a single number. Thus $(6 + 8 - 3) \times 5$ signifies that the value of $6 + 8 - 3$, that is 11, is to be multiplied by 5. A *vinculum*, drawn above the expression thus, $\overline{6 + 8 - 3} \times 5$, signifies the same thing.

The product and multiplicand signify the same things; for the *nature* of a thing is not changed by *repeating its quantity.*

The multiplier is an *abstract* number *in the process*, since it then merely signifies the required number of expressions of the multiplicand. It is often a *concrete* number *in the statement of the question.*

Illustration.—If 1 acre of land costs 100 dollars, 7 *acres* will cost 7 *times* 100 *dollars;* that is, 700 *dollars.* Here the product is dollars like the multiplicand, and the multiplier is an abstract number, though concrete as given.

WRITTEN EXERCISES IN MULTIPLICATION.

Case I.

Art. 37. When the multiplier consists of one figure; or of a number small enough to be used all at once.

Ex. 1. Multiply 365 by 8. Ans. 2920.

Written Process.	Mental Process.
365 8 2920	8 times 5 units = 40 units = 4 tens 0 units. Write 0 under the units, and keep in mind the 4 tens for carrying. Then, 8 times 6 tens = 48 tens; add in the 4 tens *carried*, making 52 tens = 5 hundreds 2 tens. Write the 2 tens and

Questions.—36. What is the multiplicand? What is the multiplier? What is the product? What are multiplier and multiplicand called? Why? What is the sign of multiplication? What does a parenthesis or vinculum signify? What do multiplicand and poduct signify? Why? What kind of number is the multiplier?

PROCESS ILLUSTRATED.

365
8

40
480
2400

2920

carry the 5 hundreds. Then 8 times 3 hundreds = 24 hundreds: add in the 5 hundreds *carried*, making 29 hundreds = 2 thousands 9 hundreds. Write the whole.

MENTAL AND RECITATION MODEL.

8 times 5 are 40. Write 0 and carry 4. 8 times 6 are 48 and 4 are 52. Write 2 and carry 5. 8 times 3 are 24 and 5 are 29. Write the whole.

RULE.—*Beginning with the units, multiply in order each figure of the multiplicand by the multiplier, writing the right hand figure of each product, and carrying the left as in addition.*

EXAMPLES FOR PRACTICE.

	(2.)	(3.)	(4.)	(5.)
Multiplicand.	9847	2637	5841	40609
Multiplier.	2	9	7	3
Product.	19694	23733	40887	121827

	(6.)	(7.)	(8.)
Multiply	1020300	7508604	30940857
By	6	5	4
Ans.	6121800	37543020	123763428

9. Multiply 30450670089 by 9. Ans. 274056030801.
10. Multiply 21703489650 by 8. Ans. 173627917200.
11. Multiply 47360590182 by 7. Ans. 331524131274.
12. Multiply 69285031704 by 6. Ans. 415710190224.
13. Multiply 50910827346 by 5. Ans. 254554136730.
14. Multiply 78054093621 by 4. Ans. 312216374484.
15. Multiply 82910730645 by 3. Ans. 248732191935.
16. Multiply 95108207436 by 2. Ans. 190216414872.
17. A car traveled 579 miles in one day: at that rate how far would it travel in 9 days? Ans. 5211 miles.
18. If one common soldier receives 8 dollars a month, how many dollars a month would pay 15674 soldiers?

Ans. 125392.

QUESTION.—37. What is the rule when the multiplier does not exceed 12?

Case II.

Art. 38. When the multiplier consists of more than one figure or of a number too large to be used all at once.

Ex. 1. Multiply 946 by 24. Ans. 22704.

Written Process.

946
24
3784
1892
22704

Mental Process.

4 times 946 + 20 times 946 = 24 times 946. First multiply by 4, obtaining 3784; next multiply by 2 tens. Since 2 tens × 6 is the same as 6 times 2 tens = 12 tens = 1 hundred 2 tens, write 2 under the tens, and carry the 1 hundred to the next product, &c. Finally draw a line under the products; and add them.

Process Illustrated.

$$946 \times 4 = 3784$$
$$946 \times 20 = 18920$$
$$946 \times (4 + 20) = 22704$$

Rule.—*Write the multiplier under the multiplicand, units under units, tens under tens, &c.; and draw a line under them.*

Beginning with the units, multiply in order each figure of the multiplicand by each significant figure of the multiplier, writing the right-hand figure of each product, and carrying the left, as in addition.

Write the products obtained by the several figures of the multiplier, in separate lines, so that the right-hand figure of each shall be under that figure of the multiplier which produced it.

Add the several products, to obtain the whole product.

Proof.—First Method. To detect error, carefully repeat the multiplication, or multiply the multiplier by the multiplicand. If the results do not agree, inspect the process cautiously again.

Second Method. Add together the figures of the multiplicand, dropping from the reckoning every *nine* as fast as formed. Put the last excess over *nine* at the right of the multiplicand. Do the same with the multiplier and with the product. Multi-

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Questions.—38. How do we write the multiplier? How do we multiply? How do we write the products? How do we obtain the whole product? What is the first method of proof?

ply the excess of the multiplicand by that of the multiplier, and reject the nines from this product. The excess of this product should equal that of the general product.

Example (2.)

654	6 excess		
349	7		
5886	42	6 excess	
2616			
1962			
228246		6 excess	

EXPLANATION.

$6 + 5 = 11$. Drop 9. 2 remain. $2 + 4 = 6$ = excess of multiplicand.

$3 + 4 = 7$. Drop the next nine and 7 = excess of multiplier.

$7 \times 6 = 42$. $4 + 2 = 6$ = excess.

Now, in the product $2 + 2 + 8 = 12$. Drop nine. 3 remain. $3 + 2 + 4 = 9$. Drop it and 6 remain.

REMARK.—None of these methods *prove* the work to be right, though their application will very probably reveal error, if it exists.

The casting out of *nines* as above would fail to detect such wrong figures in the product as would give the same excess as the right ones. Such a combination of wrong figures is improbable.

EXAMPLES FOR PRACTICE.

	(3.)	(4.)
Multiply	35796	1246853
By	408	9007
	286368	8727971
	143184	11221677
Ans.	14604768	11230404971

	(5.)	(6.)	(7.)
Multiply	12013	12013	5964
By	1204	1204	1111
	48052	48052	65604
	24026	144156	65604
	12013		
Ans.	14463652	14463652	6626004

QUESTIONS.—38. What is the second method of proof?

8. A railroad 259 miles long, cost 34618 dollars per mile; what was the whole cost? Ans. 8966062 dollars.

9. Multiply 151413 by 123. Ans. 18623799.

10. At 63 gallons per hogshead, how many gallons would 729 hogsheads hold? Ans. 45927 gallons.

11. Multiply 708099605 by 4012.

Ans. 2840895615260.

12. At 684 bricks per load, how many bricks could be drawn n 58 loads? Ans. 39672.

13. How many are $(3412 \times 85) - (2508 \times 63)$?

Ans. 132016.

14. How many are $(5987 \times 16) + (5987 \times 61)$?

Ans. 460999.

15. How many are $(6009 \times 69) + (9006 \times 96)$?

Ans. 1279197.

16. If a person saves 365 dollars annually, how many dollars would he save in 25 years? Ans. 9125.

17. If a person spends 475 dollars annually, how many dollars would he spend in 36 years? Ans. 17100.

18. Multiply twenty millions fifty-five thousand eighty-four by ninety-two thousand sixty-three.

Ans. 1846331198292.

19. Multiply eighty thousand seven hundred six by eight thousand sixty-seven. Ans. 651055302.

20. How many pounds of iron in the rails of a railroad 32 miles long, 1760 yards making a mile, and 65 pounds making a yard? (N.B. The track has 2 rails.) Ans. 7321600.

21. If one soldier consume 4 pounds of food in one day, how many pounds will 145698 soldiers consume in the same time?

Ans. 582792.

22. How many pounds would they consume in 365 days?

$582792 \times 365 =$ Ans. 212719080.

23. At the rate of 197916 miles in a second, how far would light travel in 509 seconds? Ans. 100739244 miles.

24. What will a farm of 456 acres cost at 46 dollars per acre?

Ans. 20976 dollars.

25. How many hours has a man 75 years old lived, 8766 ours making a year? Ans. 657450.

26. How many meals has he eaten, if he has eaten 1095 meals each year? Ans. 82125.

27. How many pounds of food and drink has he consumed, if he has taken 1438 pounds each year? Ans. 107850.

28. If each house consumed 428 bushels of coal annually

what would be the consumption in a town containing 15769 houses? Ans. 6749132 bushels.

29. In a cornfield there were 89 rows of hills, and 267 hills n a row. What was the whole number? Ans. 23763.

30. If each acre produces 46 bushels of wheat, how many oushels do 358 acres produce? Ans. 16468.

31. Multiply 10200809 by 705006. Ans. 7191631549854.

32. Multiply 368427 by 10905. Ans. 4017696435.

33. If each person breathed 2357 gallons of air in one day, how many gallons would 989879564 persons breathe in the same time? Ans. 2333146132348.

34. Multiply 987654321 by 123456789.
Ans. 121932631112635269.

Art. 39. To multiply by 1 with a cipher or ciphers annexed.

Ex. 1. Multiply 58 by 10: by 100. Ans. 580: 5800.

Explanation. In the Decimal Notation, each removal from units one place to the left, causes a figure to express ten times more than before. (See Art. 14.) Therefore annexing one cipher to a number multiplies that number by 10, since it removes the number one place to the left; annexing two ciphers multiplies by one hundred; &c.

Rule.—*Annex to the multiplicand all the ciphers of the multiplier.*

Ex. 2. Multiply 5 by 10. Ans. 50.
Ex. 3. Multiply 69 by 100. Ans. 6900.
Ex. 4. Multiply 187 by 1000. Ans. 187000.
Ex. 5. Multiply 4236 by 10000. Ans. 42360000.

Ex. 6. If 100 men subsisted 21 days on a certain quantity of food, how long could 1 man subsist on the same?
Ans. 2100 days.

Art. 40. A *composite number* is a number produced by factors greater than unity. Thus, 32 is a composite number, since it can be produced by 4×8, or 2×16, or $2 \times 2 \times 2 \times 2 \times 2$.

The product is the same in every order of using the factors. Thus, $4 \times 8 = 8 \times 4 = 32$.

Questions.—39. How do we multiply by 10, 100, 1000, &c.? Why?—40. What is a composite number? Does a change of the order of the factors change the product?

Art. 41. To multiply by a composite number.

CASE I.

When there is no cipher on the right of the multiplier.

Ex. 1. What cost 108 tons of iron, at 45 dollars a ton?

Ans. 4860 dollars.

WRITTEN PROCESS.

108
5

540
9

4860

MENTAL PROCESS.

45 is 9 times 5. At 5 dollars a ton, 108 tons would cost 108 times 5 dollars, or 540 dollars. At 45 dollars a ton, they would cost 9 times 540 dollars, or 4860 dollars.

RULE.—*Find a convenient set of factors of the multiplier. Multiply the multiplicand by one of these factors, this product by another, and so on till each factor of the set has been used. The last product is the one required.*

REMARK.—This method does not always save labor, especially in large multipliers and with large factors.

EXAMPLES FOR PRACTICE.

2. Multiply 8457 by $63 = 7 \times 9$. Ans. 532791.
3. Multiply 9065 by $132 = 11 \times 12$. Ans. 1196580.
4. What cost 1328 head of cattle at 45 dollars per head?

Ans. 59760 dollars.

5. What cost 567 horses at 108 dollars apiece?

Ans. 61236 dollars.

6. Does multiplying 98597 by 1001, using the factors 13, 11, and 7, take less labor than the common method?

CASE II.

When there are ciphers on the right of either number or both.

Ex. 1. What cost 900 wagons at 80 dollars apiece?

Ans. 72000 dollars.

WRITTEN PROCESS.

900
80

72000

MENTAL PROCESS.

This is a case of using factors. Since $900 = 9 \times 100$, and $80 = 8 \times 10$, we can find first the product of the significant figures 8 and 9, and then multiply this by annexing the ciphers. (See Art. 39.)

RULE.—*Multiply as if there were no right-hand ciphers, and annex to this product as many ciphers as are on the right hand of both factors.*

EXAMPLES FOR PRACTICE.

	(2.)	(3.)
Multiply	438906	758900000
By	2400	36
	1755624	45534
	877812	22767
Answer.	1053374400	27320400000

	(4.)	(5.)
	5360000	95200
	7200	580000
	1072	7616
	3752	4760
	38592000000	55216000000

6. Multiply 129000 by 830. Ans. 107070000.
7. Multiply 70600 by 45000. Ans. 3177000000.
8. Find the product of 3470 by 90000. Ans. 312300000.
9. Multiply two hundred and eight millions by forty-three thousand. Ans. 8944000000000.
10. Multiply nine millions eight hundred by seven hundred sixty thousand. Ans. 6840608000000.
11. Multiply 5360000 by 84. Ans. 450240000.
12. Multiply 98000000 by 53. Ans. 5194000000.
13. Multiply 792 by 14000000. Ans. 11088000000.
14. Multiply 460 by 90000. Ans. 41400000.
15. If nine hundred millions of people make one generation on the earth, how many people would make ninety-eight generations? Ans. 88200000000.
16. If light moves 5878410000000 miles in 1 year, how far would it move in 6000 years? Ans. 35270460000000000 miles.
17. If a clock tick 31557600 times in 1 year, how many times would it tick in 40 years? Ans. 1262304000.

CHAPTER V.

DIVISION.

Art. 42. DIVISION is the process by which we find how many times one number contains another.

ILLUSTRATION.—1st. How many boys must pay 7 cents each, to pay the price of a melon, which is 28 cents?

ANSWER.—As many boys as 7 cents are contained times in 28 cents: that is, 4 boys. Here 28 cents are divided into *parts* of 7 *cents each*, and we seek the *number of such parts*.

2d. Four boys shared equally the cost of a melon, which was 28 cents; how many cents did each pay?

ANSWER.—As many cents as 4, the *number* of boys, is contained times in 28, the *number* of cents: that is, 7 cents. Here 28 cents are divided into 4 equal parts, and we seek *the number of cents in each part*.

3d. When the four boys shared the melon equally, how much had each boy?

ANSWER.—The melon was divided into as many equal parts as there were boys, viz. 4 parts, *called fourths:* and each boy had as many such parts as the *number* of boys is contained times in the *number* of parts: that is, 1 part, or *one-fourth* (written $\frac{1}{4}$) of the melon. Here the expression $\frac{1}{4}$ *indicates* the division of 1 into 4 equal parts, and *expresses* the value of one of those parts.

DIVISION is a short process of finding the number of times one number can be subtracted from another

ILLUSTRATION.—To find the number of times 8 can be subtracted from 72, we need not write 72 and perform repeated subtractions. We can remember that 8 is contained in 72 *nine* times; and if we commit to memory such facts in regard to small numbers, we can use them in operating with large numbers. Hence, to facilitate division, earn the following

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QUESTIONS.—42. What is division? For what is it a short process?

DIVISION TABLE.

1 in 0 no time	2 in less no time	3 in less no time
1 in 1 1 time	2 in 2 1 time	3 in 3 1 time
1 in 2 2 times	2 in 4 2 times	3 in 6 2 times
1 in 3 3 times	2 in 6 3 times	3 in 9 3 times
1 in 4 4 times	2 in 8 4 times	3 in 12 4 times
1 in 5 5 times	2 in 10 5 times	3 in 15 5 times
1 in 6 6 times	2 in 12 6 times	3 in 18 6 times
1 in 7 7 times	2 in 14 7 times	3 in 21 7 times
1 in 8 8 times	2 in 16 8 times	3 in 24 8 times
1 in 9 9 times	2 in 18 9 times	3 in 27 9 times
4 in less no time	**5 in less no time**	**6 in less no time**
4 in 4 1 time	5 in 5 1 time	6 in 6 1 time
4 in 8 2 times	5 in 10 2 times	6 in 12 2 times
4 in 12 3 times	5 in 15 3 times	6 in 18 3 times
4 in 16 4 times	5 in 20 4 times	6 in 24 4 times
4 in 20 5 times	5 in 25 5 times	6 in 30 5 times
4 in 24 6 times	5 in 30 6 times	6 in 36 6 times
4 in 28 7 times	5 in 35 7 times	6 in 42 7 times
4 in 32 8 times	5 in 40 8 times	6 in 48 8 times
4 in 36 9 times	5 in 45 9 times	6 in 54 9 times
7 in less no time	**8 in less no time**	**9 in less no time**
7 in 7 1 time	8 in 8 1 time	9 in 9 1 time
7 in 14 2 times	8 in 16 2 times	9 in 18 2 times
7 in 21 3 times	8 in 24 3 times	9 in 27 3 times
7 in 28 4 times	8 in 32 4 times	9 in 36 4 times
7 in 35 5 times	8 in 40 5 times	9 in 45 5 times
7 in 42 6 times	8 in 48 6 times	9 in 54 6 times
7 in 49 7 times	8 in 56 7 times	9 in 63 7 times
7 in 56 8 times	8 in 64 8 times	9 in 72 8 times
7 in 63 9 times	8 in 72 9 times	9 in 81 9 times
10 in less no time	**11 in less no time**	**12 in less no time**
10 in 10 1 time	11 in 11 1 time	12 in 12 1 time
10 in 20 2 times	11 in 22 2 times	12 in 24 2 times
10 in 30 3 times	11 in 33 3 times	12 in 36 3 times
10 in 40 4 times	11 in 44 4 times	12 in 48 4 times
10 in 50 5 times	11 in 55 5 times	12 in 60 5 times
10 in 60 6 times	11 in 66 6 times	12 in 72 6 times
10 in 70 7 times	11 in 77 7 times	12 in 84 7 times
10 in 80 8 times	11 in 88 8 times	12 in 96 8 times
10 in 90 9 times	11 in 99 9 times	12 in 108 9 times

Let the pupil vary and extend the following

Mental Exercises.

1. How many times 2 in 0? In 4? In 8? In 12? &c.

2. How many times 2 in 1? Ans. No (0) times and 1 remains. In 3? In 5? In 7? In 9? In 11? In 13? &c.

3. How many times 3 in 0? In 3? In 6? In 9? &c.

4. How many times 3 in 2? Ans. 0 times and 2 remain. In 4? In 5? In 7? In 8? In 10? In 11? In 13? &c.

5. How many times 4 in 0? In 4? In 8? In 12? &c.

6. How many times 4 in 3? Ans. 0 times and 3 remain In 2? In 1? In 5? In 7? In 6? In 9? In 10? &c.

7. How many times 5 in 0? In 5? In 10? In 15? &c

8. How many times 5 in 4? Ans. 0 times and 4 remain. In 3? In 2? In 1? In 6? In 7? In 8? In 9? &c.

9. How many times 6 in 0? In 12? In 18? In 24? &c.

10. How many times 6 in 5? Ans. 0 times and 5 remain. In 4? In 3? In 2? In 1? In 7? In 8? In 9? &c.

11. How many times 7 in 0? In 7? In 14? In 21? &c.

12. How many times 7 in 6? Ans. 0 times and 6 remain. In 5? In 4? In 3? In 2? In 1? In 8? In 9? &c.

13. How many times 8 in 0? In 8? In 16? In 24? &c.

14. How many times 8 in 7? Ans. 0 times and 7 remain. In 6? In 5? In 4? In 3? In 2? In 1? In 9? &c.

15. How many times 9 in 0? In 9? In 18? In 27? &c.

16. How many times 9 in 8? Ans. 0 times and 8 remain. In 7? In 6? In 5? In 4? In 3? In 2? In 1? &c.

17. How many times 5 in 5? In 50? In 500? In 5000? &c.

18. How many times 9 in 9? In 90? In 900? In 9000? &c.

19. How many times can you subtract 4 from 16?

20. How many times can you subtract 12 from 89, and how many would remain?

21. If a boy has 37 cents in his pocket, and pays out 5 cents 7 times, how many has he left?

Art. 43. To divide one number *by* another is to seek the *number of times* the latter must be taken in addition to equal the former. Thus, to divide 27 *by* 3 is to seek *how many times* 3 must be taken to equal 27.

Comparing division with multiplication, we see that

In *multiplication*, the *times* are *given*, the *sum is required*;

In *division*, the *sum* is given, the *times* are required.

The *dividend* is that one of the two numbers employed in dividing which contains the other number; or, simply, the dividend is the number to be divided.

The *divisor* is the number of whose being contained in the dividend we seek the number of times; or, simply, the divisor is the number *by* which we divide.

The *quotient* is the result of the process, and signifies the number of times the divisor is contained in the dividend.

The *remainder* is the excess, whenever the divisor is not con-

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Questions.—43. What is it to divide one number by another? Compare dividing with multiplying. What is the dividend? What is the divisor? What is the quotient? What is the remainder?

tained an exact number of times in the dividend. It is of the same kind as the dividend, being a *part* of it.

In the *process*, the dividend and divisor must be *alike*, and the quotient is an *abstract* number; but in the *statement of the question*, these distinctions are not observed. Thus, it is absurd to say, "4 boys are contained in 28 cents, 7 cents times."

The signs of division are,

1st. A horizontal line having a dot above and below it, thus, $\div$.

2d. A horizontal line having the dividend above and the divisor below it.

3d. An upright line, having the divisor on its left, and the dividend on its right.

Thus, 12 divided by 4 is indicated by $12 \div 4$. or $\frac{12}{4}$, or 4)12 .

The *object* of dividing may be either

To divide the dividend into *given equal parts*, to find their *number;* or, to divide the dividend into a *given number* of equal parts, to find their *value;* that is, either knowing their value to find their number, or, knowing their number, to find their value.

Art. 44. The equal parts that make a quantity have different names.

A *half* of a quantity is one of *two* equal parts that make that quantity.

A *third* of a quantity is one of *three* equal parts that make that quantity.

A *fourth* of a quantity is one of *four* equal parts that make that quantity.

A *fifth* of a quantity is one of *five* equal parts that make that quantity, &c. &c.

Hence a *whole* quantity is composed of *two halves*, *three thirds*, *four fourths*, *five fifths*, *six sixths*, &c., of that quantity.

Writing the dividend over the divisor indicates both the division of the dividend into as many equal parts as there are units in the divisor, and the *value* of one of those parts.

Thus $\frac{12}{4}$ signifies 12 divided into 4 equal parts, and also *one fourth of twelve*, or *twelve fourths* of *one;* or *three.*

Questions.—43. What kind of number must the remainder be? Why? What kind of numbers must dividend, divisor, and quotient be? Why? What the signs of division? What may the object in dividing be?—44. What names have the equal parts of a quantity? Of how many such equal parts is a quantity composed? What is signified by writing the dividend over be divisor?

The number is read *twelve fourths*, or *twelve over four*.

Such a number is called a *fraction*, because it signifies the *breaking* of a number into *equal parts*, and expresses the value of one of those parts.

In reference to the *wholeness* of what is signified, numbers are of three classes, viz., *integral*, *fractional*, and *mixed*.

An *integer* is a number expressing one or more units: as, 1, 8, 27, &c. Integers are also called *whole numbers*.

A *fraction* is a number expressing one or more *parts* of a unit as, $\frac{1}{8}$, $\frac{3}{5}$, $\frac{6}{7}$, &c.

A *mixed number* is a number composed of an integer and a fraction: as, $3\frac{1}{2}$, which is read *three and one half*.

Let the pupil extend the following

Mental Exercises.

1. What is 1 *half* of 4? 6? 8? 10? 12? 14? 16? &c.
2. What is 1 *third* of 1? 3? 6? 9? 12? 15? 18? &c.
3. What is 1 *fourth* of 1? 2? 3? 4? 8? 12? 16? &c.
4. What is 1 *fifth* of 1? 5? 10? 15? 20? 25? 30? &c.
5. What is 1 *sixth* of 5? 6? 12? 18? 24? 30? 36? &c.
6. What is 1 *seventh* of 1? 2? 3? 7? 14? 21? 28? &c.
7. What is 1 *eighth* of 1? 7? 8? 16? 24? 32? 40? &c.
8. What is 1 *ninth* of 1? 2? 4? 5? 9? 18? 27? &c.
9. What is 1 *tenth* of 3? 7? 9? 10? 20? 30? 40? &c.
10. What is 1 *eleventh* of 8? 11? 22? 33? 44? 55? &c.
11. What is 1 *twelfth* of 5? 7? 11? 12? 24? 36? &c.
12. How many *halves* in 1 thing? 2 like things? 3 like things? &c.
13. How many *thirds* in 1 thing? 2 like things? 3 like things? &c. &c. &c. &c.
14. Henry gave his brother $\frac{7}{12}$ of his orange: how much had he left?

Mental Process.—Since 12 twelfths of an orange are the *whole* of it, if 7 twelfths be given away, there will remain $12 - 7 = 5$ twelfths of the orange.

15. If a person contribute $\frac{1}{10}$ of his income to benevolent purposes, how much does he retain?
16. If one-seventh of our days is devoted to rest, how many are devoted to labor?

Questions.—44. Why is such a number called a fraction? How are numbers classed as to the *wholeness* of the thing signified? What is an integer? A fraction? A mixed number?

17. If a person spends $\frac{1}{8}$ of his income for rent, $\frac{1}{8}$ for clothing, and $\frac{4}{8}$ for other purposes, how much does he save?

MENTAL PROCESS.—$\frac{1}{8} + \frac{1}{8} + \frac{4}{8} = \frac{6}{8}$, all that is spent. Then the part not spent is $\frac{8}{8} - \frac{6}{8} = \frac{2}{8}$ of his income.

18. If $\frac{2}{7}$ of a crop of potatoes rot in the ground, and $\frac{3}{7}$ rot after gathering, how many escape decay?

19. If $\frac{3}{11}$ of an army are killed in battle, and $\frac{1}{11}$ desert, and $\frac{3}{11}$ are sick in the hospital, how many remain in service?

20. How many are $\frac{3}{4}$ of 20?

MENTAL PROCESS.—One-fourth of 20 is 5; and *three*-fourths must be *three times five*, or 15.

21. How many are $\frac{7}{8}$ of 32? of 48? of 64? of 88?

22. If a bushel of wheat cost 84 cents, what cost $\frac{5}{12}$ of a bushel?

MENTAL PROCESS.—If 1 bushel cost 84 cents, $\frac{1}{12}$ of a bushel costs $\frac{1}{12}$ of 84 cents, that is 7 cents; and if $\frac{1}{12}$ of a bushel cost 7 cents, $\frac{5}{12}$ of a bushel will cost 5 times 7 cents; that is 35 cents.

23. If 1 yard of broadcloth cost 12 dollars, how much does $\frac{2}{3}$ of a yard cost? $\frac{3}{4}$ of a yard? $\frac{5}{6}$ of a yard? $\frac{1}{2}$ of a yard?

24. If $\frac{8}{9}$ of a ton of hay cost 16 dollars, what costs 1 ton?

MENTAL PROCESS.—If eight-ninths of a ton cost 16 dollars, one-ninth will cost $\frac{1}{8}$ of 16 dollars, that is 2 dollars: and if $\frac{1}{9}$ of a ton cost 2 dollars, $\frac{9}{9}$ or 1 ton will cost 9 times 2 dollars, that is 18 dollars.

25. At 45 dollars for $\frac{5}{7}$ of an acre of land, what is the price per acre?

26. How many persons could receive half a pound of sugar each from $3\frac{1}{2}$ pounds?

MENTAL PROCESS.—As many as there are halves in three and a half. Since 1 pound is 2 half pounds, 3 pounds are 3 times 2 half pounds, that is 6 half pounds; and the other half pound added makes 7 half pounds. Ans. 7 persons.

27. How many quarter yards in $5\frac{1}{4}$ yards? In $2\frac{3}{4}$ yards?

28. How many eighths of an inch in $6\frac{3}{8}$ inches? In $9\frac{5}{8}$ inches?

29. How many gallons are held by 10 half gallon bottles?

MENTAL PROCESS.—If 2 half gallons make 1 gallon, 10 half gallons will make as many gallons as 2 is contained times in 10, that is 5 gallons.

30. How many units in $\frac{12}{3}$? $\frac{18}{6}$? $\frac{24}{4}$? $\frac{72}{8}$? $\frac{96}{12}$?

31. At $6\frac{1}{4}$ cents per yard, what cost 8 yards of muslin?

Mental Process.—If 1 yard cost $6\frac{1}{4}$ cents, 8 yards will cost 8 times $6\frac{1}{4}$ cents, which is equal to 8 times 6 cents plus 8 times $\frac{1}{4}$ of a cent. 8 times 6 cents are 48 cents; 8 times $\frac{1}{4}$ of a cent are $\frac{8}{4}$ of a cent, equal to 2 cents; 48 cents + 2 cents = 50 cents, the answer.

32. At $12\frac{1}{2}$ cents a pound, what cost 6 pounds of coffee? 4 pounds? 2 pounds?

33. At 10 cents a peck, what cost $2\frac{1}{2}$ pecks of potatoes?

Mental Process.—If 1 peck cost 10 cents, 2 pecks will cost 2 times 10 cents, that is 20 cents; and $\frac{1}{2}$ of a peck will cost $\frac{1}{2}$ of 10 cents, that is 5 cents; therefore $2\frac{1}{2}$ pecks will cost 20 cents + 5 cents = 25 cents.

34. At 8 cents a pound, what cost $5\frac{1}{2}$ pounds of starch? $7\frac{1}{4}$ pounds? $8\frac{3}{4}$ pounds? $9\frac{1}{2}$ pounds?

35. John spent 18 cents for lemons at $2\frac{1}{4}$ cents apiece: how many did he buy?

Mental Process.—He bought as many as $2\frac{1}{4}$ cents are contained times in 18 cents, or as many as 9, the number of quarters in $2\frac{1}{4}$, is contained times in 72, the number of quarters in 18; that is 8 lemons.

36. If 22 yards of cloth be cut into pieces of $5\frac{1}{2}$ yards each, how many pieces would there be?

WRITTEN EXERCISES IN DIVISION.

Process I.—Short Division.

Art. 45. When the divisor consists of one figure, or of a number, the operations with which are easily retained in the mind.

When the *mental processes* performed in written division are not expressed in the written process, but merely their *results*, the method is called *Short Division.*

Ex. 1. Divide 120580 by 6. Ans. $20096\frac{4}{6}$.

Written Process.	Mental Process.
6) 120580	6 in 1 hundred thousand, no hundreds of thousands of times, and 1 hundred thousand remain. Unite with it the 2 *tens of thousands*, making
$20096\frac{4}{6}$	

Questions.—45. What is short division? When is it used?

PROOF.

$$\begin{array}{r} 20096\tfrac{4}{6} \\ 6 \\ \hline 120576 \\ 4 = 6 \times \tfrac{4}{6} = \tfrac{24}{6} \\ \hline 120580 = \text{Dividend.} \end{array}$$

12 tens of thousands. Then 6 in 12 tens of thousands, 2 tens of thousands of times. Write 2 under the tens of thousands. 6 in 0 no times. Write 0 under 0. 6 in 5 *hundreds* no *hundreds* of times, and 5 hundreds remain. Write 0 under the hundreds, and unite the 8 tens with them, making 58 tens. 6 in 58 tens 9 tens of times, and 4 tens remain. Write 9 under the tens, and unite the units with the remaining 4 tens, making 40. 6 in 40 6 times and 4 remain. Write 6 under the units, and *represent* the 4 remainder as divided into 6 equal parts, by writing the divisor under it, making $\frac{4}{6}$, which is read *four-sixths*.

Mental and Recitation Model.

6 in 1 impossible; prefix 1 to 2, making 12. 6 in 12 2 times. Write 2. 6 in 0 no times. Write 0. 6 in 5 no times and 5 remain. Write 0 and prefix 5 to 8, making 58. 6 in 58 9 times and 4 remain. Write 9 and prefix 4 to 0, making 40. 6 in 40 6 times and 4 remain. Write 6 and the divisor under the remainder, making $\frac{4}{6}$.

RULE.— *Write the divisor at the left of the dividend, drawing one line between them, and another under the dividend.*

Divide successively the figures of the dividend from left to right, writing the quotient under the figure divided, and mentally prefixing each remainder to the next figure for the next division.

When the divisor is greater than the figure or figures to be divided, write a cipher in the quotient, and prefix the figure or figures as a remainder to the next, for the next division. To the quotient, annex the last remainder, with the divisor under it.

PROOF.—To detect error, carefully repeat the process, or multiply divisor and quotient together and add the remainder to the product. The result should equal the dividend.

NOTE.—Since the quotient shows how many *times* the dividend contains the divisor, it is plain that the divisor, taken as many *times* as there are units in the quotient, must equal the dividend. Hence also, multiplication may be proved by dividing the product by one factor, obtaining the other.

QUESTIONS.—45. What is the rule? What is the method of proof? Why?

EXAMPLES FOR PRACTICE.

(2.) 2) 151003 — 75501½

Proof. 151003

(3.) 3) 503010 — 167670

Proof. 503010

(4.) 4) 40706050 — 10176512¾

Proof. 40706050

(5.) 5) 11459085 — 2291817

Proof. 11459085

(6.) 6) 74747492905 — 12457915484⅙

(7.) 7) 987654321 — 141093474⅜

(8.) 8) 7000060000 — 875007500

(9.) 9) 53214210310 — 5912690034⅘

(10.) 11) 1020304050607 — 92754913691 6/11

(11.) 12) 112131415161718 — 9344284596809 10/12.

12. Find the sum of 918273645 ÷ 5 + 6135043002 ÷ 6. Ans. 1206161896.

13. Find the difference of 7056904832 ÷ 4 — 511261135 ÷ 7. Ans. 1691188903.

14. If a township of 64000 acres were divided equally among 9 persons, how large a portion would each receive? Ans. 7111⅑ acres.

15. Divide 1430150 by 13. Ans. 110011 7/13.

16. Eleven heirs share equally an estate of 131516 dollars: how many dollars does each receive? Ans. 11956 dollars.

17. How many persons must contribute each 8 dollars to furnish a hall at the cost of 776 dollars? Ans. 97 persons.

18. Seven partners share equally the loss of a ship and cargo valued at 458948 dollars. What was the loss of each? Ans. 65564 dollars.

19. If an army of 75428 men were separated into 4 equal divisions, how many would be in each? Ans. 18857.

20. Three heirs inherited property which sold at 658473 dollars. Sharing equally, what did each receive?

Ans. 219491 dollars.

21. Two men bought a factory for 147600 dollars. What must each pay if they share equally? Ans. 73800 dollars.

22. In how many days would a person, going 12 miles a day travel around the globe, 24858 miles? Ans. $2071\frac{6}{12}$ days.

Art. 46. The *average* of several numbers is that number which, taken as many times as there are numbers, would equal the sum of the numbers.

Rule.—*To find the average of several quantities, divide their sum by their number.*

Ex. 23. A's age is 47, B's 63, C's 76, D's 31, E's 54, and F's 42 years. What is the average of their ages?

Ans. $52\frac{1}{6}$ years.

24. Seven clerks receive the following salaries: A 1500 dollars, C 1200, E 900, F 700, H 500, J 450, and K 350. What is the average of their salaries? Ans. 800 dollars.

25. Stephen Ball sold horses at the following prices: two at 125 dollars apiece, four at 225 dollars apiece, two at 87 dollars apiece, and three at 75 dollars apiece. What did they average him? Ans. $140\frac{9}{17}$ dollars.

26. What is the average of 790654, 58442, 3031, 750075, 47080, and 273322? Ans. 320434.

27. If a thermometer stands 60 degrees in the morning, 95 at noon, and 76 in the evening, what is the average heat of the day? Ans. 77 degrees.

28. A weighs 159 pounds, B 181, C 145, and D 134. What is their average weight? Ans. $154\frac{3}{4}$ pounds.

Process II.—Long Division.

Art. 47. When the divisor consists of more than one figure, or of a number, the operations with which are not easily retained in the mind.

When the *mental processes* performed in dividing are expressed in the written process, the method is called *Long Division.*

Questions.—46. What is the average of several numbers? How is it found?

1. Illustration with One Figure.

Dividend.

Divisor, 5)975(195 Quotient.

5

47

45

25

25

Analysis of the Process.

5)900 + 70 + 5 | 100
500 | 90
| 5
400 | 195
70
470
450
20
5
25
25

Explanation.

In short division we should say, 5 in 9 once and 4 remain, mentally performing the multiplication of 1×5, and the subtraction of 5 from 9. In long division we write 1 at the right hand in the quotient's place, then put the product of 1×5 under 9, the figure divided, then draw a line, subtract, and write the remainder 4. In short division we should mentally prefix 4 to 7, making 47. In long division we write 7 at the right hand of 4, making 47. Then 5 in 47 9 times. Write 9 as the next quotient figure and the product 9×5 under 47, and subtract, leaving 2. Bring down the next figure 5, making 25. Then 5 in 25 five times. Write 5 in the quotient, and $5 \times$ the divisor $5 = 25$ under 25. As 0 remains, and there are no more figures in the dividend, the division is finished.

2. Illustration with Two Figures.

Dividend.

Divisor, 24)34560(1440 Quotient.

24

105

96

96

96

0

Explanation.

This is performed like the preceding illustration. But when the divisor is so large that we cannot tell how many times it is contained in the figures divided, we must *make a trial*, by using the left-hand figures of each. Thus, not perceiving the times 24 is contained in 105, we say 2 in 10 *five* times; but trying 5×24 and finding it more than 105, we try 4, which is one less. The product 96, being less than 105, we can subtract and proceed in the same way with the other figures. Finally 24 in 0 no times: write 0 in the quotient.

3. Illustration with Three Figures

Dividend.

Divisor, 491)99695(203$\frac{22}{491}$ Quotient.
982

1495
1473

22

Explanation.

This is performed like the preceding illustrations, except that, after 9 is brought down to 14 in the first remainder, making 149, it will not contain 491. Then say, 491 in 149 no times; write 0 in the quotient and bring down 5, making 1495, and proceed as usual.

Rule.—*Draw a line at the left and right of the dividend, and write the divisor at the left.*

Find how many times the divisor is contained in the fewest left hand figures of the dividend that will contain it, and write the quotient at the right hand of the dividend.

Subtract the product of the divisor by this quotient figure from the figure or figures divided, and at the right of the remainder write the next figure of the dividend.

Divide as before the number thus formed, proceeding as stated till every figure of the dividend has been used.

When the number formed by the remainder and the next figure of the dividend does not contain the divisor, write 0 *in the quotient and bring down the next figure of the dividend.*

To the quotient annex the last remainder with the divisor under it.

Note.—1. In large numbers, it is best to *try* for the quotient figure with only the left-hand figures of divisor and dividend.

Note.—2. If a product is greater than the corresponding partial dividend, try a less quotient figure than that which produced it.

Note.—3. If a remainder is greater than the divisor, try a greater quotient figure than that which produced it.

Proof.—*First Method.*—Proceed as directed in Short Division.

Second Method.—Add, as they stand, the remainder and the several partial products. The sum should be equal to the dividend. (See note, p. 55.)

Questions.—47. What is long division? What is the rule for long division? What is the first method of proof? What is the second?

Third Method.—Subtract the remainder from the dividend and divide the difference by the quotient. The result should equal the divisor.

Fourth Method.—Multiply the excess over *nines* (see Ex. 5) in the figures of the divisor, by that in the figures of the quotient. To the excess in the figures of this product, add the excess in the figures of the remainder. The excess in the figures of this sum should equal that in the figures of the dividend.

EXAMPLES FOR PRACTICE.

(4.)

```
37)69009(1865
   37
   ---
   320
   296
   ---
    240
    222
    ---
     189
     185
     ---
       4
```

PROOF BY MULTIPLICATION.

```
1865 Quotient.
  37 Divisor.
------
13055
5595
   4 Remainder.
------
69009 Dividend.
```

(5.)

```
98)325761(3324
  +294
  ----
   317
  +294
  ----
    236
   +196
   ----
     401
    +392
    ----
      +9
    ----
Proof by 325761
Addition.
```

(5.)

Proof by casting out nines.

Excess in quotient	= 3
Excess in divisor	= 8
Product	24
Excess in product	6
Excess in remainder	0
Excess in sum	6
Excess in dividend	6

NOTE.—The *proof by addition* is of little value, because it does not detect error in the partial products, if the subtractions are correct.

QUESTIONS.—47. What is the third method of proof? What is the fourth?

(6.)

456)3652188(8009
3648

4188
4104

84

(6.)

Proof by Division.

3652188
84 remainder.

Quotient, 8009)3652104(456
32036

44850
40045

48054
48054

7. Divide 28476 by 14. Ans. 2034.
8. Divide 585086 by 15. Ans. $39005\frac{11}{15}$.
9. Divide 121737 by 16. Ans. $7608\frac{9}{16}$.
10. Divide 1616615 by 17 Ans. 95095.
11. Divide 365472 by 18. Ans. 20304.
12. Divide 361413 by 19. Ans. $19021\frac{14}{19}$.
13. Divide 8684415 by 123. Ans. 70605.
14. Divide 36666504 by 456. Ans. 80409.
15. Divide 25259835 by 789. Ans. 32015.
16. Divide 8560234 by 91. Ans. $94068\frac{46}{91}$.
17. Divide 7953247 by 82. Ans. $96990\frac{67}{82}$.
18. Divide 4162387 by 73. Ans. 57019.
19. Divide 3409792 by 64. Ans. 53278.
20. Divide 4959185 by 55. Ans. 90167.
21. Divide 24058035 by 4005. Ans. 6007.
22. Divide 1880498233 by 1912. Ans. $983524\frac{345}{1912}$.
23. Divide 1498686882 by 4466. Ans. 335577.
24. Divide 300880248 by 9084. Ans. 33122.
25. Divide 83810205 by 12345. Ans. 6789.

26. Divide ninety-four thousand eight hundred thirty-three, by three hundred sixty-nine. Ans. 257.

27. If in one year the expenditures of government were sixty-four millions, five hundred twenty-nine thousand, eight hundred ten dollars, how much would that be a day, allowing 365 days in a year? Ans. 176794 dollars.

28. The whole population of Pennsylvania in 1850 was two millions, three hundred eleven thousand, three hundred eighty six, and there were sixty-three counties: what was the average per county? Ans. $36688\frac{42}{63}$.

29. Ohio had 87 counties, and one million nine hundred eighty thousand, three hundred twenty-nine people: what was the average per county? Ans. $22762\frac{3}{8}\frac{5}{7}$.

30. In a library of 259 books, there were 125874 pages: what was the average per book? Ans. 486 pages.

31. The common version of the Bible contains 31173 verses: how many verses per day must a reader average to read through the book in 365 days? Ans. $85\frac{1}{3}\frac{4}{6}\frac{8}{5}$ verses.

32. An estate of eighty-five thousand eight hundred dollars was inherited by a widow and four children: what was the share of each child after the widow had taken one-third?

Ans. 14300 dollars.

33. If the population of a county is 138290, and the expenses of punishing criminals in that county in one year are twenty-seven thousand six hundred fifty-eight dollars, what is the average cost for each inhabitant? Ans $\frac{1}{5}$ of a dollar.

34. What is the value of the expression $\left(\frac{48-13}{7}\right)\times\left(\frac{14\times3}{2}\right)+(16\div\overline{2\times4})-7$? Ans. 100.

35. What is the value of the expression $(18\times3)\div\left(\frac{4\times27}{18}\right)+2\times\overline{8+3\times(6-4)}$? Ans. 37.

Art. 48. To divide by 1 with a cipher, or ciphers annexed.

Ex. 1. It cost a family of ten persons 986 dollars in one year to live. What was the average per person?

Ans. $98\frac{6}{10}$ dollars.

ILLUSTRATION.

$1|0\ )\ 98|6 = 98\frac{6}{10}$.

EXPLANATION.

In Decimal Notation, each approximation one place towards the place of *units*, causes a figure to express ten times less than before. Therefore cutting off one figure from the right of a number has that effect upon the remaining figures, and the figure cut off can be *represented* as divided by 10. The result is, therefore, a quotient produced by dividing by 10.

RULE.—*Cut off from the right of the dividend as many figures as there are ciphers in the divisor. The part of the dividend on the left of the line is the quotient; that on the right of it the remainder.*

QUESTIONS.—48. How do we divide by 1 with ciphers annexed? Why?

EXAMPLES FOR PRACTICE.

2. Divide 3862 by 10. Ans. $386\frac{2}{10}$.
3. Divide 5274 by 100. Ans. $52\frac{74}{100}$.
4. Divide 61988 by 1000. Ans. $61\frac{988}{1000}$.
5. Divide 547623942 by 100000. Ans. $5476\frac{23942}{100000}$.

Art. 49. To divide by a composite number.

CASE I.

When 10 is not one of the factors.

Ex. 1. Divide 1933 by 72. Ans. $26\frac{61}{72}$.

ILLUSTRATION.

12) 1933

6) 161 twelves + 1 unit.

26 seventy-twos + 5 twelves.
$5 \times 12 + 1 = 61$. True Remainder.

EXPLANATION.

In this operation, we use any convenient set of factors of 72, dividing by one factor, then that quotient by another, and so on, till every factor of the set has been used. To find the *true remainder*, observe that 1933 units, divided into groups of 12 units each, make 161 *twelves*, and one unit remains. Then these 161 twelves, divided into groups of *six twelves* each, make 26 *seventy-twos*, and five twelves remain. Now, the *true remainder* must be the sum of these remainders; that is *five twelves* + one = 61.

Other factors of 72 may be used as divisors, in any order, with the same results.

RULE.—*Take any convenient set of factors of the divisor, and divide the dividend first by one factor, then that quotient by another, and so on till every factor of the set has been used as a divisor. The last quotient is the true one.*

Multiply each remainder by all the divisors preceding the one that produced it, and add together these products with the first remainder. This sum is the true remainder.

EXAMPLES FOR PRACTICE.

2. Divide 1811423 by $84 = 12 \times 7$. Ans. 21564 | 47.
3. Divide 537069 by $132 = 11 \times 12$. Ans. 4068 | 93.

QUESTIONS.—49. How do we divide by a composite number when 10 is not a factor? How do we get the true remainder? Why?

4. Divide 642879 by $88 = 11 \times 8$. Ans. 7305 | 39.
5. Divide 511299 by $105 = 5 \times 3 \times 7$.
Ans. 4869 | 54.
6. Divide 106998 by $135 = 5 \times 9 \times 3$. Ans. $792\frac{78}{135}$.

Case II.

When there are ciphers on the right of the divisor.

Ex. 1. A carriage-maker sold 60 carriages for 23218 dollars; what was that apiece? Ans. $386\frac{58}{60}$ dollars.

Written Process.

60) 23218
386|58

Mental Process.

This is a case of using factors. Since $60 = 6 \times 10$, we can first cut off one figure, to divide by 10, and then divide by 6. This gives the first remainder 8 *units*, and the second 5 *tens*, making the *true remainder* 58.

Rule.—*Cut off the ciphers from the right of the divisor, and as many figures from the right of the dividend. Then divide the remaining figures of the dividend by the remaining figures of the divisor. To the remainder annex the figures cut off from the dividend, to obtain the true remainder.*

Examples for Practice.

2. Divide 7856934 by 2400. Ans. $3273\frac{1734}{2400}$.

24 | 00) 78569 | 34 ($3273\frac{1734}{2400}$
72
65
48
176
168
89
72
1734

3. Divide 24689753 by 9800. Ans. $2519\frac{3553}{9800}$.
4. Divide 5270368 by 90000. Ans. $58\frac{50368}{90000}$.
5. Divide 95456723 by 2100000. Ans. $45\frac{956723}{2100000}$.
6. Divide 638419000 by 45000. Ans. $14187\frac{4}{45}$.

Question.—49. How do we divide when the divisor ends with ciphers?

CHAPTER VI.

CONTRACTIONS.*

Art. 50. In large and tedious operations in multiplication and division, it is expedient to seek the shortest possible method of arriving at the result. The following are some of the cases in which the labor may be lessened.

CONTRACTIONS IN MULTIPLICATION.

CASE I.

When the multiplier is a convenient part of 100, 1000, &c.

Ex. 1. Multiply 9564 by 25. Ans. 239100.

WRITTEN PROCESS.	MENTAL PROCESS.
4)956400 239100	Because 25 is $\frac{1}{4}$ of 100, 25 times a number is $\frac{1}{4}$ of a 100 times that number. Hence annexing two ciphers multiplies 9564 by 100, and dividing that product by 4 produces 25 times 9564.

RULE.—*Annex to the multiplicand the ciphers of* 100, 1000, *&c., and take that part of the result, which the multiplier is of* 100, 1000, *&c.*

TABLE OF PARTS.

Of 100.				Of 1000.			
$12\frac{1}{2} = \frac{1}{8}$	$66\frac{2}{3} = \frac{2}{3}$			$125 = \frac{1}{8}$	$750 = \frac{3}{4}$		
$14\frac{2}{7} = \frac{1}{7}$	$75 = \frac{3}{4}$			$142\frac{6}{7} = \frac{1}{7}$	$833\frac{1}{3} = \frac{5}{6}$		
$16\frac{2}{3} = \frac{1}{6}$	$83\frac{1}{3} = \frac{5}{6}$			$166\frac{2}{3} = \frac{1}{6}$	$875 = \frac{7}{8}$		
$25 = \frac{1}{4}$	$87\frac{1}{2} = \frac{7}{8}$			$250 = \frac{1}{4}$	$1125 = 1\frac{1}{8}$		
$33\frac{1}{3} = \frac{1}{3}$	$112\frac{1}{2} = 1\frac{1}{8}$			$333\frac{1}{3} = \frac{1}{3}$	$1142\frac{6}{7} = 1\frac{1}{7}$		
$37\frac{1}{2} = \frac{3}{8}$	$114\frac{2}{7} = 1\frac{1}{7}$			$375 = \frac{3}{8}$	$1166\frac{2}{3} = 1\frac{1}{6}$		
$50 = \frac{1}{2}$	$116\frac{2}{3} = 1\frac{1}{6}$			$625 = \frac{5}{8}$	$1250 = 1\frac{1}{4}$		
$62\frac{1}{2} = \frac{5}{8}$	$133\frac{1}{3} = 1\frac{1}{3}$			$666\frac{2}{3} = \frac{2}{3}$	$1333\frac{1}{3} = 1\frac{1}{3}$		

EXAMPLES FOR PRACTICE.

2. Multiply 7632 by $112\frac{1}{2}$. Ans. 858600.

WRITTEN PROCESS.	MENTAL PROCESS.
8)763200 95400 858600	Since $112\frac{1}{2}$ is once and one-eighth $(1\frac{1}{8})$ of a hundred, if we multiply 7632 by 100 and add $\frac{1}{8}$ of this product to itself, the result is the product of 7632 by $112\frac{1}{2}$.

* This chapter, excepting that part which relates to cancellation, can be omitted, if desirable, till the pupil is more advanced.

QUESTIONS.—50. What should we seek to do, in large operations? How can we lessen labor, when the multiplier is a convenient part of 100, 1000, &c.?

3. $2598 \times 125 = 324750$.
4. $30427 \times 166\frac{2}{3} = 5071166\frac{2}{3}$.
5. $54558 \times 116\frac{2}{3} = 6365100$.
6. $6708 \times 1166\frac{2}{3} = 7826000$.
7. $80808 \times 87\frac{1}{2} = 7070700$.
8. $12246 \times 83\frac{1}{3} = 1020500$.
9. $564 \times 133\frac{1}{3} = 75200$.
10. $705 \times 25 = 17625$.
11. $1545 \times 1333\frac{1}{3} = 2060000$
12. $2172 \times 33\frac{1}{3} = 72400$.
13. $578 \times 875 = 505750$.
14. $3248 \times 62\frac{1}{2} = 203000$

Case II.

Art. 51. When the multiplier is any number of *nines*.

Ex. 1. Multiply 54637 by 999. Ans. 54582363.

Process Illustrated.

$$54637000 = 54637 \times 1000$$
$$54637 = 54637 \times 1$$
$$54582363 = 54637 \times 999$$

Mental Process.

999 is one less than 1000. Hence, subtracting once 54637 from 1000 times 54637 will give 999 times 54637.

Rule.—*Annex to the multiplicand as many ciphers as there are nines in the multiplier, and subtract the multiplicand from the result.*

Examples for Practice.

2. Multiply 827364 by 9999. Ans. 8272812636.
3. Multiply 90306 by 99999. Ans. 9030509694.
4. Multiply 37914 by 999999. Ans. 37913962086.

Case III.

Art. 52. When all the figures of the multiplier are the same.

Ex. 1. Multiply 39485 by 7777. Ans. 307074845.

Written Process.

$$\begin{array}{r} 394850000 \\ 39485 \\ \hline 9)394810515 \\ \hline 43867835 \\ 7 \\ \hline 307074845 \end{array}$$

Mental Process.

Since 7 is $\frac{7}{9}$ of 9, we can multiply as if the multipliers were all *nines*, and take $\frac{7}{9}$ of that product.

Question.—51. How do we proceed when the multiplier is all nines? Explain the process

RULE.—*Multiply as if the figures were nines, and take that part of the product which the figure is of nine.*

EXAMPLES FOR PRACTICE.

2. $123987456 \times 444444 =$ Ans. 55105480894464.
3. $789321654 \times 333333 =$ Ans. 263106954892782.
4. $27374757 \times 888888 =$ Ans. 24333093000216.
5. $9989769 \times 555555 =$ Ans. 5549866116795.

CASE IV.

Art. 53. When the multiplier has two significant figures, and begins or ends with 1.

RULE.—*Consider the multiplicand as a product by the* 1, *and place the other product under it as much to its right or left as is required by the nature of the multiplier.*

ILLUSTRATIVE PROCESSES.

(1.)	(2.)	(3.)
1854×17	9581×108	6435×1005
12978	76648	32175
31518	1034748	6467175

(4.)	(5.)	(6.)
379×61	468×701	1269×5001
2274	3276	6345
23119	328068	6346269

EXAMPLES FOR PRACTICE.

7. Multiply 234567 by 19. Ans. 4456773.
8. Multiply 354657 by 18. Ans. 6383826.
9. Multiply 958473 by 106. Ans. 101598138.
10. Multiply 5239 by 901. Ans. 4720339.
11. Multiply 3745 by 71. Ans. 265895.
12. Multiply 16278 by 8001. Ans. 130240278.

CASE V.

Art. 54. When one portion of the multiplier is a factor of another.

QUESTIONS.—52. How do we proceed when the figures of the multiplier are the same? 53. How when it is of two digits, and begins or ends with 1?

Ex. 1. Multiply 327654 by 729. Ans. 238859766.

Written Process.	Mental Process.
327654 729 2948886 23591088 238859766	Since 72 is 8 times 9, if we multiply by 8 the partial product, resulting from using 9, we arrive at once at the product by 72, the other figures of the multiplier. The sum of these two products must be equal to the sum of the three products by 9, 2, and 7. Here we consider the 72 as *tens*, and the first figure of its product must be written in the column of tens.

Rule.—*Whenever it will save labor, multiply any convenient partial product by that factor which, multiplied into its producing figure, would make it equal another part of the multiplier. The partial product thus produced would answer for that of said other part of the multiplier. So use the whole multiplier, and add the partial products.*

Examples for Practice.

2. Multiply 7563124 by 14412. Ans. 108999743088.

Note.—Here multiply first by 12, then that product by 12.

3. Multiply 8403215 by 108369. Ans. 910648006335.

Note.—First 9, then 4 times that for 36, then 3 times that for 108.

4. Multiply 90847316 by 378637. Ans. 34398155188292.

Note.—First 7, then 9 times that, then 6 times that.

5. Multiply 4591328 by 6613212. Ans. 30363425425536.

Note.—First 12, then 11 times that, then half of that.

CONTRACTIONS IN DIVISION.

Case I.

Art. 55. When the divisor is a convenient part of 10, 100, 1000, &c.

Ex. 1. Divide 473657 by $12\frac{1}{2}$. Ans. $37892\frac{56}{100}$.

Written Process.	Mental Process.
473657 8 37892|56	Since 8 times $12\frac{1}{2}$ are 100, it is plain that 100 is contained in 8 times the dividend as many times as $12\frac{1}{2}$ is contained times in the dividend. Hence, multiply the dividend by 8, and divide the product by 100.

Questions.—54. How do we proceed when one portion of the multiplier is a factor of another? 55. How do we proceed when the divisor is a convenient

RULE.—*Multiply the divisor and dividend by that factor which would make the divisor* 10, 100, 1000, *&c., and then divide.*

EXAMPLES FOR PRACTICE.

2. Divide 337448 by $16\frac{2}{3}$. Ans. $20246\frac{88}{100}$.
3. Divide 795861 by 125. Ans. $6366\frac{888}{1000}$.
4. Divide 902728 by $2\frac{1}{2}$. Ans. $361091\frac{2}{10}$.

NOTE.—$2\frac{1}{2}$ is $\frac{1}{4}$ of 10.

5. Divide 840127 by $14\frac{2}{7}$. Ans. $58808\frac{89}{100}$.
6. Divide 721424 by $333\frac{1}{3}$. Ans. $2164\frac{272}{1000}$.

CASE II.

Art. 56. When the divisor and dividend have the same factor involved in them.

If the dividend and divisor are both divided by the same number, the quotient of the resulting numbers is the same as that of the numbers before such division.

ILLUSTRATION.

$6)36 \div 12 = 3$
$\quad 6 \div \ 2 = 3$
$4)36 \div 12 = 3$
$\quad 9 \div \ 3 = 3$

Here 36 and 12 can both be divided by 6, and the remaining factor 6 of the dividend contains the remaining factor 2 of the divisor still 3 times. So if we divide both by 4, the remaining factor 9 of the dividend contains the remaining factor 3 of the divisor still 3 times.

CANCELLATION is a process designed to shorten the labor of dividing, by removing the factors common to divisor and dividend.

The process of cancellation is usually represented by drawing a line across the number from which a factor is rejected, and writing the remaining factor adjacent to the number thus crossed.

Ex. 1. A merchant sold to a farmer 25 yards of cloth at 4 dollars a yard, and took his pay in wheat at 4 dollars a sack. How many sacks did he receive? Ans. 25 sacks.

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part of 10, 100, 1000, &c.? 56. If dividend and divisor are both divided by the same number, what is the quotient of the result? What is cancellation? How is the process represented?

WRITTEN PROCESS. EXPLANATION.

$$\frac{4 \times 25}{4} = 25$$

Instead of finding the cost of the whole piece of cloth by multiplying 25 by 4, and again dividing by 4 to find the number of sacks of wheat, it is only necessary to *indicate* these processes, and then remove or *cancel* the factor 4 from both divisor and dividend. The quotient is at once seen, since the divisor is reduced to 1.

Ex. 2. Sold 12 loads of coal, each containing 42 bushels, at 6 cents a bushel, and took pay in cotton cloth, each piece containing 27 yards, at 8 cents a yard. How many pieces did I receive? Ans. 14 pieces.

WRITTEN PROCESS. MENTAL PROCESS.

$$\frac{\cancel{4} \quad 14 \quad \cancel{2}}{\cancel{12} \times \cancel{42} \times \cancel{6}}$$

$$\frac{\cancel{27} \times \quad 8}{\cancel{9} \qquad \cancel{2}}$$

$$\cancel{3}$$

I received as many pieces as the number of times that the cost of the coal contains the cost of one piece. Hence, arrange the factors of the cost of the coal as a dividend, and those of the cost of one piece of cloth as the divisor. Then cancel as follows:—3 from 12 and 27 leave 4 and 9; cross 12 and 27. Then 3 from 42 and 9 leave 14 and 3; cross 42 and 9. Then 3 from 6 and 3 leave 2 and 1; cross 6 and 3. Then cross the 2 above and the 2 below, and 14 is the only number left. Another method could have been pursued, with the same result.

Ex. 3. Divide $25 \times 4 \times 24$ by 21×15. Ans. $7\frac{3}{21}$.

WRITTEN PROCESS. EXPLANATION.

$$\frac{5 \qquad 8}{\cancel{25} \times 4 \times \cancel{24}} = \frac{5 \times 4 \times 8}{7 \times 3} = \frac{160}{21} = 7\tfrac{3}{21}.$$

$$\frac{\cancel{21} \times \cancel{15}}{7 \quad 3}$$

5 from 25 and 15 leave 5 and 3; cross 25 and 15. Then 3 from 24 and 21 leave 8 and 7; cross 24 and 21. Since no other factor can be taken both from dividend and divisor, we must divide the product of the remaining factors $5 \times 4 \times 8$ by that of the remaining factors 7×3.

RULE.—*Write the factors of the dividend above a line with the sign of multiplication between them; and those of the divisor below the line in the same manner.*

Notice what number above contains the same factor as any number below. Conceive this factor taken out of both numbers; cross them; and set their remaining factors near them.

Do thus till there is no factor found in both dividend and

QUESTION.—What is the rule for canceling?

divisor; then multiply all the factors that are left in the dividend for a dividend, and all that are left in the divisor for a divisor. Divide with these products as usual.

NOTE.—The pupil is earnestly recommended to resort to this method of contraction, whenever it will lessen labor without obscuring the written process.

EXAMPLES FOR PRACTICE.

4. Divide $9 \times 15 \times 48$ by $6 \times 5 \times 4$. Ans. 54.

ANOTHER METHOD.

$$\begin{array}{r|l} 2\,6 & 9\,3 \\ 5 & 1\,5\,3 \\ 4 & 4\,8\,1\,2\,6 \\ \hline & 3 \times 3 \times 6 = 54 \end{array}$$

Many prefer writing the factors of the dividend on the right of a line and those of the divisor on the left, as in this process. The pupil should adopt that method which is most convenient to him.

5. Divide $24 \times 36 \times 60$ by $8 \times 9 \times 10$. Ans. 72.

6. Divide $18 \times 30 \times 96$ by $2 \times 10 \times 16$. Ans. 162.

7. Divide $25 \times 75 \times 125$ by $25 \times 25 \times 25$. Ans. 15.

8. Exchanged 8 kegs of butter, each containing 36 pounds, at 18 cents a pound, for 64 sacks of potatoes, each containing 3 bushels. What did the potatoes cost a bushel?

Ans. 27 cents.

9. Exchanged 15 barrels of pork, each containing 200 pounds, at 10 cents a pound, for 16 barrels of fish, each containing 220 pounds. What did the fish cost a pound? Ans. $8\frac{8}{11}$ cents.

10. How many barrels of apples, each containing 3 bushels, at 50 cents a bushel, must be given for 9 boxes of raisins, each containing 25 pounds, at 16 cents a pound? Ans. 24 barrels.

QUESTIONS INVOLVING THE FOUR FUNDAMENTAL RULES.

1. M. Kidd bought 532 acres of land, at 12 dollars per acre, and sold it at 23 dollars per acre; what did he gain?

Ans. 5852 dollars.

2. F. Mills bought a farm of 325 acres, at 25 dollars per acre, and paid 2575 dollars down. How much remained to be paid? Ans. 5550 dollars.

3. S. Parker bought 96 cords of oak wood, at 6 dollars per cord, 24 cords of hickory, at 8 dollars per cord, and 17 cords of pine wood, at 3 dollars per cord. What did the whole cost?

Ans. 819 dollars.

4. Wm. Rich sold from his trees in one year 488 bushels of peaches, at 2 dollars a bushel, 29 bushels of cherries, at 3 dollars a bushel, and 143 barrels of apples, at 2 dollars a barrel. What did he get for all? Ans. 1349 dollars.

5. Bought at A. Camp's store $5\frac{1}{2}$ yards of broadcloth, at 8 dollars per yard, 12 yards satinet, at $1\frac{1}{4}$ dollars per yard, and $1\frac{1}{2}$ yards satin, at 6 dollars per yard. I offered him a hundred dollar bill; how much change should I receive?

Ans. 32 dollars.

6. A. walked 15 miles one day, 22 the next, 25 the next, and 18 the fourth. What was the average? Ans. 20 miles a day.

7. Having to travel 2840 miles by railroad, I went 525 miles the first day, 462 the second, 397 the third, and 540 the fourth. How many miles remained to be traveled? Ans. 916 miles.

8. In 1 day there are 24 hours; how many hours in 7 days?

9. In 1 pound of sugar there are 16 ounces; how many ounces in 2000 pounds?

10. In 1 mile there are 5280 feet; how many feet in $16\frac{2}{3}$ miles? Ans. 88000 feet.

11. Fourteen thousand three hundred eighty-seven, and another number, amount to nineteen thousand one hundred and three. What is that other number? Ans. 4716.

12. Two thousand one hundred seventy-five multiplied by another number produces 180525; what is the other factor?

Ans. 83.

13. The quotient is 709, the remainder 142, and the dividend 3579174; what is the divisor? Ans. 5048.

14. J. Dale opened the year's business with 8642 dollars, and closed it with 10235 dollars. What was his whole or net gain? His average gain per day?

Ans. 1593 dollars; $4\frac{3}{6}\frac{3}{5}$ dollars.

15. G. Perry made in a year's business 1846 dollars, but saved from his gains only 720 dollars; what were his average daily expenses? Ans. $3\frac{3}{3}\frac{1}{6}\frac{1}{5}$ dollars.

16. At a certain mill, 18 hands are paid 2 dollars per day, 76 hands $1\frac{1}{2}$ dollars per day, 128 hands $1\frac{1}{4}$ dollars per day, and 35 hands 1 dollar per day. What is paid for labor per day? What is the average per hand?

Ans. 345 dollars per day; $1\frac{88}{257}$ dollars average per hand.

17. T. Starr bought 896 barrels of flour, at 4 dollars per barrel; 51 barrels spoiled so that they were thrown away; 145, being less injured, he sold at 3 dollars per barrel; the remainder he sold at 7 dollars per barrel. Did he gain or lose, and how much? Ans. Gained 1751 dollars.

18. C. Allen bought an ox weighing 1260 pounds, at 4 cents a pound. On slaughtering and dressing it, he sold the hide, weighing 100 pounds, at $6\frac{1}{2}$ cents a pound, the fore-quarters, each weighing 180 pounds, at 7 cents a pound, and the hind-quarters, each weighing 160 pounds, at 9 cents a pound. Did he gain or lose, and how much? Ans. Gained 1010 cents.

19. What is the difference between $(2 \times 9) + 70$, and 2×79? Ans. 70.

20. How many days would it take to count one million, counting fifty thousand per day? Ans. 20 days.

21. D. Boyd bought of R. Kennedy a house and farm for 4280 dollars, for which he paid Kennedy 4 horses, valued at 125 dollars apiece, 15 tons of hay, at 23 dollars per ton, 720 bushels of wheat, at $1\frac{1}{4}$ dollars per bushel, and the balance in cash. How much money did he pay? Ans. 2535 dollars.

22. Obtain the product of 17359824×14472189, by four partial products. Ans. 251234653934736.

23. E. Mercer bought an equal number of pairs of boots and shoes; for the shoes he gave 2 dollars a pair, and for the boots 4 dollars a pair; he gave 450 dollars for all. What was the number of pairs of each? Ans. 75 pairs.

24. The planet Neptune is calculated to be about 2856000000 miles from the sun; how long would it take a locomotive to travel that distance, at the rate of 596 miles a day?

Ans. $4791946\frac{184}{596}$ days.

25. How many years would that be, at 365 days per year? (Excluding the fraction.) Ans. $13128\frac{2}{3}\frac{6}{6}\frac{6}{5}$ years.

26. A pipe leading into a cistern delivers 65 gallons per hour, and another discharges from the cistern on the average 28 gallons per hour; how long will it take the cistern to gain 16872 gallons? Ans. 456 hours.

27. If one pipe can fill a cistern holding 720 gallons in 12 hours, and another empty it in 18 hours, how long will it take the cistern to fill, if both are left open? Ans. 36 hours.

CHAPTER VII.

FEDERAL, OR UNITED STATES MONEY.

Art. 57. FEDERAL MONEY, or United States Money, is the legal currency of the United States of North America.

Its denominations are Eagles, Dollars, Dimes, Cents, and Mills.

TABLE.

10 Mills (marked m.)	make	1 Cent,	marked	ct.
10 Cents	"	1 Dime,	"	d.
10 Dimes	"	1 Dollar,	"	$
10 Dollars	"	1 Eagle,	"	E.

Mills.		Cents.		Dimes.		Dollars.		Eagle.
10	=	1						
100	=	10	=	1				
1000	=	100	=	10	=	1		
10000	=	1000	=	100	=	10	=	1

The *coins* of the United States are :—

1st. Of alloyed gold, the Double-Eagle, Eagle, Half-Eagle, Quarter-Eagle, Three-Dollars, and Dollar. The mixture is 18 parts gold, 1 part silver, and 1 part copper.

2d. Of alloyed silver, the Dollar, Half-Dollar, Quarter-Dollar, Dime, Half-Dime, and Three-Cent-piece. The mixture is 9 parts silver and 1 part copper; except the Three-Cent-piece, which is 3 parts silver and 1 part copper.

3d. Of copper, or alloyed copper, the Cent and Half-Cent.

NOTE.—The weight of the Eagle is 258 grains Troy, and that of the other gold coins is in proportion to their values. The Half-Dollar weighs 192 grains; Quarter-Dollar 96 grains; Dime $38\frac{2}{5}$ grains; Half-Dime $19\frac{1}{5}$ grains; Three-Cent-piece $11\frac{52}{100}$ grains: old copper Cent 168 grains; and the cent of copper and nickel, 72 grains. The old silver dollar, now mostly superseded by the gold dollar, weighed $412\frac{1}{2}$ grains.

QUESTIONS.—57. What is Federal Money? What are its denominations? Repeat the table. What are the coins of the United States?

Art. 58. The Dollar is the unit of United States Money In business transactions the only denominations used are dollars, cents, and mills.

United States Money is written like simple numbers, separating dollars from cents by a point, called the *separatrix*, and sometimes cents from mills by a comma. Thus, 187 dollars, 14 cents, and 5 mills are written \$187.14,5, or \$187.145. This facility results from the value of a greater denomination being tenfold that of the next less, as in simple numbers.

Art. 59. To read numbers in United States Money.

RULE.—*Call all on the left of the separatrix dollars, the next two figures cents, the next figure mills, and the next figure tenths of a mill.*

NOTE.—For further information on the figures at the right of mills, see Decimal Fractions, Art. 145.

EXERCISES.

Pronounce, or write in words, the following numbers:—

	\$ cts. m.		\$ cts. m.		\$ cts. m.
1.	17.25,3.	4.	3101.10,13.	7.	200408.11,11.
2.	458.00,5.	5.	5002.02,07.	8.	3050709.05,09.
3.	6983.04,8.	6.	27640.00,00.	9.	85043206.00, 1.

Art. 60. To write numbers in United States Money.

RULE.—*First write the dollars, then two figures for cents, and one figure for mills. When cents are not given, or only one figure of them is given, fill the vacant places with ciphers.*

EXERCISES.

Write in figures the following numbers.

1. Sixty-five dollars, six cents. Ans. \$65.06.
2. Three hundred and two dollars, twenty cents, five mills.
3. Fourteen dollars, four cents, four mills.
4. One hundred and one dollars, one mill.
5. Forty-eight dollars, fifty cents, five mills.

QUESTIONS.—58. What is the unit of Federal Money? What are its business denominations? How is it written? Why?—59. How do we read numbers in Federal Money?—60. How do we write numbers in Federal Money?

REDUCTION OF UNITED STATES MONEY.

Art. 61. Reduction of United States Money is changing a number expressed in one of its denominations, so as to express its value in another.

Case I.

To reduce to a lower denomination.

Cents become *mills* by multiplying by 10, or annexing *one* cipher.

Dollars become *cents* by multiplying by 100, or annexing *two* ciphers.

Dollars become *mills* by multiplying by 1000, or annexing *three* ciphers.

Ex. 1. Reduce \$416.17 to cents. Ans. 41617 cents

Explanation.		Suggestion.
\$416	= 41600 cents.	This result may be reached by simply removing the separatrix.
17 cents =	17 cents.	
Ans.	41617 cents.	

Ex. 2. Reduce \$25.09,8 to mills. Ans. 25098 mills.

Explanation.		Suggestion.
\$25	= 25000 mills.	This result may be reached by simply removing the separatrix and comma.
9 cents =	90 mills.	
8 mills =	8 mills.	
Ans.	25098 mills.	

Rules.—1. *To reduce dollars and cents to cents, remove the separatrix.*

2. *To reduce dollars and cents to mills, remove the separatrix and annex one cipher.*

3. *To reduce dollars, cents, and mills to mills, remove both points.*

Questions.—61. What is reduction of Federal Money? What is its first case? How are cents reduced to mills? Dollars to cents? Dollars to mills? How do we reduce dollars and cents to cents? To mills? Dollars, cents, and mills to mills?

EXAMPLES FOR PRACTICE.

3. How many cents in $675? How many mills?
4. How many mills in $0.25?
5. How many cents in $698.07? How many mills?
6. How many mills in $144.00,4?

CASE II.

To reduce to a higher denomination.

RULES—1. *Mills become cents by dividing by* 10, *or cutting off one right-hand figure.*

2. *Mills become dollars by dividing by* 1000, *or cutting off three right-hand figures.*

3. *Cents become dollars by dividing by* 100, *or cutting off two right-hand figures.*

EXAMPLES FOR PRACTICE.

1. How many cents in 195284 mills? 43062 mills? 475063 mills?
2. How many dollars in 62114 mills? 108243 mills? 261182 mills?
3. How many dollars in 118117 cents? 642023 cents? 161252 cents?

Art. 62. ADDITION OF UNITED STATES MONEY.

RULE.—*Write dollars under dollars, cents under cents, and mills under mills.*

Add as in simple addition, and place the separatrix in the amount directly under that in the numbers added.

EXAMPLES FOR PRACTICE.

(1.)	(2.)	(3.)	(4.)
$ cts. m.	$ cts. m.	$ cts. m.	$ cts. m
28.13,5	75.87,5	450.50	999.09,9
19.05	8.25	84.98,8	888.08,8
4.00,3	63.95	9.09,9	777.07,7
59.62,5	27.03	37.37,6	666.06,6
$110.81,3	$175.10,5	$581.96,3	$3330.33,

QUESTIONS.—61. How do we reduce mills to cents? Mills to dollars? Cents to dollars?—62. What is the rule for addition of United States Money?

5. E. Ross bought a stock of groceries for $347.50; dry goods for $823.12; paint for $37.87,5; cutlery for $62.65; crockery for $125.37,5; and stationery for $15.88. What was the amount of his purchases? Ans. $1412.40.

6. R. Davis sold in six days goods to the following amounts: $117.12,5; $84.05; $159.09,5; $104.15; $93.18; and $114.44,3. What was the amount of the week's sales? Ans. $672.04,3.

7. L. Carter's monthly expenses for one year were $63.25: $81.54; $59.78,5; $94.06,25; $61.11; $49.33,3; $72.75; $67.10,25; $57.13; $75.27; 88.66,6; and $60.27,9. How much did he spend that year? Ans. $830.27,8.

8. N. Hale gave 12½ cents for paper, 25 cents for a slate, $1.12½ for a book, $0.62½ for drawing-pencils, and 37½ cents for ink. How much did he give for all? Ans. $2.50.

Art. 63. SUBTRACTION OF UNITED STATES MONEY.

RULE.— *Write the less number under the greater, dollars under dollars, cents under cents, and mills under mills.*

Subtract as in simple subtraction, and place the separatrix in the remainder directly under that in the numbers above.

EXAMPLES FOR PRACTICE.

	(1.)	(2.)	(3.)
	$ cts. m.	$ cts. m.	$ cts.
Minuend	892.42	201.81,2	475.75
Subtrahend	97.64,5	154.95,5	87.88
Remainder	$794.77,5	$46.85,7	$387.87

4. Starting with $75 in my pocket, I spent in one store $9.63, in another $15.84,5, in another $27.42, and in another $8.37,5. How much had I left? Ans. $13.73.

5. From five cents take five mills. Ans. $0.04,5.

6. From one hundred dollars take one cent and one mill. Ans. $99.98,9.

7. From ninety-nine dollars, take nine dollars nine cents and nine mills. Ans. $89.90,1.

8. S. Leonard's effects were valued at $68125, and his debts were $15175.37,5. What was he worth?

Ans. $52949.62,5.

QUESTION.—63. What is the rule for subtraction of United States Money?

9. Bought cloth for \$8.62,5, trimmings for \$2.25, and gloves for 87½ cents. Offered a double eagle. How much change should I receive? Ans. \$8.25.

10. What is the difference between 235675 mills, and 23567 mills? Ans. \$212.10,8.

Art. 64. MULTIPLICATION OF UNITED STATES MONEY.

Ex. 1. What cost 9 hats, at \$3.375 apiece? Ans. \$30.375.

Written Process.	Mental Process.
\$ 3.375 9 \$30.375	If 1 hat costs \$3.375, or 3375 mills, 9 hats will cost 9 times 3375 mills; that is, 30375 mills. This reduced to dollars, by Art. 61, Case II., becomes \$30.375.

Rule.—*Multiply as in simple numbers. The product is in the lowest denomination of the multiplicand, and should be reduced to higher denominations as the case requires.*

Note 1.—To obtain the cost of several things, multiply the price of one thing by the number of things.

Note 2.—Since 100 cents make a dollar, when one factor is a convenient part of a dollar, we may often lessen labor, as in Case I., Art 50.

Examples for Practice.

2. What cost 3 cows, at \$35.375 apiece? Ans. \$106.125.
3. What cost 4 pairs of boots, at \$4.625 a pair? Ans. \$18.50.
4. What cost 5 gallons of molasses, at 37½ cents a gallon? Ans. \$1.875.
5. What cost 6 pounds of butter, at \$0.1875 per pound? Ans. \$1.125.
6. What cost 7 bushels of corn, at 35 cents a bushel? Ans. \$2.45.
7. What cost 8 tons of pig-iron, at \$40.50 a ton? Ans. \$324.
8. What are the united wages of 68 men, at \$1.25 per day? Ans. \$85.
9. A speculator bought 1896 barrels of flour, at \$4.50 per

Question.—64. What is the rule for multiplication of United States Money?

barrel, 2545 bushels of wheat, at 87½ cents per bushel, and 1575 bushels of oats, at 37½ cents per bushel. He sold the flour at $6.25 per barrel, the wheat at $1.18 per bushel, and the oats at 31 cents per bushel. How much did he make?

Ans. $3991.85.

BILLS.

Art. 65. A BILL is a written statement of things bought, given by the seller to the buyer.

A bill is *receipted* when the signature of the seller is put to the statement that payment has been received.

Carry out the cost of the items, and verify the answer in each of the following

EXAMPLES.

(1.) *Baltimore, January* 11, 1859.

J. C. BRADFORD, *Esq.*,

Bought of AMES & FISHER,

5	*Sofas*,	@	$44.50	$222.50
24	*Chairs*,	"	5.25	126.00
12	"	"	.875	10.50
2	*Mirrors*,	"	35.375	70.75
	Received payment,			$429.75

AMES & FISHER.

(2.) *Cleveland, Dec.* 22, 1858

Mr. N. B. MARSHALL,

Bought of A. M. WHITMORE & Co.,

108	*gallons Oil*,	@	$ 1.125	
62	" *Alcohol*,	"	.75	
95	" " *Syrup*,	"	.625	
8	*chests Green Tea*,	"	23.75	
4	" *Black* "	"	8.375	
				$450.87,5
	Received payment,		A. M. WHITMORE & Co.	

QUESTIONS.—65. What is a bill? When is a bill receipted?

(3.) *Pittsburgh, March* 9, 1859.

Mr. D. R. CLINTON,

Bought of COOPER & LESLIE,

6 *sets Chisels,*	@	$9.25
35 *Hammers,*	"	0.625
24 *Saws,*	"	1.125
18 *sets Knives and Forks,*	"	5.75
65 *Shovels,*	"	0.625
12 *sets Fire-irons,*	"	9.875
18 *pairs Scissors,*	"	0.375
215 *pounds Chain,*	"	.07
		$388.80

Received payment, COOPER & LESLIE.

Art. 66. DIVISION OF UNITED STATES MONEY.

Ex. 1. If 45 pounds of coffee cost $5.625 what did 1 pound cost? Ans. $0.12,5.

WRITTEN PROCESS.

```
45)5.625(125
   45
  ---
  112
   90
  ---
  225
  225
  ---
```

MENTAL PROCESS.

If 45 pounds cost $5.625, that is, 5625 mills, 1 pound will cost $\frac{1}{45}$ of 5625 mills, that is, 125 mills, equal to $0.12,5.

Ex. 2. B. Lane paid $20 for 275 pounds of sugar: how much was that a pound? Ans. $0.07,2+.

WRITTEN PROCESS.

```
275)20.000(0.072+
   1925
  ----
    750
    550
  ----
    200
```

MENTAL PROCESS.

One pound cost $\frac{1}{275}$ of $20. Now, since 20 does not contain 275, it is plain that 1 pound will cost less than $1. Hence, if we reduce $20 to mills, the quotient will be in mills. It is 72 mills and a remainder, which is indicated by +. This reduced to dollars is $0.07,2+.

QUESTIONS.—66. What is the rule for division of United States Money? How do we find the price of one thing? How do we find the number of things?

Ex. 3. At 9 cents a yard, how many yards of cotton cloth can I buy for $8.375? Ans. $93\frac{5}{9}$ yards.

Written Process.

$$
\begin{array}{r}
90)8375(93\frac{5}{9}\\
\underline{810}\\
275\\
\underline{270}\\
5
\end{array}
$$

Mental Process.

Nine cents = 90 mills, and $8.375 = 8375 mills. There will be as many yards as 90 mills are contained times in 8375 mills; that is, $93\frac{5}{9}$ yards.

Rule.—*When the dividend represents money, and the divisor does not, reduce the dividend to cents or mills, if it is too small to contain the divisor, and divide as in simple numbers. The quotient is in the lowest denomination of the dividend, and should be reduced to higher denominations if the case requires it.*

When the divisor and dividend both represent money, they should both be reduced to the lowest denomination mentioned in either of them, and then treated as simple numbers.

Note 1.—To obtain the price of one thing, divide the *cost* of the given number of things by their number.

Note 2.—To find the number of things, divide the cost of all of them by the price of one.

Examples for Practice.

4. Gave $60 for 96 pounds of black tea; how much was that per pound? Ans. $0.62,5.

5. Gave $9 for 48 pounds of butter; find the price per pound. Ans. $0.18,75.

6. At 15 cents per yard for calico, how many yards can be bought for $5.25? Ans. 35 yards.

7. At $3.375 per yard for broadcloth, how many yards can be bought for $151.875? Ans. 45 yards.

8. A certain office brought its incumbent $42575 in one year: how much was that per day? Ans. $116.64,3+.

9. H. Gibson's daily receipts in the store for a week were $37.75, $42.875, $55.14, $49.95, $34.19, and $61.88. What was the average per day? Ans. $46.96,4$\frac{1}{8}$.

10. How many hands, at an average of $1.25 wages per day, will consume $187.50 per day? Ans. 150 hands.

MISCELLANEOUS EXERCISES IN UNITED STATES MONEY.

1. What are the sums of the following counting-room columns?

(1.) $ cts	(2.) $ cts. m.	(3.) $ cts.
48.65	321.125	2425.
4.05	18.04	125.
321.98	6.27	66.25
27.15	401.012	232.37
8.00	214.14	45.92
15.23	75.75	25.25
72.42	22.375	13.39
145.50	16.625	10.21
83.75	——— 1075.337	12.
19.02	308.83	250.
20.00	452.13	108.
6.67	984.875	11.05
205.91	4.50	8.08
3.03	2.18	16.
39.01	10.	97.14
55.25	550.	559.50
10.10	63.625	64.
9.04	——— 2376.14	18.25
426.60	12.08	4.80
13.33	9.90	33.
49.66	385.65	265.
8.83	28.02	42.16
70.27	240.50	88.
6.05	180.17	175.75
14.21	14.25	52.
6.50	5.75	1.25
.25	3.03	10.50
284.42	——— 879.35	486.28
19.15	28.63	14.10
1.	7.05	9.88
10.	.95	.44
.88	564.18	95.25
9.91	22.75	8.19
487.63	8.373	.50
25.	15.42	4.38
	5.10	
33754	——— 652.453	
	$4983.28	

4. Bought 17 lemons, at 2½ cents apiece, 9 pounds of almonds, at 23 cents a pound, and 25 pounds of raisins, at 16 cents a pound. What was the cost of the whole? Ans. $6.49,5.

5. If one pound of coffee costs 12½ cents, how much will 56 pounds cost? Ans. $7.

6. If A earns $60 a month, and spends $35.75 of it, how much will he save in a year? Ans. $291.

7. A farmer sold his marketing for $16.875, and bought 8 pounds of coffee at 13 cents a pound, 3 pounds of tea at 62½ cents a pound, half a pound of spice at 75 cents a pound, and 45 pounds of sugar at 8 cents a pound. How much had he left? Ans. $9.985.

8. B. Hall employed in harvest 9 hands 10 days at $1.625 per day, and it cost him 33 cents a day to "keep" each hand. His harvest sold for $983.16: how much did he clear?

Ans. $807.21.

9. At 325 cents per cord, how many cords of wood will $139.75 buy? Ans. 43 cords.

10. Sold 84 thousand feet of pine boards at $18.50 per thousand, and took pay in wheat at 75 cents per bushel; how many bushels of wheat did I receive? Ans. 2072 bushels.

11. C's estate, valued at $84720, was divided equally among 5 children, after his wife had taken one-third; how much did each receive? Ans. Wife, $28240. Each child, $11296.

12. A railroad 243 miles long cost $10404336.60. What did it average per mile? Ans. $42816.20.

13. S. March bought 3 chests of black tea, each containing 45 pounds, at 35 cents a pound, and 9 chests of green tea, each containing 72 pounds, at 56 cents a pound. What was the average cost of his teas per pound? Ans. $0.52+.

14. If a man's average annual earnings, from 21 years of age to 55, were $1125.66, what average per year would that be for his whole life, ending at 55 years? Ans. $695.86,2+.

15. Wm. Eaton sold 63 acres of land, at $18.75 per acre; 31 acres at $25.62,5 per acre; 124 acres at $40 per acre; 80 acres at $37.50 per acre; 160 acres at $22.50 per acre; and 120 acres at $15 per acre. What price per acre did he average?

Ans. $26.53,2+.

16. David Fisk began business one day with $50.75 in the drawer. He first received $8.12½, then paid out $4.25, then received $15.37½, then paid 12½ cts., then received $21.91, then $11.42, then paid $5.05, then $1, and closed. How much had he then, and what was his gain?

Ans. Had $97.15,5; Gain, $46.40,5.

CHAPTER VIII.

REDUCTION.

Art. 67. A DENOMINATE number is a concrete number expressed in *measuring* units.

Denominate numbers, in reference to the number of denominations expressed in them, are *simple* or *compound*.

A *simple* denominate number is one that is expressed in measuring units of one kind or denomination.

A *compound* number is a denominate number expressed in measuring units of more than one kind or denomination.

Thus the expression 2 *feet* 7 *inches*, taken together, is a compound number, because it expresses length in more than one kind of measuring unit.

Art. 68. REDUCTION is converting a concrete number expressed in measuring units of one or more kinds, into a number expressing the same value in other measuring units. It is of two kinds,—viz.: *descending* and *ascending*.

Descending Reduction is converting a number into another whose measuring units are of less value than its own; as *gallons* to *pints*. It is performed by multiplication.

Ascending Reduction is converting a number into another whose measuring units are of greater value than its own; as *inches* to *miles*. It is performed by division.

ENGLISH MONEY

Art. 69. English or Sterling Money is the legal currency of England. Its measuring units or denominations are *pounds*, *shillings*, *pence*, and *farthings*.

QUESTIONS.—67. What is a denominate number? How are denominate numbers classified? What is a simple denominate number? What is a compound number?—68. What is Reduction? Of what kinds is it? What is descending Reduction? How is it performed? What is ascending Reduction? How is it performed?—69. What is English Money? What are its units?

TABLE.

4 farthings (qr. or f.)	make	1 penny,	(d.)
12 pence	"	1 shilling,	(s.)
20 shillings	"	1 pound or sovereign.	(£)

f.		d.		s.		£		$ cts. m.
4	=	1					=	$0.02,0\frac{1}{6}$
48	=	12	=	1			=	0.24,2
960	=	240	=	20	=	1	=	4.84

NOTE 1.—Other denominations sometimes used. 21 shillings make 1 *guinea;* 2 shillings make 1 *florin;* 5 shillings make 1 *crown.*

NOTE 2.—Farthings are often expressed as *quarters* of a penny; thus, 1 far. = $\frac{1}{4}$d.; 2 far. = $\frac{1}{2}$d.; 3 far. = $\frac{3}{4}$d.

DESCENDING REDUCTION.

Art. 70. To reduce a number to a lower denomination.

Ex. 1. In 14£ how many pence? Ans. 3360 pence

WRITTEN PROCESS.

14
20

280
12

3360

MENTAL PROCESS.

If 1£ is 20 s., 14£ will be $14 \times 20 = 280$s
If 1s. is 12d., 280s. will be $280 \times 12 = 3360$d.

Ex. 2. In 29£ 7s. 4d. 3qr. how many farthings?

Ans. 28195.

WRITTEN PROCESS.

29£ 7s. 4d. 3qr.
20
---
587 shillings.
12

7048 pence
4

28195 farthings.

MENTAL PROCESS.

In 29£ 7s. there are 29×20 shillings + 7 shillings = 587s. In 587s. 4d. there are 587×12 pence + 4 pence = 7048 d. In 7048d. 3qr. there are 7048×4 farthings + 3 farthings = 28195qrs.

QUESTIONS.—69. What is the table of English Money?—70. How do we reduce to a lower denomination?

RULE.—*Multiply the highest denomination given, by that number of the next lower denomination which makes one of the denomination multiplied. If the number to be reduced is compound, add to this product that part of the number which is of the same denomination as the product.*

Convert this result in like manner to the next lower denomination, and so on till the required denomination is reached.

ASCENDING REDUCTION.

Art. 71. To reduce a number to a higher denomination.

Ex. 1. Convert 28195 farthings to pounds.

Ans. 29£ 7s. 4d. 3qr.

WRITTEN PROCESS.

4) 28195qrs.

12) 7048d. 3qrs.

20) 587s. 4d.

29£ 7s.

MENTAL PROCESS.

In 28195qrs. there are as many pence as 4qrs., which make 1 penny, are contained times in 28195qrs.; that is, 7048d. and 3qrs. remain. In 7048d. there are as many shillings as 12d., which make 1s., are contained times in 7048d; that is, 587s. and 4d. remain. In 587 shillings there are as many pounds as 20s., which make 1£, are contained times in 587s.; that is, 29£ and 7s. remain. Since the remainders are such parts of 28195qrs. as are not reduced to pounds, they, united with the 29£, express the whole value of 28195 farthings.

RULE.—*Divide the given denomination by that number of it which makes one of the next higher. Convert this quotient in like manner to the next higher denomination, and so on till the required denomination is reached. The answer is composed of the last quotient and all the remainders.*

NOTE.—Each remainder must be marked in the answer as of the same denomination with that dividend from which it came.

EXERCISES IN ENGLISH MONEY.

Reduce	Reduce
1. 4£ 9s. 7d. 3f. to qrs.	2. 4303f. to pounds.
3. 12£ 0s. 3d. 2f. to qrs.	4. 11534f. to pounds.
5. 855£ to pence.	6. 205200d. to pounds.
7. 63£ 15s. 11d. to pence.	8. 15311d. to pounds.
9. 75£ 13s. to qrs.	10. 72624qrs. to pounds.
11. 16 guineas to pence.	12. 4032d. to guineas.
13. 629s. to qrs.	14. 30192qrs. to shillings.

QUESTIONS.—71. How do we reduce to a higher denomination? How must each remainder be marked?

LONG, OR LINEAR MEASURE.

Art. 72. Linear Measure is used in estimating and expressing distance.

TABLE.

12 inches (in.)	make	1 foot,	(ft.)
3 feet	"	1 yard,	(yd.)
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet	"	1 rod, pole, or perch,	(rd.)
40 rods	"	1 furlong	(fur.)
8 furlongs, or 320 rods	"	1 mile,	(m.)
3 miles	"	1 league,	(lea.)

in.	ft.	yd.	rd.	fur.	m.	lea.
12 =	1					
36 =	3 =	1				
198 =	$16\frac{1}{2}$ =	$5\frac{1}{2}$ =	1			
7920 =	660 =	220 =	40 =	1		
63360 =	5280 =	1760 =	320 =	8 =	1	
190080 =	15840 =	5280 =	960 =	24 =	3 =	1

NOTE 1.—Other denominations sometimes used are, 12 lines make 1 inch; 3 inches make 1 palm; 4 inches make 1 hand; 9 inches make 1 span; 18 inches make 1 cubit; 3 feet make 1 pace; 6 feet make 1 fathom.

NOTE 2.—The standard unit of linear measure adopted by the United States Government is *the yard*, which is the same length as the imperial yard of Great Britain. The mile of the United States is the same as that of Great Britain.

NOTE 3.—For estimating certain kinds of work, the foot is divided *duodecimally* in succession; thus,

12 inches or primes (′)	make	1 foot,
12 seconds (″)	"	1 inch,
12 thirds (‴)	"	1 second,
12 fourths (⁗)	"	1 third.

EXERCISES.

1. Reduce 5lea. 2m. 7fur. 34rds. 11ft. 9in. to inches.

QUESTIONS.—72. What is linear measure? Repeat the table. What other denominations are sometimes used? What is the standard of the United States? What is the mile of the United States? Repeat the table of duodecimals.

WRITTEN PROCESS.

5lea. 2m. 7fur. 34rds. 11ft. 9in.
3

17 m.
8

143 fur.
40

2)5754 rds.
$16\frac{1}{2}$

2877
34524
5754
11

94952 ft.
12

1139433 inches.

2. Reduce 1139433 inches to leagues.

WRITTEN PROCESS.

12)1139433

$16\frac{1}{2}$)94952—9 in.
2 2

33)189904

40)5754 — $\frac{22}{2}$ = 11 ft.

8)143 — 34 rds.

3)17 — 7 fur.

5 — 2 m

Ans. 5lea. 2m. 7fur. 34rds. 11ft. 9in.

NOTE.—To multiply by $16\frac{1}{2}$, first take $\frac{1}{2}$ of the multiplicand; this multiplies by $\frac{1}{2}$; then multiply by 16, and add these products.

NOTE.—To divide by $16\frac{1}{2}$, first reduce both dividend and divisor to halves, and then divide. As the remainder 22 is half feet, it is divided by 2 to get the true remainder in feet.

3. 69m. 1fur. 13rd. 5ft. 6in. to inches.
4. 4382400in. to miles.
5. 18m. 19rd. to feet.
6. 95353$\frac{1}{2}$ft. to miles.
7. 9ft. 7′ 8″ 11‴ to thirds.
8. 16667‴ to feet.
9. 4621m. 7fur. 37rd. 12ft. 9in. to inches.
10. 292849479in. to miles.

ENGINEERS' AND SURVEYORS' MEASURE.

Art. 73. Engineers' and Surveyors' Measure is a kind of linear measure used in surveying for roads, boundaries of land, &c.

QUESTIONS.—72. How do we multiply by $16\frac{1}{2}$? How do we divide by $16\frac{1}{2}$? 73. What is Engineers' and Surveyors' measure?

TABLE.

$7\frac{92}{100}$ inches (in.)	make	1 link,	(l.)
25 links	"	1 pole or rod,	(p.)
4 poles, or 66 feet	"	1 chain,	(ch.)
10 chains	"	1 furlong,	(fur.)
8 furlongs or 80 chains	"	1 mile,	(m.)

in.	l.	p.	ch.	fur.	m.
$7\frac{92}{100}$ =	1				
198 =	25 =	1			
792 =	100 =	4 =	1		
7920 =	1000 =	40 =	10 =	1	
63360 =	8000 =	320 =	80 =	8 =	1

NOTE.—A surveyor's chain is called Gunter's chain, and is composed of 100 links, the chain being 4 rods, or 66 feet, long.

Engineers sometimes use a chain 100 feet long, containing 120 links, each 10 inches long.

EXERCISES.

Reduce	Reduce
1. 44ch. 3p. 19l. to links.	2. 4494l. to chains.
3. 6m. 61ch. 3p. to poles.	4. 2167p. to miles.
5. 17ch. 23l. to links.	6. 1723l. to chains.

CLOTH MEASURE.

Art. 74. Cloth Measure is a kind of linear measure used in measuring dry goods.

TABLE.

$2\frac{1}{4}$ inches (in.)	make	1 nail,	(na.)
4 nails, or 9 inches	"	1 quarter of a yard,	(qr.)
4 quarters	"	1 yard,	(yd.)

in.	na.	qr.	yd.
$2\frac{1}{4}$ =	1		
9 =	4 =	1	
36 =	16 =	4 =	1

NOTE.—Other denominations sometimes used are, 3 qrs. make 1 Flemish ell; 4 qrs. $1\frac{1}{2}$ in. make 1 Scotch ell; 5 qrs. make 1 English ell; 6 qrs. make 1 French ell.

QUESTIONS.—73. Repeat the table. Describe the surveyors' chain.—74. What is cloth measure? Repeat the table. What other denominations are sometimes used?

Exercises.

Reduce	Reduce
1. 31yd. 3qr. 2na. to nails.	2. 510na. to yards.
3. 419¾yds. to quarters.	4. 1679qrs. to yards.

SURFACE OR SQUARE MEASURE.

Art. 75. Square Measure is used in estimating and expressing quantity of surface. It is so called because the unit used in expressions of surface is a *square*. A square is a plane surface contained by four equal sides, each perpendicular to its adjacent sides.

A square takes its name from the length of its side. Thus, a *square inch* is a square each of whose sides is an *inch long;* a *square foot* is a square whose side is a *foot* long, &c. A surface inclosed by lines is called an *area;* and the number expressing its quantity is called its *area*.

Table.

144 square inches (sq. in.) make 1 square foot, (ft.)
9 square feet " 1 square yard, (yd.)
30¼ square yards, or 272¼ square feet " 1 square rod, or pole, or perch, (P.)

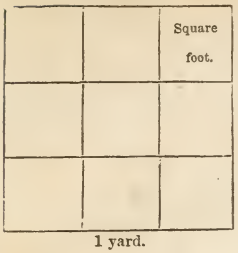
Also, for measuring the surface of land,
40 square rods, or poles, (perches,) make 1 Rood, (R.)
4 Roods (or 10 square chains) " 1 Acre, (A.)
640 Acres " 1 square mile, (sq. mi.)

sq. in.	ft.	yd.	P.	R.	A.	sq. mi.
144 =	1					
1296 =	9 =	1				
39204 =	272¼ =	30¼ =	1			
1568160 =	10890 =	1210 =	40 =	1		
6272640 =	43560 =	4840 =	160 =	4 =	1	
4014489600 =	27878400 =	3097600 =	102400 =	2560 =	640 =	1

Note 1.—Any four-sided surface, whose adjacent sides are perpendicular to each other, contains as many *square units* as is the product of similarly named linear units in its length multiplied into those in its breadth.

Questions.—75. What is square measure? Why so called? What is a square? Whence does a square take its name? Repeat the table. How many square units in a four-sided surface of perpendicular sides?

ILLUSTRATION.



In this figure, if the small squares represent each a *square foot*, the side of each is a *linear foot;* and the continuous sides of three of them would make three *linear feet* or one *linear yard.* It would also take three rows to make a *linear yard* in breadth; that is, a square 1 yard long and 1 yard broad, or a *square yard.* Hence, the square feet in a square yard are 3 times 3 = 9. In the same way it can be shown that 144 sq. in. make 1 sq. ft., &c.

NOTE 2.—A surface a certain number of *units square* is not the same number of *square units;* thus a surface 3 *feet square* contains 9 *square feet.* But a surface 1 unit square is 1 square unit of the same name. Thus a surface 1 foot square is 1 square foot.

NOTE 3.—Let the pupil illustrate upon his slate, or the blackboard, every other statement in the table.

EXERCISES.

1. Reduce 11A. 2R. 37P. 198ft. 19in. to square inches.

WRITTEN PROCESS.

11A. 2R. 37P. 198ft. 19in.
4
46 roods.
40
1877 poles.
272¼ NOTE.—To multiply by ¼, divide the multiplicand by 4.
469¼
3754
13139
3754
198 = feet given to be added in.
511211¼ feet.
144
36 NOTE.—To multiply 144 by ¼, divide it by 4.
2044844
2044844
511211
19 = inches given to be added in.
73614439 inches. Ans.

QUESTION.—75. Repeat and explain Note 2.

2. Reduce 73614439 sq. in. to acres.

WRITTEN PROCESS.

144) 73614439

272¼) 511211 feet + 55 inches.
4 4

1089) 2044844

40) 1877 poles + $\frac{72}{4}$ = 197¾ feet.

4) 46R. + 37 poles.

11A + 2 roods.

Ans. 11A. 2R. 37P. 197¾ft. 55in.

NOTE.—This answer is the same *value* as that form of it in Ex. 1, and can be changed to that form by taking a sufficient number of inches (36) from 55in. to make up the ¼ft. lacking to make 198ft. This leaves 19in. and fills up 197¾ft. to 198ft.

3. How many square feet in a township 6 *miles square?*

4. How many square miles in 1003622400 square feet?

5. How many perches in a tract of land 10 miles square?

6. How many square miles in 10240000 perches?

7. How many square feet in a lot 8 rods long and 5 rods broad? Ans. 10890.

8. In buying a city lot 25 feet front and 147 feet deep, the purchaser agreed to cover it with bank-bills of the denomination of $5; how much did he pay for his lot, if the bills were 7 inches long and 3 inches broad? Ans. $126000.

SOLID OR CUBIC MEASURE.

Art. 76. Solid or Cubic Measure is used in estimating and expressing the bulk or volume of bodies.

It is called cubic-measure because the unit used in expressions of volume or bulk is *a cube.*

A *cube* is a body having six surfaces, which are squares.

The length, breadth, and height of a cube are equal.

A cube takes its name from the length of one of its edges Thus, a *cubic inch* is a cube each of whose edges is an inch long; a *cubic foot* is a cube each of whose edges is a foot long, &c

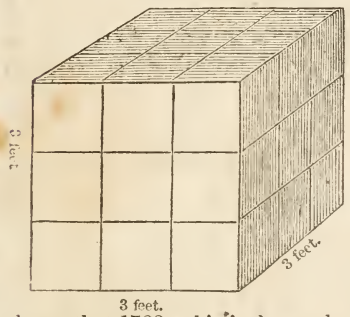
QUESTIONS.—76. What is solid or cubic measure? Why so called? What is a cube? Whence does a cube take its name?

TABLE.

1728 cubic inches (cu. in.)	make 1 cubic foot,	(cu. ft.)
27 cubic feet	" 1 cubic yard,	(cu. yd.)

NOTE 1.—Any six-sided body, whose adjacent edges are perpendicular to each other, contains as many cubic units as is the product of similarly named linear units in its length, multiplied into those in its breadth, into those in its thickness.

ILLUSTRATION.



In this figure, if each small block represents *a cubic foot*, the edge of each is a *linear* foot; and the continuous edges of three of them would make a linear yard. It would also take three rows of blocks, that is, 9 blocks, to cover the base-surface. Finally, it would take 3 such layers, of 9 blocks each, to make the whole 1 yard high. Hence the cubic feet in a cubic yard are 3 times 3 times 3 = 27. In the same way it can be shown that 1728 cubic inches make 1 cubic foot.

NOTE 2.—A *cord* of wood is 128 cu. ft., and is conveniently represented by a pile 8 feet long, 4 feet wide, and 4 feet high.

A *cord foot* is that part of such a pile which makes 1 foot of its length; being 1 foot × 4 feet × 4 feet = 16 cu. ft. Hence 8 cord feet make 1 cord.

NOTE 3.—A load or *tun* of rough timber is such a quantity as would, if hewed, yield 40 cu. ft. A load or tun of square timber is 50 cu. ft. A ton of shipping is 42 cu. ft.

NOTE 4.—A perch of masonry is 24¾ cu. ft., and is conveniently represented by a mass 1 rod (16½ feet) long, 1 foot high, and 1½ feet thick. This is regarded as 25 cu. ft. in practice.

EXERCISES.

1. What is the volume, in cubic feet, of a room 15 feet long, 11 feet wide, and 9 feet high? Ans. 1485 cu. ft.

2. How many cords in a pile of wood 16 rods long, 4 feet wide, and 4 feet high? Ans. 33 cords.

3. How many cubic inches in a box 4½ feet long, 3 feet wide, and 2 feet high? Ans. 46656 cu. in.

QUESTIONS.—76. Repeat the table. How many cubic units are there in any six-sided body of perpendicular edges? Prove it. What is a cord of wood? A cord foot? A ton of timber? A perch of masonry?

4. How many cubic yards in 438048 cu. in.?

Ans. 9 cu. yd. 10 cu. ft. 864 cu. in.

5. How many cubic yards of earth were taken from a cellar 24 feet long, 18 feet wide, and 6 feet deep?

Ans. 96 cubic yards.

6. How many cubic feet in 7 cords 3 cord feet?

Ans. 944 cubic feet.

TROY WEIGHT.

Art. 77. Troy Weight is used in weighing coins, precious metals, jewels, and liquors. It is much used in philosophical experiments.

TABLE.

24 grains (gr.)	make	1 pennyweight,	(pwt.)
20 pennyweights	"	1 ounce,	(oz.)
12 ounces	"	1 pound,	(lb.)

grs.		pwt		oz.		lb.
24	=	1				
480	=	20	=	1		
5760	=	240	=	12	=	1

NOTE 1.—The standard unit of weight adopted by the United States Mint is the *Troy Pound*, of 5760 grains. It is the same as the imperial Troy pound of Great Britain. It is equal to the weight of $22\frac{7944}{10000}$ cubic inches of distilled water, at $39\frac{83}{100}$ degrees of Fahrenheit's thermometer, the barometer standing at 30 inches.

NOTE 2.—Precious stones are weighed by *Diamond Weight*, as follows:—

16 parts make 1 carat grain,	=	$\frac{4}{5}$ Troy grain.
4 carat grains 1 carat,	=	$3\frac{1}{5}$ Troy grains.

EXERCISES.

Reduce	Reduce
1. 3lbs. 14pwts. 17grs. to grs.	2. 17633grs. to lbs.
3. 10lbs. 10oz. 21grs. to grs.	4. 62421grs. to lbs.
5. 16pwts. 9grs. to grs.	6. 393grs. to pwts.
7. 75 lbs. to pwts.	8. 18000pwts. to lbs.
9. 362oz. to grs.	10. 173760grs. to oz.
11. 8lbs. 9oz. 10pwts. 11grs. to grs.	12. 50651grs. to lbs.

QUESTIONS.—77. What is Troy weight? For what is it used? What is the United States standard of weight? How are precious stones weighed?

APOTHECARIES' WEIGHT.

Art. 78. Apothecaries' Weight is used in compounding medical prescriptions.

TABLE.

20 grains (gr.)	make	1 scruple,	(℈)
3 scruples	"	1 dram,	(ʒ)
8 drams	"	1 ounce,	(℥)
12 ounces	"	1 pound,	(℔)

grs.		℈		ʒ		℥		℔
20	=	1						
60	=	3	=	1				
480	=	24	=	8	=	1		
5760	=	288	=	96	=	12	=	1

In this weight the grain, ounce, and pound are the same as in Troy weight.

EXERCISES.

Reduce	Reduce
1. 4℔ 7℥ 5ʒ 2℈ 17grs. to grs.	2. 26757grs. to ℔s.
3. 9℥ 1℈ to grs.	4. 4340grs. to ℥
5. 5ʒ 15 grs. to grs.	6. 315grs. to ʒ
7. 8℔ 2℈ to ℈	8. 2306℈ to ℔s.

AVOIRDUPOIS WEIGHT.

Art. 79. Avoirdupois Weight is used in estimating the weight of almost all articles taken in large quantity. It is used in ordinary commercial transactions with groceries, drugs, the baser metals, &c.

TABLE.

16 drams (dr.)	make	1 ounce,	(oz.)
16 ounces	"	1 pound,	(lb.)
25 pounds	"	1 quarter,	(qr.)
4 quarters	"	1 hundredweight,	(cwt.)
20 hundredweight	"	1 ton,	(T.)

drs.		oz.		lb.		qr.		cwt.		T.
16	=	1								
256	=	16	=	1						
6400	=	400	=	25	=	1				
25600	=	1600	=	100	=	4	=	1		
512000	=	32000	=	2000	=	80	=	20	=	1

QUESTIONS.—78. What is apothecaries' weight? Repeat the table.—79. What is avoirdupois weight? Repeat the table. What is the standard of avoirdupois weight in the U.S.? Compare Troy with avoirdupois weight.

NOTE 1.—The standard avoirdupois pound of the United States is the weight of $27\frac{7015}{10000}$ cubic inches of distilled water, at $39\frac{83}{100}$ degrees of Fahrenheit's thermometer, the barometer standing at 30 inches. It is the same as the imperial avoirdupois pound of Great Britain.

NOTE 2.—The avoirdupois pound is heavier than the Troy or apothecaries' pound; but the Troy ounce is heavier than the avoirdupois ounce, and the dram of apothecaries' weight is heavier than the dram of avoirdupois weight.

COMPARISON OF WEIGHTS.

1lb. avoir.	= 7000 Troy grs.	=	1lb. 2oz. 11pwt. 16grs. Troy.
1lb. Troy or ap.	= 5760 " "	=	13oz. $2\frac{144}{175}$ dr. av.
1oz. Troy or ap.	= 480 " "		
1oz. av.	= $437\frac{1}{2}$ " "		
1dr. ap.	= 60 " "		
1dr. av.	= $27\frac{11}{32}$ " "		

NOTE 3.—In Great Britain it is common to estimate the hundredweight at 112lbs., and the quarter at 28lbs. In this country the quarter is generally estimated at 25lbs., the cwt. at 100 lbs., and the ton at 2000 lbs.

EXERCISES.

Reduce	Reduce
1. 14T. 5cwt. 3qr. 23lb. to lbs.	2. 28598lbs. to tons.
3. 9cwt. 17lbs. to drs.	4. 234752drs. to cwt.
5. 196T. 21lbs. to oz.	6. 6272336oz. to tons.

LIQUID OR WINE MEASURE.

Art. 80. Wine Measure is used in measuring the bulk of all kinds of liquids.

In some places beer, ale, and milk are not measured by wine measure, but by a measure of their own.

TABLE.

4 gills (gi.)	make	1 pint,	(pt.)			
2 pints	"	1 quart,	(qt.)			
4 quarts	"	1 gallon,	(gal.)	= 231 cu. in.		

gi.		pt		qt.		gal.
4	=	1				
8	=	2	=	1		
32	=	8	=	4	=	1

NOTE 1.—The following vessels are seldom used as units of measure. Casks being generally made without reference to accuracy of capacity, their contents are usually gauged and sold by the gallon. A *barrel*

QUESTIONS.—79. What is a cwt. in Great Britain? 80. What is wine measure? Repeat the table. What vessels are also described?

varies from 28 to 32 gallons, by the laws of the various States; 31½ gals. often make 1 barrel, (bbl.); 42 gals., 1 tierce; 63 gals., 1 hogshead (hhd.); 84 gals., 1 puncheon; 126 gals., 1 pipe or butt; 2 pipes, 1 tun. Also, in England, 10 gals. make 1 anker; 18 gals., 1 runlet.

NOTE 2.—The standard unit of liquid measure adopted by the United States Government is the English *Winchester wine gallon*, of the capacity of 231 cu. in.

NOTE 3.—The standard unit of liquid measure of Great Britain is the *imperial gallon*, of the capacity of $277\frac{274}{1000}$ cu. in. It was adopted in 1826.

NOTE 4.—Apothecaries' fluid measure is a subdivision of the Winchester wine gallon, for compounding medical prescriptions, as follows:—

60 minims (℥.)	make	1 fluidrachm,	(f℥)
8 fluidrachms	"	1 fluidounce,	(f℥)
16 fluidounces	"	1 pint,	(O.)
8 pints	"	1 gallon,	(Cong.)

O. signifies *octarius*, or one-eighth of a gallon; and cong. is for *congius*, the Latin for a gallon.

EXERCISES.

Reduce	Reduce
1. 27gal. 3qt. 1pt. 3gi. to gills.	2. 895gi. to gallons.
3. 9gal. 1pt. to gills.	4. 292gi. to gallons.
5. 2hhd. to cubic inches.	6. 29106 cu. in. to hhd.
7. 3cong. 7pt. 15f℥ to f℥.	8. 4088f℥ to cong.
9. 7qts. 1pt. to ℥.	10. 115200℥ to qts.

BEER MEASURE.

Art. 81. Beer Measure is sometimes used in measuring malt liquors and milk. It is seldom used in the United States.

TABLE.

2 pints (pt.)	make	1 quart,	(qt.)
4 quarts	"	1 gallon,	(gal.) = 282 cu. in.

NOTE 1.—In England 9 gals. make 1 firkin; 2 firkins, 1 kilderkin; 2 kilderkins, 1 bbl.

NOTE 2.—The English imperial gallon is of the same capacity for all liquids.

DRY MEASURE.

Art. 82. Dry Measure is used in measuring the bulk of

QUESTIONS.—80. What is the U.S. standard of liquid measure? What is that of Great Britain? What is apothecaries' fluid measure? Repeat the table.—81. What is beer measure? Repeat the table.—82. What is dry measure?

solid matters in a more or less separable form; as grains, fruits, coal, lime, &c.

TABLE.

2 pints (pt.)	make	1 quart	(qt.)		
8 quarts	"	1 peck	(pk.)		
4 pecks	"	1 bushel	(bu.)	= $2150\frac{42}{100}$ cu. in.	

pts.		qt.		pk.		bu.
2	=	1				
16	=	8	=	1		
64	=	32	=	4	=	1

NOTE 1.—The standard unit of Dry Measure, adopted by the Government of the United States, is the English *Winchester bushel*, a cylindrical vessel 8 in. deep and $18\frac{1}{2}$ in. in diameter. Its capacity is $2150\frac{42}{100}$ cu. in.

NOTE 2.—The standard unit of Dry Measure in Great Britain is the *imperial bushel*, having a capacity of $2218\frac{192}{100}$ cu. in. Hence the imperial *dry* gallon is the same as the imperial *liquid* gallon, viz., $277\frac{274}{1000}$ cu. in.

NOTE 3.—In England 8 bu. make 1 *quarter;* and 36 bu. of coal make 1 chaldron.

NOTE 4.—In Pennsylvania and some other States 2688 cu. in. *of bituminous coal* make 1 bushel.

EXERCISES.

Reduce	Reduce
1. 25bu. 3pk. 7qt. 1pt. to pints.	2. 1663pts. to bushels.
3 7bu. 4qt. to pints.	4. 456pts. to bushels.
5 2pk. 1pt. to pints.	6. 33pts. to pecks.
7. 12bu. to quarts.	8. 384qts. to bushels.

COMPARISON OF MEASURES.

1 U.S. wine gallon	= 231	cubic	inches.
1 " beer gallon	= 282	"	"
1 " dry gallon	= $268\frac{4}{5}$	"	"
1 Imperial gallon (both dry & liq.)	= $277\frac{274}{1000}$	"	"
1 U.S. wine quart	= $57\frac{3}{4}$	"	"
1 " beer quart	= $70\frac{1}{2}$	"	"
1 " dry quart	= $67\frac{1}{5}$	"	"
1 Imperial quart	= $69\frac{3185}{10000}$	"	"
1 U.S. bushel	= $2150\frac{42}{100}$	"	"
1 Imperial bushel	= $2218\frac{192}{100}$	"	"

From these numbers other denominations can be compared.

Repeat the table. What is the U. S. standard unit? The English standard? What other English denominations are mentioned?

ANGULAR OR CIRCULAR MEASURE.

Art. 83. Circular Measure is used in measuring angles by means of the subdivisions of the circumference of a circle.

A *plane angle*, called simply *an angle*, is the inclination of two straight lines which meet.

The point at which they meet is the *vertex* of the angle.

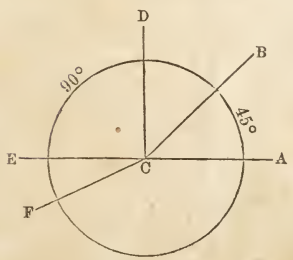
A *circle* is a plane figure bounded by a curved line, called the *circumference*, which is in every part equally distant from a point called the centre, in the same plane.

The inclination of two straight lines to each other is measured by describing a circumference from the vertex of the angle as a centre, and reckoning the number of equal parts of the circumference which are included between the lines. The divisions of the circumference are given in the following

TABLE.

60 seconds (″)		make		1 minute,		(′)
60 minutes		"		1 degree,		(°)
360 degrees		"		1 circumference,		(C.)
″		′		°		C.
60	=	1				
3600	=	60	=	1		
1296000	=	21600	=	360	=	1

ILLUSTRATION.



(*a*) The lines CE and CD, including *a quarter* of a circumference between them, have their inclination expressed by 90°, which is ¼ of 360°; and this angle is called a *right angle*. In this case the lines CE and CD are said to be perpendicular to each other.

(*b*) The lines CA and CB include ⅛ of a circumference between them, and their inclination is expressed by 45°, which is ½ of 360°. In this case the lines CA and CB are said to form an *acute angle* with each other. All angles less than 90° are *acute*.

QUESTIONS.—83. In what is circular measure used? What is an angle? A circle? How is an angle measured? Repeat the table. Illustrate a right angle. An acute angle.

(*c*) The lines CA and CF include more than a quarter of a circumference between them, and their inclination ACF is called an *obtuse* angle.

All angles greater than 90° and less than 180° are *obtuse*.

NOTE 1.—A degree, minute, or second is not of the same length in circles of different sizes, but each is still *the same part of a circumference*.

NOTE 2.—In astronomy, the circumference of the Zodiac is divided nto 12 equal parts, called *signs*, and 30° make 1 sign.

NOTE 3.—The earth's circumference on a meridian is about 24855 miles. This, divided by 360, gives $69\frac{1}{24}$ miles for the average length of a degree of latitude; $\frac{1}{360}$ of the circumference at the equator is about $69\frac{1}{7}$ miles.

EXERCISES.

Reduce	Reduce
1. 275° 39′ 48″ to seconds.	2. 992388″ to degrees.
3. 56° 17′ to seconds.	4. 202620″ to degrees.
5. 98° 27″ to seconds.	6. 352827″ to degrees.

MEASURE OF TIME.

Art. 84. The *natural* units of time are days and years. These have been artificially divided and arranged as in the following

TABLE.

60 seconds (sec.)	make	1 minute, (min.)
60 minutes	"	1 hour, (hr.)
24 hours	"	1 day, (da.)
7 days	"	1 week, (wk.)
12 calendar months	"	1 year, (yr.)
365 days (52 wk. 1 da.)	"	1 common year, (c. yr.)
366 days (52 wk. 2 da.)	"	1 leap-year.
365 da. 5hr. 48m. $49\frac{7}{10}$sec.	"	1 solar year.
100 years	"	1 century.

sec.	min.	hr.	da.	wk.	c. yr.
60 =	1				
3600 =	60 =	1			
86400 =	1440 =	24 =	1		
604800 =	10080 =	168 =	7 =	1	
31536000 =	525600 =	8760 =	365 =	$52\frac{1}{7}$ =	1

NOTE 1.—The names of the calendar months are January, February, March, April, May, June, July, August, September, October, November,

---

QUESTIONS.—83. Illustrate an obtuse angle. Is every degree of the same length? How is the Zodiac divided? What is the length of a degree of latitude?—84. What are the natural units of time? Repeat the table. What are the names and order of the calendar months?

December. The number of days in each month may be remembered by the aid of the following lines:—

Thirty days have September,
April, June, and November:
And all the rest have thirty-one,
Save February, which alone
Has twenty-eight; and we assign
To this, in leap-year, twenty-nine.

Or,

The fourth, eleventh, ninth, and sixth,
Have thirty days to each affixed:
All the rest have thirty-one,
Except the second month alone,
To which we twenty-eight assign,
Till leap-year gives it twenty-nine.

NOTE 2.—Every year, whose number is not divisible by 4 without a remainder, consists of 365 days: every year divisible by 4 but not by 100, consists of 366 days: every year divisible by 100 but not by 400, consists of 365 days; and every year divisible by 400, consists of 366 days. Years of 366 days are called *leap-years* or *bissextile years*.

The *Julian* calendar, instituted by Julius Cæsar B.C. 44, merely made a year of 366 days after three years of 365 days each, thus making the average of the years $365\frac{1}{4}$ days.

NOTE 3.—The true solar year is 5h. 48m. $49\frac{7}{10}$sec., or nearly 6 hours, longer than the civil year of 365 days. Hence, one year in four, another day is added to make up the deficiency. But, in allowing 6 hours for each year, an over-allowance of 11m. $10\frac{3}{10}$sec. is made. This, in 100 years, is about $\frac{3}{4}$ of a day too much. Hence, the hundredth year has only 365 days. But in this we drop about 6 hours too much; hence the 400th year is 366 days. This rule allows no larger error than a day in about 3875 years. It was instituted by Pope Gregory XIII., in A.D. 1582, to avoid the confusion produced by the imperfect Julian calendar. At the same time, the error of about 10 days, made by the Julian calendar, was corrected by reckoning the 5th of October, in the *Old Style*, as the 15th of October, in the *New Style*. This, called the *Gregorian* calendar, was adopted in England A.D. 1752, in Germany A.D. 1777, and in France A.D. 1805.

NOTE 4.—The earth turns from west to east $\frac{1}{24}$ of 360°; that is, 15° of longitude, every hour. This is 1° of longitude for 4 minutes of time, and 1′ of longitude for 4 seconds of time. Hence, the time of any place on the earth is one hour earlier than that of a place east of it, for every 15° of difference of longitude; and later than that of a place west of it by the same rule. Thus the difference of longitude between New York and New Orleans is about 15° 0′ 44″, equal, in time, to 1 hour 3 seconds. When, therefore, it is 12 o'clock at New Orleans, it is 3 seconds past 1 o'clock at New York.

QUESTIONS.—What are the numbers of days in the months? Give the rule for the lengths of the years. Explain the reasons for the rule. What is the relation of time to longitude?

TABLE,

SHOWING THE NUMBER OF DAYS FROM ANY DAY OF ONE MONTH TO THE SAME DAY OF ANY OTHER MONTH WITHIN A YEAR OF THE FORMER.

FROM ANY DAY OF	TO THE SAME DAY OF											
	Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
January,	365	31	59	90	120	151	181	212	243	273	304	334
February,	334	365	28	59	89	120	150	181	212	242	273	303
March,	306	337	365	31	61	92	122	153	184	214	245	275
April,	275	306	334	365	30	61	91	122	153	183	214	244
May,	245	267	304	335	365	31	61	92	123	153	184	214
June,	214	245	273	304	334	365	30	61	92	122	153	183
July,	184	215	243	274	304	335	365	31	62	92	123	153
August,	153	184	212	243	273	304	334	365	31	61	92	122
September,	122	153	181	212	242	273	303	334	365	30	61	91
October,	92	123	151	182	212	243	273	304	335	365	31	61
November,	61	92	120	151	181	212	242	273	304	334	365	30
December.	31	62	90	121	151	182	212	243	274	304	335	365

EXAMPLE. How many days from Dec. 3d to Aug. 3d of the next year?

Find December on the left; trace its line into the column headed Aug., where you find 243 days, the answer. If an example includes the end of bissextile February, its answer must have one more day than that in the table.

This table is much used in calculating interest and in other business.

EXERCISES.

Reduce

1. 13wk. 6da. 14h. 45min. 31sec. to seconds.
3. 349da. 12h. to minutes.
5. 18h. 25sec. to seconds.

Reduce

2. 8433931sec. to weeks.
4. 503280min. to days.
6. 64825sec. to hours.

7. If January, 1859, comes in on Saturday, on what day of the week will February come in? March? April? June? July? August? September? October? November? December?
8. Leap-year 1860 comes in on Sunday; on what days of the week do its months come in?
9. What are the remaining leap-years of this century?
10. Will the year 1900 be a leap year? Why?
11. In 1860 how many days from Jan. 25th to November 25th?
12. In 1862 how many days from Feb. 1st to July 1st?
13. How many days from Oct. 13th, 1871, to June 13th, 1872?

QUESTION.—How do we employ the table for intervening days?

14. If Pittsburgh is 79° 58′ west of Greenwich, what time is it at Greenwich when it is noon at Pittsburgh?

Ans. 19min. 52sec. past 5 o'clock P.M.

15. Will the year 2000 be a common or leap year? Why?

MISCELLANEOUS TABLE.

12 things	make	1 dozen.
12 dozen, or 144 things	"	1 gross.
12 gross, or 1728 things	"	1 great gross.
20 things	"	1 score.
100 lbs. of fish	"	1 quintal.
196 lbs. of flour	"	1 barrel.
200 lbs. of beef or pork	"	1 barrel.

Of Paper.

24 sheets	"	1 quire.
20 quires, or 480 sheets	"	1 ream.
2 reams	"	1 bundle.
5 bundles, or 10 reams	"	1 bale.

Of Books.

A folio (fol.) is made of sheets folded in	2 leaves.
A quarto (4to)	4 "
An octavo (8vo)	8 "
A duodecimo (12mo)	12 "
An 18mo	18 "
A 24mo	24 "
A 36mo	36 "

MISCELLANEOUS EXERCISES IN REDUCTION.

1. B. Swain walked in one day 23mi. 6fur. 24p. How many paces did he take, allowing 3 feet for each pace?

Ans. 41932 paces.

2. How many leagues in 933139 inches?

Ans. 4lea. 2mi. 5fur. 32rd. 13ft. 7in.

3. W. Worth's farm borders on the public road for 1mi. 2fur. 3ch. 2p. How many rails, each 11 feet long, would fence it, the fence being 5 rails high? Ans. 3105.

4. From E. Davis's house to his store is 78ch. 39li. How many miles per day does he walk in going this distance four times a day? Ans. 3mi. 73ch. 2rd. 6li.

5. How many acres in the Great African Desert, if it is 3000 miles long and 1000 miles broad? Ans. 1920000000 acres.

6. If 6 lbs. make 1 square foot of sheet lead, how many

pounds would it take to cover the inside of a cistern 9 feet long, 7 feet broad, and 8 feet deep? Ans. 1914lbs.

7. A room, whose walls measure 380 square feet, is to be papered with pieces 8yds. long and 19in. broad. How much will the paper cost, at 25 cents a piece? Ans. $2.50.

8. How many yards of carpeting one yard broad will carpet a room 15 feet long and 12 feet wide,—allowing for the hearth, which is 6 feet long and 18 inches wide? Ans. 19 yards.

9. Allowing a bushel of wheat to be produced on a piece of land 2 rods square, at what rate is that per acre?

Ans. 40 bushels per acre.

10. If bricks are 8 dollars a thousand, and a brick is 9 inches long by 4in. broad, how much will the bricks cost that will pave a sidewalk 100 rods long by 10 feet broad? Ans. $528.

11. Pure water weighs 1000 avoirdupois ounces per cubic foot: how many tons of water could be held by a box 12 feet long, 10 feet wide, and 8 feet deep? Ans. 30 tons.

12. How many U. S. wine gallons would the same box contain? Ans. $7181\frac{69}{231}$ gallons.

13. How many perches of masonry in a wall 75 feet long, 50 feet high, and 2 feet thick? Ans. 300.

14. Marble weighs about 170lbs. per cubic foot. What is the weight of a slab 6 feet long, 3 feet broad, and 4 inches thick? Ans. 1020 lbs.

15. At $16 per ounce, what would 43lbs. of gold-dust be worth? Ans. $8256.

16. At 5 cents an ounce, what cost 18lbs. 4oz. of mercury?

Ans. $14.60.

17. At 3 cents per grain, what cost 5lbs. 3oz. of gold?

Ans. $907.20.

18. How many square yards of canvas in a piece 36 yards long and 1yd. 1qr. broad? Ans. 45 sq. yds.

19. How many coats, each taking 1yd. 3qr. 2na. of broadcloth, can be cut from a piece 30 yards long? Ans. 16 coats.

20. If a lump of silver, weighing 12lbs. 3oz. 13pwt. 3gr., were put in one scale, how much weight avoirdupois must be put in the other scale to balance it? Ans. 10lbs. 2oz.

21. If a person walked 4 miles an hour for 10 hours a day, how long would it take him to walk around the globe, 24856 miles? Ans. 1yr. 36wk. 4da. 4hr.

22. A druggist bought 5lbs. of ipecacuanha. How many prescriptions could he fill from it, each calling for ℥ss ℈ss 10gr.?

Ans. 100.

23. How many ounces Troy in a diamond of 1680 carats?
Ans. $11\frac{96}{480}$.

24. L. Emerson imported from England 5T. 11cwt. 3qrs. 20lb. of coffee, English weight, at a cost of $1120, and sold 50 U. S. cwt. of it here, for $600, and the rest at 13cts. per lb. How much did he gain? Ans. $459.68.

25. E. Howell bought a hhd. of molasses, containing 88gals., at 45cts. per gal., and sold it at 15 cents per quart. How much did he gain? Ans. $13.20.

26. A cubic foot of oil of turpentine weighs 870 ounces avoirdupois. What is the weight of 48 barrels of it, each containing 36 gallons, allowing 60lbs. for the weight of each barrel?
Ans. 7T. 14cwt. 1qr. 15lbs. 10oz.

27. From 3gal. 1pt. alcohol, how many prescriptions can be filled, each calling for 5f℥? Ans. 80.

28. What cost 3bu. 3pk. 5qt. of timothy seed, at 7 cents per quart? Ans. $8.75.

29. N. Hale bought 8000 bushels of wheat, at 75 cents per bu. and sold it in England at $8 per quarter. What was his gain after deducting $500 for expenses? Ans. $1500.

30. How many seconds of arc in 9 signs 25° 21′ 44″?
Ans. 1063304.

31. How many signs in 897243 seconds of arc?
Ans. 8s. 9° 14′ 3″.

32. Sound moves about 1120 feet per second. How long would the sound of a cannonade be in going 200 miles?
Ans. 15min. 43sec. nearly.

33. How many seconds has a man 62 years old lived, if 365da. 5h. 48m. 49sec. make a year? Ans. 1956529598 sec.

34. If you counted 95 each minute, how many dollars' worth of cents could you count in a common year, working 10 hours each day, and not counting on 52 Sabbaths?
Ans. 178410.

35. If the heart of a person 68 years old has averaged 72 beats per minute, how many beats has it made, allowing $365\frac{1}{4}$ days to a year? Ans. 2575100160.

36. If 45 drops make 1 fluidrachm, how long would it take to fill a 63 gallon hogshead, at the rate of 1 drop per second?
Ans. 33da. 14hr. 24min.

37. If the wheel of a locomotive 15 feet 6in. in circumference averages 118 turns a minute, how long would it take to go 32 miles? Ans. 1hr. 32min. nearly.

CHAPTER IX.

ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION OF COMPOUND NUMBERS.

ADDITION OF COMPOUND NUMBERS.

Art. 86. ADDITION of Compound Numbers is the process of uniting two or more compound numbers, of the same kind, into one sum.

Ex. 1. J. Ross, when in London, paid 12£ 10s. 6d. 3qr. for a suit of clothes, 18£ 4s. 0d. 1qr. at a hotel, 9s. 6d. for cab hire, and 20£ 5s. 4d. 2qr. for a ticket to America. How much did he expend? Ans. £51 9s. 5d. 2qr.

WRITTEN PROCESS.

£	s.	d.	qr.
12	10	6	3
18	4	0	1
	9	6	
20	5	4	2
Ans. 51	9	5	2

MENTAL PROCESS.

Since like things only can be added, we write like denominations in the same column. The sum of the qrs. is 6. Dividing this by 4, to reduce to pence, gives 1 penny, and 2 qrs. remain. Write 2 under the qrs., and reckon the 1 penny in the column of pence, making its sum 17d. Dividing 17d. by 12, to reduce to shillings, gives 1 shilling, and 5d. remain. Write 5 under the pence, and reckon the 1s. in the column of shillings, making its sum 29s. Dividing 29s. by 20, to reduce to pounds, gives £1, and 9s. remain. Write 9 under the shillings, and reckon the £1 in the column of pounds, making its sum £51. Write this entire.

RULE.—*Write the numbers so that those of the same denomination may be in the same column, units under units, tens under tens, &c., and draw a line under them.*

Add the numbers of the lowest denomination, and divide their sum by that number of their denomination which makes one of the next higher.

Write the remainder under the column added, and add the quotient to the column of the next higher denomination.

Do thus with all the denominations to the highest, of which write the whole sum.

PROOF.—To detect error, carefully review the work.

QUESTIONS.—86. What is addition of compound numbers? How is it performed?

LINEAR MEASURE.

(2.)

mi.	fur.	rd.	yd.	ft.	in.
25	2	31	5	2	0
4	5	28	0		11
17	7	0	4		11
10	3	39	4	2	9
8	6	16	3	2	8
67	1	37	$2\frac{1}{2}$	0	3
			$\frac{1}{2}=1$		6
67	1	37	2	1	9

(3.)

mi.	fur.	rd.	ft.	in.
31	0	39	12	11
9	7	38	10	0
19	0	37	10	8
24	5	36	9	5
2	1	35	8	0
87	1	28	$1\frac{1}{2}$	0
			$\frac{1}{2}=6$	
87	1	28	1	6

(4.)

ft.	′	″	‴	⁗
9	7	5	4	11
	10	8	6	9
3	2	0	5	7
		11	9	4
6	1	7	2	3
19	10	9	4	10

(5.)

ft.	′	″	‴	⁗
12	4	1	3	5
	9	6	8	7
		2	11	3
	10	5	1	6
11	11	11	11	11

SURVEYORS' MEASURE.

(6.)

mi.	fur.	ch.	p.	li.
9	2	9	3	19
17	7	5	2	23
4	4	4	2	2
15	7	8	0	24
13	0	6	1	18
60	7	4	3	11

(7.)

mi.	fur.	ch.	p.	li.
12	2	2	2	12
13	3	3	3	13
18	4	2	1	15
	7	9	3	24
		8	3	23

CLOTH MEASURE.

(8.)

yd.	qr.	na.	in.
12	3	3	2
9	2	2	$1\frac{1}{2}$
14	3	2	$1\frac{1}{2}$
37	2	1	$0\frac{1}{2}$

(9.)

yd.	qr.	na.	in.
17	1	2	$1\frac{1}{2}$
24	3	2	2
37	3	3	$1\frac{1}{2}$

SQUARE MEASURE.

(10.)

A.	R.	P.	ft.	in.
18	2	35	258	119
49	3	27	295	143
5	1	8	217	82
6	3	39	49	9
80	3	32	4	65
				$\frac{1}{4}$=36
80	3	32	4	101

(11.)

A.	R.	P.	yd.	ft.	in.
3	3	33	28	8	95
12	2	22	19	7	116
	1	39	29	8	
		6	16	6	53
17	0	23	4	3	120
				$\frac{1}{4}$=2	
					$\frac{1}{4}$=36
17	0	23	4	6	12

CUBIC MEASURE.

(12.)

yd.	ft.	in.
349	26	1387
88	18	456
9	9	95
122	10	9
16	15	247
586	25	466

(13.)

Cords.	ft.	in.
18	117	994
162	92	840
8	104	18
125	3	64
9	123	293

TROY WEIGHT.

(14.)

lb.	oz.	pwt.	gr.
8	11	19	16
27	3	7	21
18	5	11	8
4	2	15	12
37	9	10	23
96	9	5	8

(15.)

lb.	oz.	pwt.	gr.
42	8	17	22
6	10	14	5
63	6	6	4
		4	5
	8	9	10

APOTHECARIES' WEIGHT.

(16.)

lb.	℥.	ʒ.	℈.	gr.
2	9	7	2	19
6	2	5	1	15
	11	6	2	9
		5	1	5
				13
10	1	2	0	1

(17.)

lb.	℥.	ʒ.	℈.	gr.
	10	4	1	8
5	7	6	2	
	3	5		18
			2	16
11				

AVOIRDUPOIS WEIGHT.

(18.)

T.	cwt.	qr.	lb.	oz.	dr.
42	15	3	22	13	14
	12		18	8	11
			98	6	15
6	19		43		
	12	2	24		
51	1	1	6	13	8

(19.)

T.	cwt.	qr.	lb.	oz.	dr.
	18		21	12	13
49	6		8	13	
	17	1	6	15	15
31	2	3	88	5	
	10		95		

LIQUID MEASURE.

(20.)

Gal.	qt.	pt.	gi.
28	3	1	3
36	2	1	2
18	3	1	
	3	1	2
		1	1
85	2	1	

(21.)

Gal.	qt.	pt.	gi.
49	2	1	
	1	1	3
75	3	1	1
	3	1	3
12	2	1	

APOTHECARIES' FLUID MEASURE.

(22.)

Cong.	O.	f℥.	f℈.	℥.
			6	50
		14	5	
	6	12	4	
6	2	8		
			7	30
7	2	4	7	20

(23.)

Cong.	O.	f℥.	f℈.	℥.
	5	14	7	45
	3	8	4	30
2	4			
		12	4	
			6	25

DRY MEASURE.

(24.)

bu.	pk.	qt.	pt.
8	3	6	1
11	2	5	
45	3		
		7	1
	3	4	1
67	1	7	1

(25.)

bu.	pk.	qt.	pt.
59	3	2	1
127	2	6	
9	1	1	1
	3	7	1
25	2		

CIRCULAR MEASURE.

(26.)

C.	°.	′	″
	275	55	42
1	180		
	90	28	37
	270	30	30
	45	45	12
3	142	40	1

(27.)

C. Signs.	°	′	″
5	27	16	29
1 9	15		
	325	59	25
1	180		
2	22	32	42

TIME.

(28.)

c. yr.	da.	hr.	min.	sec.
65	91	14	45	21
53	128	3	36	48
22	351	21	13	59
47	173	12		
8	291	23	18	24
197	307	2	54	32

(29.)

wk.	da.	hr.	min.	sec.
8	6	22	54	39
4	5	11		
7	4	19	45	
	1	23	59	59
		15	15	15

SUBTRACTION OF COMPOUND NUMBERS.

Art. 87. SUBTRACTION of Compound Numbers is the process of finding the difference between two compound numbers of the same kind.

Ex. 1. T. Dunham arrived at Liverpool with £98 10s. 4d., and spent there £51 9s. 5d. 2qr. How much remained when he left the place? Ans. £47 0s. 10d. 2qr.

WRITTEN PROCESS.

£	s.	d.	qr.
98	10	4	0
51	9	5	2
47	0	10	2

MENTAL PROCESS.

Since a number must be like that from which it is taken, write farthings under farthings, pence under pence, &c. Since we cannot take 2qrs. from 0qrs., we add 4qrs. to 0qrs., making 4 qrs., *to render subtraction possible.* Then, 2qrs. from 4qrs. leave 2qrs: write the difference 2 under the qrs. Now, since we have added *the value* of 1 penny to the upper number, we must *keep the difference between the compound numbers unchanged,* by adding 1 penny to the 5d. of the lower number, making 6d. Then, since 6d. cannot be taken from 4d., we add 12d. to 4d., making 16d., and then take 6d.

QUESTIONS.—87. What is subtraction of compound numbers? How is it performed?

from 16d., leaving 10d. Write the difference 10 under the pence. Since 12d., *the value of* 1s., was added to the upper number, add 1s. to the 9s. of the lower number, making 10s.: 10s. from 10s. leave 0s.: write 0 under the shillings. Then £51 from £98 leave £47. The above plan is often called *borrowing and carrying.*

RULE.— *Write the denominations of the less number under the same denominations of the greater, and draw a line under them.*

Then, beginning with the lowest denomination, subtract in order each lower number from that above it, if possible, and write the difference below.

When any lower number is greater than that above it, add as many units of its kind to the upper number as make one of the next higher denomination; then subtract the lower number and add 1 *to the next lower number before subtracting it.*

PROOF.—To detect error, carefully review the work.

EXAMPLES FOR PRACTICE.

ENGLISH MONEY.

(2.)

£	s.	d.	qr
321	17	6	1
283	18	8	3
37	18	9	2

(3.)

£	s.	d.	qr.
527	15	4	2
359	16	3	3

LINEAR MEASURE.

(4.)

mi.	fur.	rd.	ft.	in.
79	5	26	9	6
32	6	34	12	8
46	6	31	$12\frac{1}{2}$	10
			$\frac{1}{2} = 6$	
46	6	31	13	4

(5.)

mi.	fur.	rd.	yd.	ft.	in.
36	4	31	3	2	4
19	5	35	4	1	9

(6.)

ft.	′	″	‴	⁗
18	5	4	7	11
3	8	6	9	4
14	8	9	10	7

(7.)

ft.	′	″	‴	⁗
15	8	0	7.	5
	11	10	9	8

SURVEYORS' MEASURE.

(8.)

mi.	fur.	ch.	p.	li.
14	5	6	2	18
9	6	5	3	21
4	7	0	2	22

(9.)

mi.	fur.	ch.	p.	li.
37	4	7	1	4
18	7	8	2	14

CLOTH MEASURE.

(10.)

yd.	qr.	na.	in.
31	1	2	1
13	2	1	2
17	3	0	$1\frac{1}{4}$

(11.)

yd.	qr.	na.	in.
25	2	1	
18	3		1

SQUARE MEASURE.

(12.)

A.	R.	P.	ft.	in.
520	2	23	159	91
80	3	35	265	127
439	2	27	$165\frac{1}{4}$	108
			$\frac{1}{4}=36$	
439	2	27	166	

(13.)

A.	R.	P.	yd.	ft.	in.
160	1	19	24	6	75
40	2	25	27	7	101

CUBIC MEASURE.

(14.)

yds.	ft.	in.
63	23	1548
18	25	1696
44	24	1580

(15.)

Cords.	ft.	in.
182	109	1327
95	118	1598

TROY WEIGHT.

(16.)

lb.	oz.	pwt.	gr.
13	9	15	19
6	8	18	
7	0	17	19

(17.)

lb.	oz.	pwt.	gr.
17			
8	11	19	23
8	0	0	1

APOTHECARIES' WEIGHT.

(18.)

lb	℥	ʒ	℈	gr.
1				
	6	4	2	15
	5	3	0	5

(19.)

lb	℥	ʒ	℈	gr.
1				
	11	7	2	19
				1

AVOIRDUPOIS WEIGHT.

(20.)

T.	cwt.	qr.	lb.	oz.	dr.
25	11	0	24	8	8
	17		24	9	
24	13	3	24	15	8

(21.)

T.	cwt.	lb.	oz.	dr.
37	16	65	13	
	18	90	15	7

LIQUID MEASURE.

(22.)

Gal.	qt.	pt.	gi.
37	2		
15	2	1	2
21	3	0	2

(23.)

Gal.	qt.	pt.
63		
31	2	1

APOTHECARIES' FLUID MEASURE.

(24.)

Cong.	O.	f℥.	fʒ.	♏.
1				
	4	8	4	30
	3	7	3	30

(25.)

Cong.	O.	f℥.	fʒ.	♏.
1				
	7	15	7	59
				·1

DRY MEASURE.

(26.)

bu.	pk.	qt.	pt.
278	1	3	
125	2	6	1
152	2	4	1

(27.)

bu.	pk.	qt.	pt.
525	2		
389	3	4	1

CIRCULAR MEASURE.

(28.)

C.	Signs.	°	′	″
1	6	15		
	9	20	29	30
	8	24	30	30

(29.)

C.	°	′	″
1	180		
	275	35	45

TIME.

(30.)

c.	yr.	da.	hr.	min.	sec.
3		150	18	48	30
		325	12	54	
2		190	5	54	30

(31.)

wk.	da.	hr.	min.	sec.
21	4	6		
9	5	18	28	57

Art. 88. To find the time between two dates.

Ex. 1. George Washington was born Feb. 22, 1732: how old was he when he died, Dec. 14, 1799? Ans. 67yrs. 9m. 22da.

WRITTEN PROCESS.

yr.	m.	da.
1799	12	14
1732	2	22
67	9	22

Or,

yr.	m.	da.
1799	11	14
1732	1	22
67	9	22

This second method is more consistent in the following form.

yr	m.	da.
1798	11	14
1731	1	22
67	9	22

MENTAL PROCESS.

The earlier date must be taken from the later, since it expresses a less quantity of time. In the first method we reckon Dec. 14th as the 14th day of the 12th month of the 1799th year; and Feb. 22d as the 22d day of the 2d month of the 1732d year.

In the second method we reckon Dec. 14th as 11 months 14 days, and Feb. 22d as 1 month 22 days, considering all the time as elapsed.

The *difference* of time is correctly obtained by both methods, but the second method, to be consistent, should not suppose the whole current year to have elapsed.

In subtracting, call 30da. a month, and 12m. a year.

RULE.—*Write the earlier date under the later, reckoning the months in each date either by their number in the year, or by the number of months preceding them in the year. Subtract as*

QUESTIONS.—88. How do we find the time between two dates? Is this method accurate? Why?

in compound subtraction, allowing 30 *days for a month, and* 12 *months for a year.*

NOTE.—This method seldom obtains the *exact* difference, since it allows only 360 days for a year.

EXAMPLES FOR PRACTICE.

2. From the Declaration of Independence, July 4th, 1776, how long was it to the declaration of war with England, June 18th, 1812? Ans. 35yr. 11mo. 14da.

3. L. Kendal borrowed money Aug. 15th, 1855, and paid it April 7th, 1859: how long did he keep it?

Ans. 3yr. 7m. 22da.

MULTIPLICATION OF COMPOUND NUMBERS.

Art. 89. MULTIPLICATION of Compound Numbers is the process of multiplying a compound number by a simple number.

Ex. 1. What cost 7 crates of crockery at 9£ 8s. 6d. 3qr. per crate? Ans. £65 19s. 11d. 1qr.

WRITTEN PROCESS.

£	s.	d.	qr.
9	8	6	3
			7
65	19	11	1

MENTAL PROCESS.

Seven times 3qrs. = 21qrs. Dividing 21qrs. by 4qrs., to reduce to pence, gives 5d., and 1qr. remains. Write 1 under the qrs., and remember the 5d. Then $7 \times 6d. = 42d.$; add the 5d., making 47d. Dividing by 12d. gives 3 shillings, and 11d. remain. Write 11 under the pence, and remember the 3 shillings. Then, $7 \times 8s. = 56s.$; add the 3s., making 59s. Dividing by 20s. gives 2£, and 19s. remain. Write 19 under the shillings, and remember the £2. Then, $7 \times 9£ = 63£$; add the 2£, making £65.

RULE.—*Multiply first the number in the lowest denomination, and divide this product by that number of this denomination which makes one of the next higher. Write the remainder and add the quotient to the next product. Do this through all the denominations.*

PROOF.—To detect error, carefully review the work; or, divide the product by the multiplier, to reproduce the multiplicand. To prove by the last method, the pupil must first learn to divide compound numbers.

NOTE.—The following examples, from the 4th on, correspond with those similarly numbered in Division of Compound Numbers.

QUESTIONS.—89. What is multiplication of compound numbers? How is it performed? How is the operation proved?

Examples for Practice.

(2.)

mi.	fur.	rd.	ft.	in.
9	5	29	12	6
				6
58	2	18	9	0

(3.)

mi.	fur.	ch.	p.	li.
3	4	6	2	18
				4
14	2	6	2	22

(4.)

ft.	′	″	‴
327	7	9	11
			8
2621	2	7	4

(5.)

ft.	′	″	‴	⁗
589	6	4	10	9
				9

(6.)

yd.	qr.	na.	in.
15	3	3	1
			9
143	2	3	0

(7.)

A.	R.	P.	yd.	ft.	in.
42	2	18	24	7	36
					5
213	0	14	3	0	36

(8.)

cu. yd.	cu. ft.	cu. in.
43	22	1492
		8
350	20	1568

(9.)

lb.	oz.	pwt.	gr.
25	9	15	23
			12
309	9	11	12

(10.)

lb	℥	ʒ	℈	gr.	
2	7	5	2	15	
				53	
7	11	1	2	5	Product by 3.
132	2	7	2	10	Product by 50.
140	2	1	1	15	Product by 53.

(11.)

T.	cwt.	qr.	lb.	oz.	dr.
3	9	2	8	10	5
					937
3260	2	1	24	14	13

(12.)

gal.	qt.	pt.	gi.
40	3	1	3
			863
35356	0	0	1

(13.)

Cong.	O.	f℥	f℥	ℳ	
1	4	12	5	10	× 769
				10	
15	7	14	3	40	Product by 10.
				10	
159	7	0	4	40	Product by 10 × 10 = 100.
				7	
1119	1	4	0	40	Product by 7 × 100 = prod. by 700
95	7	6	6	0	= 6 times the above "prod. by 10" = prod. by 60
14	3	1	6	30	= 9 times the first quantity = prod. by 9
1229	3	12	5	10	= Product by 769

WRITTEN PROCESS FOR (11).

16)4685

292 — 13

16)9370

16)9662

603 — 14

25)7496

25)8099

323 — 24

4)1874

4)2197

549 — 1

2 | 0)8433 | 2

449 — 2

2811

3260

(14.)

bu.	pk.	qt.	pt.
24	3	6	1
			839
20935	2	5	1

(15.)

C.	°	′	″
1	338	51	15
			43
83	170	43	45

(16.)

c.	yr.	da.	hr.	min.	sec.	
5	180	23	41	53		× 96 = 8 × 12
				12		
65	346	20	22	36		= Prod. by 12.
				8		
527	219	19	0	48		= Prod. by 8 × 12, or 96.

17. If B. walks 3mi. 159rd. 11ft. 9in. per day, how far does he walk in 1 working week? In 52 working weeks?
1st Ans. 20mi. 318rd. 4ft. 6in. 2d Ans. 1091mi. 230rd. 3ft.

18. How much broadcloth will 7 coats take, each taking 1yd. 3qr. 3na.? Ans. 13yd. 2qr. 1na.

19. How much land in 9 lots, each containing 3A. 150P.?
Ans. 35A. 70P.

20. How many cords of word in 10 piles, each containing 112 cu. ft. 864 cu. in.? Ans. 8C. 101ft.

21. If each silver table-spoon weighs 1oz. 15pwt. 18gr., what is the weight of a set of 6 spoons?
Ans. 10oz. 14pwt. 12gr.

22. How much calomel will it take to fill 1000 prescriptions, each taking 10grs.? Ans. 1℔ 8ʒ 6ʒ 2ϑ.

23. How many tons in 2000bbls. of flour? Ans. 196T

24. How many gallons in 25 barrels, each containing 39gal. 3qt. 1pt.? Ans. 996gal. 3qt. 1pt.

25. How many bushels in 39 sacks, each containing 2bu. 1pk. 6qt. 1pt.? Ans. 95bu. 2pk. 5qt. 1pt.

26. If the moon's average daily motion is 13° 10′ 35″, how much of her orbit does she go through in 14da.?
Ans. 184° 28′ 10″.

27. If a person averages daily 7hr. 44min. 52sec. of sleep, how much will he sleep in 100 days?
Ans. 4wk. 4da. 6hr. 46min. 40sec.

LONGITUDE AND TIME.

Art. 90. To find the difference of longitude between two places, from the difference of their time.

Ex. 1. If, when it is 3 o'clock 28min. 30sec. P.M. at Boston, it is 2 o'clock 22min. 45sec. P.M. at Chicago, what is the difference in the longitude of the two places? Ans. 16° 26′ 15″.

WRITTEN PROCESS.

3hr. 28min. 30sec.
2 22 45

1hr. 5min. 45sec. = diff. of time.
15

16° 26′ 15″ diff. of longitude.

MENTAL PROCESS.

Since, by Art. 84, Note 4, 15° of longitude correspond to 1 hour of time, 15′ of longitude to 1 min. of time, and 15″ of longitude to 1 sec. of time, there will be 15 times as many degrees, minutes, or seconds of longitude between two places, as there are hours, minutes, or seconds difference of their times.

RULE.—*Multiply the difference of time by* 15, *as in compound multiplication, calling the product of seconds of time, seconds of arc, of minutes of time, minutes of arc, and of hours of time, degrees.*

EXAMPLES FOR PRACTICE.

2. If, when it is 11 o'clock 15min. 11sec. A.M. at Washington, D.C., it is 4 o'clock 32min. 38sec. P.M. at Paris, France, what is the difference of longitude of the two places?
Ans. 79° 21′ 45″.

3. A sea-captain observed at noon one day, that his watch, set to the time of Greenwich, England, pointed to 4 o'clock 10min. 21sec. What was his longitude? Ans. 62° 35′ 15″ W.

4. A man, traveling from Augusta, Maine, to Little Rock, Arkansas, found his watch pointed to 29min. 28sec. past 1 o'clock when it was 12 o'clock. How many degrees of longitude between the two places? Ans. 22° 22′.

5. A watch, whose dial had 24 hour-marks,* being set to Greenwich time and carried in Perry's expedition, showed 17 hr. 19min. when the time at a certain place in Japan was 2 o'c. 39min. 30 sec. P. M. What was the longitude of that place?
Ans. 140° 7′ 30″ E. of Greenwich.

* The time of day is reckoned on such dials from noon to noon.

DIVISION OF COMPOUND NUMBERS.

Art. 91. DIVISION of Compound Numbers is the process either of finding how many times a compound number contains a number expressed in one or more of its denominations, or of finding a required *part* of a compound number.

Ex. 1. How many crates of crockery can be bought for £65 19s. 11d. 1qr., each crate costing £9 8s. 6d. 3qr.?

Ans. 7 crates.

WRITTEN PROCESS.

£	s.	d.	qr.
9	8	6	3
20			
188			
12			
2262			
4			
9051			

£	s.	d.	qr.
65	19	11	1
20			
1319			
12			
15839			
4			

)63357(7
·63357

MENTAL PROCESS.

As many crates can be bought as the price of one is contained times in the cost of all. Since dividend and divisor must be alike, (see Art. 43,) we reduce both to qrs., and then proceed by simple division. Another method will be explained in Art. 156, in Decimals.

Ex. 2. If 7 crates of crockery cost £65 19s. 11d. 1qr., what did 1 crate cost? Ans. £9 8s. 6d. 3qr.

WRITTEN PROCESS.

£	s.	d.	qr.
7)65	19	11	1
9	8	6	3

MENTAL PROCESS.

One crate will cost $\frac{1}{7}$ of the cost of 7 crates. Now $\frac{1}{7}$ of £65 is £9, with a remainder of £2. Write 9 under the pounds, and reduce the remainder £2 to shillings, making 40s.: add the given 19s., making 59s. Then $\frac{1}{7}$ of 59s. is 8s., with a remainder of 3s. Write 8 under the shillings, and reduce the remaining 3s. to pence, making 36d.: add the given 11d., making 47d. Then, $\frac{1}{7}$ of 47d. is 6d., with a remainder of 5d. Write 6 under the pence, and reduce the remaining 5d. to qrs., making 20qrs.: add the given 1qr., making 21qrs. Then, $\frac{1}{7}$ of 21qrs. is 3qrs. Write 3 under the qrs. Here the *written process* seems to show that divisor and dividend are *not alike*, and the quotient is a *concrete* number, contrary to Art. 43. But in writing, the true mental process is not expressed,

QUESTION.—91. What is Division of Compound Numbers?

thus, "$\frac{1}{7}$ of £65 is as many pounds as 7 is contained times in 65: but 7 is contained in 65 nine times and 2 remains: therefore $\frac{1}{7}$ of £65 is £9, with a remainder of £2:" &c. &c.

Ex. 3. If 17 heirs share equally an estate of £1982 11s. 1d. how much has each? Ans. £116 12s. 5d.

Written Process.

```
      £     s.   d.
17)1982   11   1(116
   17
   ---
    28
    17
    ---
    112
    102
    ---
     10
     20
     ---
17)211(12
   17
   ---
    41
    34
    ---
     7
     12
     --
17)85(5
   85
   --
```

Mental Process.

Each has $\frac{1}{17}$ of the estate. Since the divisor is greater than 12, and is not a composite number, the process is conducted by *long division.*

Rule.—*If the divisor is expressed in one or more denominations of the dividend, reduce both divisor and dividend to the lowest denomination in either of them; then divide as in simple numbers.*

If the division is to find a required portion of the dividend, divide first the number in its highest denomination. If there is a remainder, reduce it to the next lower denomination, and add to this product that number of the dividend which is of this lower denomination. Divide this result, proceeding with its remainder, if any, as before. Do thus through all the denominations.

Question.—91. How do we divide compound numbers?

Examples for Practice.

(4.)

	ft.	′	″	‴
8)	2621	2	7	4
	327	7	9	11

(5.)

	ft.	′	″	‴	⁗
9)	5305	9	8	0	9

(6.)

	yd.	qr.	na.	in.
9)	143	2	3	
	15	3	3	1

(7.)

	A.	R.	P.	yd.	ft.	in.
5)	213	0	14	3	0	36
	42	2	18	24	7	36

(8.)

	cu. yd.	cu. ft.	cu. in.
8)	350	20	1568
	43	22	1492

(9.)

	lb.	oz.	pwt.	gr.
12)	309	9	11	12
	25	9	15	23

(10.)

	℔	℥	ʒ	℈	gr.
53)	140	2	1	1	15

(11.).

	T.	cwt.	qr.	lb.	oz.	dr.
937)	3260	2	1	24	14	13

(12.)

	Gal.	qt.	pt.	gi.
863)	35356	0	0	1

(13.)

	Cong.	O.	f℥.	f℥.	♏.
769)	1229	3	12	5	10

(14.)

	bu.	pk.	qt.	pt.
839)	20935	2	5	1

(15.)

	C.	°	′	″
43)	83	170	43	45
	1	338	51	15

(16.)

	c. yr.	da.	h.	mi.	sec.
96)	527	219	19	0	48
	5	180	23	41	53

17. If B. walks 1091mi. 230rd. 3ft. in 52 working weeks, what is the average per day? Ans. 3mi. 159rd. 11ft. 9in.

18. If 7 coats take 13yd. 2qr. 1na. of broadcloth, what is the average per coat? Ans. 1yd. 3qr. 3na.

19. If 9 equal lots contain 35A. 70P., how much land does each lot contain? Ans. 3A. 150P

20. If 10 equal piles of wood contain 8 cords 101 feet, how many feet does each pile contain? Ans. 112 cu. ft. 864 cu. in.

21. A set of 6 silver table-spoons weighs 10oz. 14pwt. 12gr. how much does each weigh? Ans. 1oz. 15pwt. 18gr.

22. How many grains of calomel would each prescription contain, if 1000 equal ones amounted to 1℔ 8℥ 6℥ 2ℨ?
Ans. 10grs.

23. If 2000bbls. of flour weigh 196 tons, how much is that per bbl.? Ans. 196lbs.

24. If 25bbls. contain 996gal. 3qt. 1pt., what is the average?
Ans. 39gal. 3qt. 1pt.

25. If 39 sacks contain 95bu. 2pk. 5qt. 1pt., what is the average? Ans. 2bu. 1pk. 6qt. 1pt.

26. If in 14 days the moon moves through 184° 28′ 10″, what is her average daily motion? Ans. 13° 10′ 35″.

27. If a person in 100 days sleeps 4wk. 4da. 6hr. 46mi. 40sec., what does he average per day? Ans. 7hr. 44min. 52sec.

LONGITUDE AND TIME.

Art. 92. To find the difference of time between two places from their difference of longitude.

Ex. 1. The longitude of Boston is 71° 4′ 20″ W. of Greenwich, and that of Chicago is 87° 30′ 30″: what o'clock is it at Chicago when it is 3 o'clock 28min. 30sec. at Boston?
Ans. 2 o'clock 22min. 45sec.

WRITTEN PROCESS.

87° 30′ 30″
71 4 20

15)16 26 10 dif. of long.

1hr. 5mi. 45sec. = dif. of time, nearly.

3hr. 28mi. 30sec.
1 5 45

2o'c. 22mi. 45sec.

MENTAL PROCESS.

Since Chicago is west of Boston, its time must be earlier than that of Boston. (See Art. 84, Note 4.) Since in 1 hour of time a place turns 15° eastward, in 1 minute of time 15′ eastward, and in 1 sec. of time 15″ eastward, there will be $\frac{1}{15}$ as many hours as degrees, $\frac{1}{15}$ as many minutes of time as minutes of arc, and $\frac{1}{15}$ as many seconds of time as seconds of arc, in any difference of longitude.

QUESTION.—92. How do we find the difference of time between two places, from their difference of longitude?

RULE.—*Divide the difference of longitude by* 15, *as in compound division, calling the quotient of degrees hours, of minutes of arc minutes of time, and of seconds of arc seconds of time.*

EXAMPLES FOR PRACTICE.

2. The longitude of Paris, France, is 2° 20′ 22″ E. of Greenwich, and that of Washington, D.C., is 77° 1′ 30″ W. of Greenwich: what o'clock is it at Paris, when it is 11 o'c. 15mi. 11sec. at Washington? Ans. 4 o'clock 32min. 38sec. nearly.

3. What time corresponds to 62° 35′ 15″ of longitude?
Ans. 4hr. 10min. 21sec.

4. There are about 22° 22′ difference of longitude between Augusta, Me., and Little Rock, Ark. What o'clock is it in the former place, when it is 12 o'clock in the latter?
Ans. 1 o'clock 29min. 28sec.

MISCELLANEOUS EXERCISES IN COMPOUND NUMBERS.

1. E. Schott, of Penn'a., has a coal-bin, 11ft. 6in. long, 6ft 4in. wide, and 5 feet deep: how many bushels of coal will it hold? Ans. $234\frac{288}{2688}$ bushels.

2. H. Brady bought several pieces of linen, measuring as follows: 28yd. 3qr. 3na.; 31yd. 2qr. 1na.; 33yd. 1qr. 2na.; 27yd. 3qr.; 30yd. 1qr.; 34yd.; 32yd.: what was the average length of the pieces? Ans. 31yd. 0qr. 2na.

3. In an orchard 4 trees bore each 2bu. 3pk. of apples; 7 trees each 2bu. 2pk.; and 9 trees each 1bu. 3pk. 6qt.: what was the average yield per tree? Ans. 2bu. 1pk. 1qt. 1pt.

4. A company of persons told their ages, as follows: 31yr. 7mo. 13da.; 24yr. 2 mo. 21da.; 28yr. 6mo. 18da.; 23yr. 11mo.; 30yr. 1mo.; 35yr. 9mo. 3da.; and 34yr.: what was their average age? Ans. 29yr. 8mo. 25da.

5. Albany is 42° 39′ 3″ North latitude, and Galveston, 29° 15′: what is their difference of latitude? Ans. 13° 24′ 3″.

6. Cape Horn is 55° 58′ 40″ South latitude, and Cape Cod 42° 2′ 22″ North latitude: what is their difference of latitude?
Ans. 98° 1′ 2″.

7. Bought a hogshead of oil containing 76gal. 3qt. 1pt., for $58.25; and, having lost 9gal. 1qt. 1pt. by leakage, I sold the rest for $0.88 per gallon; how much did I gain?
Ans. $1.15.

8. What is the weight of $3000 in coins as follows: $2000 in half-dollars, of 192grs. each, and $1000 in quarter-dollars, of proportional weight? Ans. 200lbs.

9. How many dollars, in half-dollars, must be melted down to make a tea-set of 6 cups and saucers, each piece weighing 2oz. 9pwt. 16gr.; an urn, of 1lb. 10oz. 18pwt. 15gr.; a bowl, of 3oz. 15pwt.; a pitcher, of 4oz. 8pwt. 15gr.; and half a dozen spoons, each 19pwt. 15grs.? Ans. $83.50.

10. Bought 10A. 3R. 36P. of land, and laid it off into 4 corner lots, each 1A. 4P.; and the rest was divided into 8 equal lots: how much did each contain?

Ans. 3R. 17P. 15sq. yd. 1sq. ft. 18sq. in.

11. From a piece of land, 80 rods square, I sold a piece 40 rods square, at $75 per acre, and raised upon the rest wheat, which I sold at $1 a bushel, realizing $600. What was the produce per acre, and the total of money received?

Ans. 20bu. per A., and $1350.

12. If the equatorial circumference of the earth is 24899 miles, how far eastward is a person there carried every second by the daily revolution? Ans. 92rd. 3ft. 7+in.

13. The earth's orbit being about 596902654 miles in circumference, what is the earth's average annual motion per second, allowing 365da. 5hr. 48min. 50sec. for a year?

Ans. 19 miles, nearly.

14. If the average depth of rain that falls in a year upon a county, 36 miles long and 30 miles broad, is 3 feet, how many gallons would it be? Ans. $6756855222855\frac{5}{7}$ gallons.

15. Sighting along the side of a north-and-south wall at noon, I observed that the western edge of the sun came to it at 11o'c. 57min. 32sec., by my clock, and the eastern edge at 11o'c. 59 min. 40sec. If the centre of the sun should have come to it exactly at 12M., what was the error of the clock?

Ans. 1min. 24sec. too slow.

16. If a load in a boat displaces its own weight of water, how many bushels of coal, 76 lbs. each, would cause a flat boat. 100 ft. long and 10ft. broad, to sink 6ft. in pure water?

(See Ex. 11, p. 100.) Ans. $4934\frac{4}{19}$ bushels.

17. What day of the month and year is the third Monday of May, in a common year commencing on Friday?

Ans 17th day of May; 137th day of the year.

CHAPTER X.

PROPERTIES AND RELATIONS OF NUMBERS.

Art. 93. INTEGERS, or whole numbers, are either *odd* or *even*.

An *odd* number is a number which cannot be divided by 2 without a remainder; as 1, 3, 5, 7, 23, 35, &c.

An *even* number is a number which can be divided by 2 without a remainder; as 2, 4, 6, 8, 24, 36, &c.

Integers are either *prime* numbers, or *composite* numbers.

A *prime* number is a number which is divisible without a remainder by no integer except itself and 1; as 19, 31, &c.

A *composite* number is a number produced by factors greater than unity, and is divisible by its factors without a remainder.

Numbers are *prime to each other* when 1 is the only number contained in them all without a remainder. Thus 4, 9, and 35 are prime *to each other*, though they are not prime numbers separately.

The *prime factors* of a number are those factors of it which are prime numbers.

NOTE.—It is not customary either to call a number a factor of itself, or to call 1 a factor of any number, since we do not consider any number to be *produced* by multiplying itself by 1.

TABLE OF PRIME NUMBERS AS FAR AS 1109.

1	59	139	233	337	439	557	653	769	883	1013
2	61	149	239	347	443	563	659	773	887	1019
3	67	151	241	349	449	569	661	787	907	1021
5	71	157	251	353	457	571	673	797	911	1031
7	73	163	257	359	461	577	677	809	919	1033
11	79	167	263	367	463	587	683	811	929	1039
13	83	173	269	373	467	593	691	821	937	1049
17	89	179	271	379	479	599	701	823	941	1051
19	97	181	277	383	487	601	709	827	947	1061
23	101	191	281	389	491	607	719	829	953	1063
29	103	193	283	397	499	613	727	839	967	1069
31	107	197	293	401	503	617	733	853	971	1087
37	109	199	307	409	509	619	739	857	977	1091
41	113	211	311	419	521	631	743	859	983	1093
43	127	223	313	421	523	641	751	863	991	1097
47	131	227	317	431	541	643	757	877	997	1103
53	127	229	331	433	547	647	761	881	1009	1109

QUESTIONS.—93. What is an odd number? An even number? A prime number? A composite number? When are numbers prime to each other? What are the prime factors of a number?

Art. 94. To resolve a composite number into its prime factors.

Ex. 1. Resolve 45 into its prime factors. Ans. 3, 3, 5.

WRITTEN PROCESS.

3)45

3)15

5

Proof. $3 \times 3 \times 5 = 45$

MENTAL PROCESS.

Every composite number must be composed of prime factors. If we divide 45 by the prime factor 3, it gives a composite factor 15. If we divide 15 by its prime factor 3, it gives a prime factor 5. The first division resolves 45 into 3×15; the second resolves 3×15 into $3 \times 3 \times 5$, all of which are prime numbers.

RULE.—*Divide the given number by any prime number, except* 1, *which is contained in it without a remainder. Divide the quotient, if it is composite, in the same manner. Do thus till a quotient is found which is a prime number.*

The last quotient and all the divisors are the prime factors sought.

EXAMPLES FOR PRACTICE.

2. Resolve 210 into its prime factors. Ans. 2, 3, 5, 7.
3. What are the prime factors of 1430?
4. Resolve 1615 into its prime factors. Ans. 5, 17, 19.
5. Resolve 1127 into its prime factors.
6. What are the prime factors of 2046?
7. Resolve 128 into its prime factors. Ans. 2, 2, 2, 2, 2, 2, 2.
8. What are the prime factors of 900? Ans. 2, 2, 3, 3, 5, 5.
9. Resolve 4725 into its prime factors.
10. Resolve 5504701 into its prime factors. Ans. 31, 41, 61, 71.
11. Resolve 7007 into its prime factors.
12. Resolve 16169 into its prime factors. Ans. 19, *, *.

Art. 95. A *common divisor*, or *common measure*, of several numbers, is a number which is contained as a factor in each of them. Thus, 12, 24, and 36 have several common divisors, viz., 2, 3, 4, 6, and 12.

Art. 96. To find the common divisors of several numbers.

QUESTIONS.—94. How do we resolve a number into its prime factors?—95. What is a common divisor of several numbers?—96. How do we find the common divisors of numbers?

Ex. 1. What are the common divisors of 12, 24, and 36?

ILLUSTRATION.

$12 = 2 \times 2 \times 3$ { Therefore 12 contains without a remainder 2, or 3, or 2×2, or 2×3, or $2 \times 2 \times 3$.

$24 = 2 \times 2 \times 2 \times 3$ { Therefore 24 contains 2, or 3, or 2×2, or 2×3, or $2 \times 2 \times 3$, or $2 \times 2 \times 2$, or $2 \times 2 \times 3$.

$36 = 2 \times 2 \times 3 \times 3$ { Therefore 36 contains 2, or 3, or 2×2, or 2×3, or $2 \times 2 \times 3$, or 3×3, or $2 \times 3 \times 3$.

Inspecting the above sets of factors, we find *in all the sets* 2, 3, $2 \times 2 = 4$, $2 \times 3 = 6$, and $2 \times 2 \times 3 = 12$. Hence 2, 3, 4, 6, and 12 are the *common* divisors of 12, 24, and 36.

RULE.—*One common divisor of several numbers may be found by inspection of the numbers, or by trial.*

To find all the common divisors of several numbers, resolve each number into its prime factors; then form as many products as possible of the prime factors of each number. Such factors and products as are in all the sets are the common divisors.

EXAMPLES FOR PRACTICE.

2. Find the common divisors of 8, 24, and 72.
Ans. 2, 4, and 8.
3. What are the common divisors of 18, 48, and 72?
Ans. 2, 3, and 6.
4. What are the common divisors of 24, 96, and 144?
Ans. 2, 3, 4, 6, 8, 12, and 24.

GREATEST COMMON DIVISOR.

Art. 97. The greatest common divisor, or measure, of several numbers, is the greatest number which is contained as a factor in each of them. Thus, of the several common divisors of 12, 24, and 36, the greatest is 12.

Art. 98. To find the greatest common divisor of two or more numbers.

Ex. 1. What is the greatest divisor common to 105 and 165?
Ans 15.

QUESTIONS.—97. What is the greatest common measure of several numbers? —98. How do we find the greatest common measure of two or more numbers?

FIRST METHOD.

$105 = 3 \times 5 \times 7$
$165 = 3 \times 5 \times 11$
$3 \times 5 = 15$ Ans.

EXPLANATION.

Resolving both numbers into their prime factors, we see that 3×5 is the greatest product common to both sets of factors. Therefore 15 must be the greatest number that will divide both 105 and 165.

SECOND METHOD.

$$\begin{array}{r} 105)165(1 \\ \underline{105} \\ 60)105(1 \\ \underline{60} \\ 45)60(1 \\ \underline{45} \\ 15)45(3 \\ \underline{45} \end{array}$$

EXPLANATION.

This method consists in making successive *trials*. The greatest common divisor cannot exceed the less number, and we try *that* first. The remainder 60 shows that 105 will not measure 165. We see that $105 + 60 = 165$. Now, as 60 is the greatest measure of itself, if it *also* measures 105, it will measure 165. Try it. 60 will not measure 105, since 45 remain. The above expression,

$105 + 60 = 165$ now becomes
$45 + 60 + 60 = 165$

Now, since 45 is the greatest measure of itself, if it also measures 60 it will measure both 105 and 165. Try it: 45 will not measure 60, since 15 remain. The above expressions then become

$$105 + 60 = 165$$
$$45 + 60 + 60 = 165$$
$$45 + 45 + 15 + 60 = 165$$

Now, since 15 is the greatest measure of itself, if it also measures 45, it will measure $45 + 15$, or 60, and of course $45 + 45 + 15$, or 105; and of course 165 also. Try it: since 15 does completely measure 45, it is the greatest common divisor sought.

RULES.—I. *Resolve each number into its prime factors, and find the product of those that are common to the several numbers:*— Or,

II.—*Divide the greater number by the less, and that divisor by the remainder, if any. So continue, dividing the last divisor by the last remainder, until nothing remains.*

The last divisor will be the greatest common divisor of those two numbers.

When there are more than two numbers, first find the greatest common divisor of two of them, then of that common divisor and one of the other numbers, and so on till every number is used. The last common divisor is the greatest common divisor of all the numbers.

Examples for Practice.

2. What is the greatest measure common to 54 and 126?
Ans. 18.

3. To 40, 64, and 84? Ans. 4.

4. To 36, 63, and 99? Ans. 9.

5. What is the longest rule that will apply a whole number of times to each dimension of a house 33 feet long, 24 feet broad, and 36 feet high? Ans. 3 feet.

6. If I were to carpet alike three rooms, the first $11\frac{1}{4}$ feet broad, the second 15 feet, the third $18\frac{3}{4}$ feet, what is the greatest breadth of carpet that will cover the floor with whole breadths?
Ans. $1\frac{1}{4}$ yd. broad.

MULTIPLES.

Art. 99. A MULTIPLE of a number is that number which is divisible by it without a remainder. Thus, 24 is a multiple of 3

Any number has an unlimited number of multiples; hence there can be no *greatest multiple*. Thus, 3 may have multiples as follows:—6, 9, 12, 15, 18, 21, 24, &c., without limit.

The *least* multiple of a number is the least number divisible by it without a remainder.

A *common multiple* of two or more numbers is a number divisible by each of them without a remainder. Thus, 32 is a multiple common to 2, 4, 8, and 16.

The *least common multiple* of two or more numbers is the least number divisible by each of them without a remainder.

Art. 100. To find the least common multiple of two or more numbers.

Ex. 1. What is the least multiple common to 10, 15, and 20?
Ans. 60.

First Method.

$10 = 2 \times 5$
$15 = 3 \times 5$
$20 = 2 \times 2 \times 5$
$2 \times 2 \times 3 \times 5 = 60$ Ans.

Explanation.

Resolving the numbers into prime factors, we see that the answer must contain 2×5, because it must contain 10. It must also contain 3×5, in order to contain 15. But in 2×5, we already have one 5. Therefore we need only take $2 \times 5 \times 3$ in order to contain both 10 and 15. It must also contain $2 \times$

Questions.—99. What is a multiple? How many multiples has a number? What is the least multiple of a number? What is a common multiple of two or more numbers? What is the least common multiple of several numbers?—100. How do we find the least common multiple of two or more numbers?

2×5, in order to contain 20. But as we already have 2×5 in $2 \times 5 \times 3$, above, we need only take another 2, making $2 \times 2 \times 3 \times 5$, in order to contain 10, 15, and 20.

SECOND METHOD.

2) 10 15 20

5) 5 15 10

1 3 2

$2 \times 5 \times 3 \times 2 = 60$ Ans.

EXPLANATION.

Here we more conveniently get rid of extra factors by division. Taking 2 out of 10 and 20 leaves the factors 5 and 10, with 15, from which no factor was taken. Then taking 5 from 5, 15, and 10, leaves 1, 3, and 2. Since no prime factor is common to these remaining factors, the operation of rejecting factors is ended. Then the retained factors 2, 5, 3, and 2, give 60 the least multiple of all the numbers. Since the divisors are the retained factors, it is plain that dividing only *one* of the numbers does not reject a factor.

RULE.— *Write the numbers in a line, and divide by any prime number that is contained exactly in two or more of them. Write the quotients and undivided numbers in a line below. Divide these in the same manner. Continue thus till no number greater than* 1 *is exactly contained in any two of the numbers. Then multiply all the divisors and remaining numbers together; the product is the least common multiple.*

NOTE 1.—Another correct way is to divide by any number, whether prime or not, which is contained in as many of the numbers as any other.

NOTE 2.—When any one of the numbers is exactly contained in one of the others, it may be left out of the operation.

EXAMPLES FOR PRACTICE.

2. What is the least multiple common to 3, 4, 6, and 8?

Ans. 24.

WRITTEN PROCESS.

2) ~~3~~ 4 6 8

3 4

$2 \times 3 \times 4 = 24$ Ans.

Since 3 is contained in 6, and 4 in 8, 3 and 4 may be neglected, because the least multiple of 6 and 8 must contain them.

3. What is the least common multiple of 10, 15, 20, and 25?

Ans. 300.

4. Of 18, 27, 36, and 45? Ans. 540.

5. Of 6, 8, 9, 12, 15, 16, 18, and 20? Ans. 720.

6. Wishing to buy some live-stock, and having my choice of horses at \$80, oxen at \$50, cows at \$40, and hogs at \$5 each, what is the least money I should take to purchase all of one kind? Ans. \$400.

CHAPTER XI.

COMMON, OR VULGAR FRACTIONS.

Art. 101. A FRACTION is a number expressing one or more parts of a unit.

Fractions are of two kinds,—viz.: *Common Fractions*, often called *Vulgar Fractions;* and *Decimal Fractions.*

A *common* fraction is written by placing one number above another, with a line between them; thus, $\frac{4}{9}$ is read *four-ninths;* $\frac{3}{5}$ *three-fifths*, &c.

The number below the line is called the *denominator*, because it denotes the number of equal parts into which the unit, or thing in question, is divided. Thus, $\frac{5}{7}$ *of an acre* implies that an acre is considered as divided into 7 equal parts.

The number above the line is called the *numerator*, because it states the number of equal parts expressed by the fraction. Thus, in the expression $\frac{5}{7}$ *of an acre*, 5 denotes that 5 equal parts out of 7 are referred to in the fraction.

The numerator and denominator are called the *terms* of the fraction.

Art. 102. From Art. 44 it appears that the form of a fraction is also used to signify the division of the upper term by the lower. Thus, $\frac{8}{4}$ may either mean that a unit is divided into 4 equal parts, and that 8 of such parts are expressed, or, that the whole number 8 is divided by the whole number 4.

The value of *eight-fourths of one* is the same as that of *one-fourth of eight.*

The *value* of a fraction is the quotient obtained by dividing the numerator by the denominator. Thus, 8 *fourths* of 1 equal 2; and 1 fourth of 8 equals 2.

Hence the value of a fraction does not depend upon the simple value of the terms, but upon the number of times that the lower

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QUESTIONS.—101. What is a fraction? What are the two kinds of fractions? What is a common fraction? What is the denominator? Why? What is the numerator? Why? What are the terms of a fraction? What two ways of reading a fraction are there? What is the value of a fraction? On what does the value of a fraction depend?

term is contained in the upper. Thus, $\frac{2}{1}$, $\frac{4}{2}$, $\frac{8}{4}$, $\frac{6}{3}$, $\frac{10}{5}$, $\frac{12}{6}$, &c., have the same value, though the *terms* have different values.

Art. 103. Since $\frac{2}{1} = \frac{2 \times 2}{1 \times 2} = \frac{2 \times 3}{1 \times 3} = \frac{2 \times 4}{1 \times 4}$, &c., it follows that *multiplying both terms by the same number does not affect the value of the fraction.*

Since $\frac{12}{6} = \frac{12 \div 2}{6 \div 2} = \frac{6}{3} = \frac{6 \div 3}{3 \div 3} = \frac{2}{1}$, &c., it follows that *dividing both terms by the same numbers does not affect the value of the fraction.*

If we divide the 12, in $\frac{12}{6}$, by 2, we obtain $\frac{6}{6} = 1$, which is *half* the value of $\frac{12}{6}$. Therefore *dividing the numerator by a number divides the value of the fraction by that number.*

If we divide the 6, in $\frac{12}{6}$, by 2, we obtain $\frac{12}{3} = 4$, which is *twice* the value of $\frac{12}{6}$. Therefore *dividing the denominator by a number multiplies the value of the fraction by that number.*

If we multiply the 12, in $\frac{12}{6}$, by 2, we obtain $\frac{24}{6} = 4$, which is *twice* the value of $\frac{12}{6}$. Therefore *multiplying the numerator by a number multiplies the value of the fraction by that number.*

If we multiply the 6, in $\frac{12}{6}$, by 2, we obtain $\frac{12}{12} = 1$, which is *half* the value of $\frac{12}{6}$. Therefore *multiplying the denominator by a number divides the value of the fraction by that number.*

Art. 104. In reference to their terms, common fractions are of four kinds,—viz., *proper*, *improper*, *simple*, and *complex*.

A *proper* fraction is one whose numerator is less than its denominator; for example, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{7}$, $\frac{20}{39}$, &c. It is called *proper* because its value is less than unity, and therefore its proper form is a fraction instead of a whole number.

An *improper* fraction is one whose numerator equals or exceeds its denominator; for example, $\frac{2}{2}$, $\frac{9}{3}$, $\frac{13}{6}$, &c. It is called *improper* because the proper expression for its value would be a whole or mixed number. Thus, $\frac{12}{3}$ is properly written 4.

A *simple* fraction is one whose terms are whole numbers; for example, $\frac{5}{6}$, $\frac{11}{15}$, $\frac{3}{8}$, &c.

A *complex* fraction is one which has a fraction in one or both of its terms; for example, $\frac{\frac{2}{3}}{4}$, $\frac{6\frac{1}{5}}{\frac{1}{2}}$, $\frac{5\frac{1}{8}}{9\frac{1}{7}}$, &c.

Questions.—103. Prove the first proposition. The second. The third. The fourth. The fifth. The sixth.—104. How are common fractions classified? What is a proper fraction? What is an improper fraction? Why? What is a simple fraction? What is a complex fraction?

Art. 105. In reference to their combinations, fractions form either *compound fractions*, or *mixed numbers*.

A *compound* fraction is a fraction of a fraction. Its members may be connected either by the word *of*, or by the sign of multiplication; for example, $\frac{4}{5}$ of $\frac{7}{12}$, or $\frac{4}{5} \times \frac{7}{12}$.

A *mixed number* is a number composed of an integer and a fraction; for example, $3\frac{1}{2}$, $10\frac{1}{7}$, $5\frac{4}{11}$, &c.

REDUCTION OF FRACTIONS.

Art. 106. REDUCTION of fractions is converting fractions into others expressing the same value in other terms.

CASE I.

Art. 107. To reduce a fraction to greater terms.

Ex. 1. Reduce $\frac{7}{9}$ to 54ths. Ans. $\frac{42}{54}$.

WRITTEN PROCESS.	MENTAL PROCESS.
9)54	Whatever multiplier converts 9 into 54 must also multiply the numerator 7, that the value of $\frac{7}{9}$ may not be changed. Such multiplier is found by dividing 54 by 9.
$6 \times 7 = \frac{42}{54}$	

RULE.—*Divide the proposed by the given denominator, and multiply both terms of the fraction by the quotient.*

EXAMPLES FOR PRACTICE.

2. Reduce $\frac{8}{13}$ to 65ths. Ans. $\frac{40}{65}$.
3. Reduce $\frac{11}{15}$ to 90ths. Ans. $\frac{66}{90}$.
4. Reduce $\frac{9}{17}$ to 136ths. Ans. $\frac{72}{136}$.
5. Reduce $\frac{5}{6}$ to 72ds. Ans. $\frac{60}{72}$.
6. Reduce $\frac{4}{9}$ to 81sts. Ans. $\frac{36}{81}$.
7. Reduce $\frac{3}{7}$ to 63ds. Ans. $\frac{27}{63}$.

CASE II.

Art. 108. To reduce a fraction to lower, that is, smaller, terms.

Ex. 1. Reduce $\frac{32}{48}$ths to 6ths. Ans. $\frac{4}{6}$.

QUESTIONS.—105. How are combined fractions classified? What is a compound fraction? What is a mixed number?—106. What is reduction of fractions? How do we reduce to greater terms?

WRITTEN PROCESS. MENTAL PROCESS.

6)48

—

8)$\frac{32}{48} = \frac{4}{6}$

Whatever divisor converts 48 into 6 must also divide the numerator 32, that the value of $\frac{32}{48}$ may not be changed. Such divisor is found by dividing 48 by 6.

RULE.—*Divide the given by the proposed denominator, and both terms of the fraction by this quotient.*

EXAMPLES FOR PRACTICE.

2. Reduce $\frac{40}{65}$ths to 13ths. Ans. $\frac{8}{13}$.
3. Reduce $\frac{66}{90}$ths to 15ths. Ans. $\frac{11}{15}$.
4. Reduce $\frac{72}{136}$ths to 17ths. Ans. $\frac{9}{17}$.
5. Reduce $\frac{60}{72}$ds to 6ths. Ans. $\frac{5}{6}$.
6. Reduce $\frac{36}{81}$sts to 9ths. Ans. $\frac{4}{9}$.
7. Reduce $\frac{27}{63}$ds to 7ths. Ans. $\frac{3}{7}$.

CASE III.

Art. 109. To reduce a fraction to its lowest, that is, smallest, terms.

Ex. 1. Reduce $\frac{18}{30}$ to its lowest terms. Ans. $\frac{3}{5}$.

FIRST PROCESS. SECOND PROCESS. THIRD PROCESS.

$\frac{18}{30} = \frac{2 \times 3 \times 3}{2 \times 3 \times 5} = \frac{3}{5}$. 2)$\frac{18}{30} = \frac{9}{15}$ 3)$\frac{9}{15} = \frac{3}{5}$. 6)$\frac{18}{30} = \frac{3}{5}$.

EXPLANATION.—In the first process we decompose both terms into their prime factors and cancel out all common factors.

In the second process we do the same thing by a series of divisions. In the third process we produce the same result at once by dividing both terms by their greatest common divisor.

RULE.—*Divide both terms of the fraction by any common factor. Do the same with the new terms thus produced, and so on, till the terms are prime to each other.*

Or, *Divide both terms by their greatest common divisor.*

EXAMPLES FOR PRACTICE.

2. Reduce $\frac{72}{96}$ to its lowest terms. Ans. $\frac{3}{4}$.
3. Reduce $\frac{81}{486}$ to its lowest terms. Ans. $\frac{1}{6}$.
4. Reduce $\frac{168}{216}$ to its lowest terms. Ans. $\frac{7}{9}$.

QUESTIONS.—108. How do we reduce a fraction to smaller terms?—109. How do we reduce fractions to their smallest terms?

5. Reduce $\frac{160}{252}$ to its lowest terms. Ans. $\frac{5}{11}$.
6. Reduce $\frac{144}{208}$ to its lowest terms. Ans. $\frac{9}{13}$.
7. Reduce $\frac{120}{225}$ to its lowest terms. Ans. $\frac{8}{15}$.
8. Reduce $\frac{784}{1232}$ to its lowest terms. Ans. $\frac{7}{11}$.
9. Divide 27692 by 1248. Ans. $22\frac{59}{312}$.
10. Divide 23409 by 1836. Ans. $12\frac{3}{4}$.

Case IV.

Art. 110. To find the part that one abstract number is of another.

Ex. 1. What part of 6 is 4? Ans. $\frac{2}{3}$.

Written Process. Mental Process.

$2)\frac{4}{6} = \frac{2}{3}$. 6 is 6 units, and 4 expresses 4 *of the same units* that compose 6. Hence 4 must be $\frac{4}{6}$ of 6. This, reduced to its lowest terms, gives $\frac{2}{3}$.

Rule.—*Write that number which is considered the part, for a numerator, and the other number for a denominator. Then reduce to lowest terms.*

Examples for Practice.

2. What part of 64 is 16? Ans. $\frac{1}{4}$.
3. What part of 35 is 7? Ans. $\frac{1}{5}$.
4. What part of 51 is 17? Ans. $\frac{1}{3}$.
5. What part of 625 is 50? Ans. $\frac{2}{25}$.
6. What part of 343 is 49? Ans. $\frac{1}{7}$.
7. What part of 225 is 30? Ans. $\frac{2}{15}$.
8. What part of 1728 is 24? Ans. $\frac{1}{72}$.
9. What part of 1331 is 33? Ans. $\frac{3}{121}$.
10. What part of 630 is 35? Ans. $\frac{1}{18}$.

Case V.

Art. 111. To reduce an improper fraction to its proper terms.

Ex. 1. What is the proper expression for $\frac{49}{8}$? Ans. $6\frac{1}{8}$.

Written Process. Mental Process.

$8)49$
$6\frac{1}{8}$

Since $\frac{8}{8}$ make 1 unit, $\frac{49}{8}$ expresses as many units as $\frac{8}{8}$ are contained times in $\frac{49}{8}$: that is, as many units as 8 is contained times in 49.

Questions.—110. How do we find the part that one abstract number is of another?—111. How do we reduce an improper fraction to its proper form?

RULE.—*Divide the numerator by the denominator.*

EXAMPLES FOR PRACTICE.

2. What is the proper expression for $\frac{164}{12}$? Ans. $13\frac{2}{3}$.
3. What is the proper expression for $\frac{1728}{144}$? Ans. 12.
4. What is the proper expression for $\frac{20}{8}$? Ans. $23\frac{3}{4}$.
5. Reduce $\frac{253}{1}$ to its proper form. Ans. 253.
6. Reduce $\frac{91}{91}$ to its proper form. Ans. 1.
7. What is the value of $\frac{1000}{12}$? Ans. $83\frac{1}{3}$
8. What is the value of $\frac{578}{10}$? Ans. $57\frac{4}{5}$.
9. Reduce $\frac{1438}{18}$ to its proper form. Ans. $79\frac{8}{9}$.
10. Reduce $\frac{10802409}{123456}$ to its proper form. Ans. $8\frac{3}{4}$.

CASE VI.

Art. 112. To reduce a mixed or whole number to an improper fraction.

Ex. 1. Reduce $9\frac{3}{4}$ to fourths. Ans. $\frac{39}{4}$.

WRITTEN PROCESS.

9
4
—
36 fourths in 9.
3 fourths added in.
—
$39 = \frac{39}{4}$

MENTAL PROCESS.

Since 4 fourths make a unit, 9 units are 9 times 4 fourths, that is, 36 fourths: add the three fourths given, and the whole amount to 39 fourths, or $\frac{39}{4}$.

Ex. 2. Reduce 13 to fifths. Ans. $\frac{65}{5}$.

WRITTEN PROCESS.

13
5
—
65 fifths $= \frac{65}{5}$

MENTAL PROCESS.

Since 5 fifths make a unit, 13 units are 13 times 5 fifths; that is, 65 fifths, or $\frac{65}{5}$.

RULES.—I. For mixed numbers.

Multiply the whole number by the denominator of the fractional part, add the numerator to the product, and place the sum over the denominator.

II. For whole numbers.

Multiply by the proposed denominator, and place the product over that denominator.

QUESTIONS.—112. How do we reduce a mixed number to an improper fraction? How a whole number?

EXAMPLES FOR PRACTICE.

3. Express $79\frac{8}{9}$ as an improper fraction. Ans. $\frac{719}{9}$.
4. Reduce $57\frac{4}{5}$ to an improper fraction. Ans. $\frac{289}{5}$.
5. Express 253 in a fractional form. Ans. $\frac{253}{1}$.
6. Reduce $23\frac{3}{4}$ to an improper fraction. Ans. $\frac{95}{4}$.
7. Reduce $13\frac{2}{3}$ to the form of a fraction. Ans. $\frac{41}{3}$.
8. Express 1 as a fraction. Ans. $\frac{1}{1}$ or $\frac{2}{2}$ or $\frac{3}{3}$, &c.
9. Express 17 in ninths. Ans. $\frac{153}{9}$.
10. Express 129 in fifteenths. Ans. $\frac{1935}{15}$.
11. How many persons could share $8\frac{5}{12}$ oranges, each having one-twelfth of one? Ans. 101.
12. How many pecks of wheat in $45\frac{3}{4}$ bushels? Ans. 183.

CASE VII.

Art. 113. To reduce a compound fraction to a simple fraction.

Ex. 1. What simple fraction expresses the value of $\frac{3}{5}$ of $\frac{4}{7}$? Ans. $\frac{12}{35}$.

WRITTEN PROCESS.

$$\frac{3}{5} \times \frac{4}{7} = \frac{3 \times 4}{5 \times 7} = \frac{12}{35}$$

MENTAL PROCESS.

Since *one-fifth* of a number is obtained by dividing it by 5, and multiplying the denominator of a fraction divides it, it follows that $\frac{1}{5}$ of $\frac{4}{7} = \frac{4}{35}$. If $\frac{4}{35}$ is $\frac{1}{5}$ of $\frac{4}{7}$, then 3 fifths of $\frac{4}{7}$ must be 3 times $\frac{4}{35}$; that is, $\frac{12}{35}$.

RULE.—*Multiply all the numerators together for a new numerator, and all the denominators together for a new denominator.*

NOTE 1.—If there are whole or mixed numbers in the compound expression, first reduce them to the form of a fraction.

NOTE 2.—It is best, before multiplying, to cancel all common factors from numerator and denominator.

NOTE 3.—It is best, if canceling is not employed, to reduce the product to its lowest terms.

EXAMPLES FOR PRACTICE.

2. Reduce $\frac{2}{3}$ of $\frac{5}{9}$ of 12 to a simple fraction. Ans. $\frac{40}{9}$.

WRITTEN PROCESS.

$$\frac{2}{3} \times \frac{5}{9} \times \frac{12}{1} = \frac{120}{27} = \frac{40}{9}$$

QUESTIONS.—113. How do we reduce a compound fraction to a simple one? What is to be done with whole or mixed numbers? What is recommended to shorten the work?

3. Reduce $\frac{4}{5}$ of $\frac{3}{8}$ of $3\frac{1}{7}$ to a simple fraction. Ans. $\frac{3}{35}$.

Written Process.

$$\frac{4}{5}\times\frac{3}{\cancel{8}_{2}}\times\frac{\cancel{22}^{11}}{\cancel{7}}=\frac{33}{35}$$

4. What is the value of $\frac{5}{6}$ of $15\frac{1}{8}$ of $2\frac{1}{3}$? Ans. $29\frac{59}{144}$.
5. What is the value of $\frac{5}{9}$ of $\frac{18}{25}$ of $\frac{5}{24}$ of $\frac{12}{17}$? Ans. $\frac{1}{17}$.
6. Reduce $\frac{8}{15}$ of $6\frac{3}{4}$ of $2\frac{1}{9}$ of $\frac{4}{11}$ to a simple fraction. Ans. $1\frac{52}{55}=2\frac{42}{55}$.
7. What is the value of $\frac{11}{17}$ of $\frac{3}{8}$ of 10? Ans. $1\frac{65}{68}=2\frac{29}{68}$.
8. What is the worth of $\frac{1}{2}$ of $\frac{3}{5}$ of $\frac{8}{9}$ of a ship valued at \$30000? Ans. \$8000.
9. How much is $\frac{1}{8}$ of $24\frac{1}{4}$ of $\frac{8}{194}$ of \$8? Ans. \$1.
10. Express $\frac{3}{7}$ of $\frac{14}{27}$ of $\frac{9}{28}$ of $\frac{14}{15}$ of 5 as a simple fraction. Ans. $\frac{1}{3}$.

Case VIII.

Art. 114. To reduce two or more fractions, having different denominators, to fractions having a common denominator.

Note.—Two or more fractions are said to have *a common denominator*, when their denominators are alike.

Ex. 1. Reduce $\frac{3}{4}$ and $\frac{7}{8}$ to fractions having like denominators.

Explanation.—Here we see at once that $\frac{3}{4}$ can be reduced to *eighths* by multiplying both terms by 2. Then the fractions would be $\frac{6}{8}$ and $\frac{7}{8}$, having *the same*, that is, a *common*, denominator.

Ex. 2. Reduce $\frac{3}{4}$ and $\frac{7}{9}$ to fractions having a common denominator.

Explanation.—Here we cannot do as in example 1, because 4 is not contained in 9 without a remainder. But if we make a new denominator of 4×9, or 36, we can reduce both fractions to 36ths by Case I. But this process is the same as multiplying the numerator 3 by the denominator 9 for one new numerator, and 7 by 4 for the other.

Rule.—*Multiply each numerator into all the denominators except its own, for the new numerators: then multiply all the denominators together, and write the product as a denominator to each new numerator.*

Questions.—114. When have fractions a common denominator? How do we reduce fractions to a common denominator?

Examples for Practice.

3. Reduce $\frac{3}{5}$, $\frac{7}{10}$, and $\frac{4}{9}$ to a common denominator.

Ans. $\frac{270}{450}$, $\frac{315}{450}$, $\frac{200}{450}$.

4. Reduce $\frac{6}{7}$, $\frac{1}{2}$, $\frac{10}{11}$, and $\frac{1}{3}$ to a common denominator.

Ans. $\frac{396}{462}$, $\frac{231}{462}$, $\frac{420}{462}$, $\frac{154}{462}$.

5. Reduce $2\frac{1}{2}$, $5\frac{1}{8}$, $\frac{2}{5}$, and $1\frac{1}{4}$ to a common denominator.

Ans. $\frac{800}{320}$, $\frac{1640}{320}$, $\frac{128}{320}$, $\frac{400}{320}$.

6. Reduce $\frac{1}{2}$ of $\frac{3}{4}$, $\frac{1}{4}$ of $\frac{2}{5}$, $\frac{3}{7}$, and $\frac{3}{5}$ of $\frac{6}{7}$, to a common denominator. Ans. $\frac{14700}{39200}$, $\frac{3920}{39200}$, $\frac{16800}{39200}$, $\frac{20160}{39200}$.

Case IX.

Art. 115. To reduce two or more fractions to their least common denominator.

Note.—The *least* common denominator of two or more fractions is the smallest denominator that they all can have.

Ex. 1. Reduce $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{12}$ to their least common denominator.

Explanation.—Here we see, by inspection, that the denominator 12 contains each of the denominators, and that no number less than 12 will do this. Therefore all we have to do, in this case, is to reduce $\frac{3}{4}$ and $\frac{5}{6}$ to 12ths by Case I. The fractions then are $\frac{9}{12}$, $\frac{10}{12}$, and $\frac{7}{12}$.

Ex. 2. Reduce $\frac{3}{4}$, $\frac{5}{12}$, and $\frac{7}{16}$ to their least common denominator.

Explanation.—Here, to make the new denominators, we must find the least number that will contain all the denominators. This must be their least common multiple. (See Art. 100.) Having found this new denominator, the given fractions may be reduced to it by Case I. The following is a good form of

Written Process.

2) 4 12 16

2) 2 6 8

1 3 4

$4 \times 3 \times 2 \times 2 = 48$

$48 \div 4 = 12$; $12 \times 3 = 36$ = 1st Numerator.

$48 \div 12 = 4$; $4 \times 5 = 20$ = 2d Numerator.

$48 \div 16 = 3$; $3 \times 7 = 21$ = 3d Numerator.

Ans. $\frac{36}{48}$, $\frac{20}{48}$, $\frac{21}{48}$.

Questions.—115. When have fractions their least common denominator? How do we reduce fractions to their least common denominator? What should first be done?

RULE.—*If one denominator is a multiple of each of the others, reduce the other fractions to that denominator.*

If no denominator is a multiple of every other, find the least common multiple of all the denominators, and write it as a new denominator; then reduce the fractions to this denominator.

NOTE.—First reduce each fraction to its lowest terms; also compound fractions to simple, and mixed or whole numbers to fractions.

EXAMPLES FOR PRACTICE.

3. Reduce $\frac{5}{6}$, $\frac{4}{7}$, $\frac{7}{9}$, and $\frac{2}{3}$ to the least common denominator.
Ans. $\frac{105}{126}$; $\frac{72}{126}$; $\frac{98}{126}$; $\frac{84}{126}$.

4. Reduce $\frac{3}{4}$, $\frac{3}{5}$, $\frac{3}{6}$, $\frac{3}{10}$, and $\frac{3}{8}$ to the least common denomina tor.
Ans. $\frac{30}{40}$; $\frac{24}{40}$; $\frac{20}{40}$; $\frac{12}{40}$; $\frac{15}{40}$.

5. Reduce $2\frac{1}{2}$, $7\frac{3}{4}$, and $5\frac{5}{6}$ to the least common denominator.
Ans. $\frac{30}{12}$; $\frac{93}{12}$; $\frac{70}{12}$.

6. Reduce $\frac{1}{12}$, $\frac{1}{24}$, $\frac{1}{36}$, and $\frac{1}{48}$ to the least common denominator.
Ans. $\frac{12}{144}$; $\frac{6}{144}$; $\frac{4}{144}$; $\frac{3}{144}$.

7. Reduce $9\frac{1}{8}$, $\frac{6}{16}$, $10\frac{5}{24}$, and $3\frac{3}{4}$ to the least common denominator.
Ans. $\frac{219}{24}$; $\frac{9}{24}$; $\frac{245}{24}$; $\frac{90}{24}$.

8. Reduce $\frac{4}{5}$, 5, $\frac{5}{6}$, 6, and $\frac{1}{2}$ to the least common denominator.
Ans. $\frac{24}{30}$; $\frac{150}{30}$; $\frac{25}{30}$; $\frac{180}{30}$; $\frac{15}{30}$.

9. Reduce $\frac{2}{7}$, $\frac{3}{8}$, $\frac{4}{9}$, and $\frac{5}{10}$ to the least common denominator.
Ans. $\frac{144}{504}$; $\frac{189}{504}$; $\frac{224}{504}$; $\frac{252}{504}$.

10. Reduce $\frac{7}{20}$, $\frac{13}{40}$, $\frac{29}{80}$, and $\frac{57}{160}$ to the least common denominator.
Ans. $\frac{56}{160}$; $\frac{52}{160}$; $\frac{58}{160}$; $\frac{57}{160}$.

ADDITION OF COMMON FRACTIONS.

Art. 116. To find the sum of two or more fractions.

Ex. 1. What is the sum of $\frac{3}{11}$ and $\frac{5}{11}$? Ans. $\frac{8}{11}$.

PROCESS INDICATED.

$$\frac{3}{11} + \frac{5}{11} = \frac{3+5}{11} = \frac{8}{11}$$

MENTAL PROCESS.

Since 3 and 5 make 8, it is plain that 3 elevenths and 5 elevenths make 8 elevenths.

Ex. 2. What is the sum of $\frac{2}{5}$ and $\frac{3}{8}$? Ans. $\frac{31}{40}$.

PROCESS INDICATED.

$$\frac{2}{5} + \frac{3}{8} = \frac{16}{40} + \frac{15}{40} = \frac{16+15}{40} = \frac{31}{40}$$

MENTAL PROCESS.

Since the things to be added must be alike, (Art. 26,) it is plain that the sum of $\frac{2}{5}$ and $\frac{3}{8}$ is not $\frac{2+3}{5}$ nor $\frac{2+3}{8}$ nor $\frac{2+3}{5+8}$; and that $\frac{2}{5}$ and $\frac{3}{8}$ must be reduced *so as to represent like parts of a unit*, before they can be united in one sum. This is reducing them to a common denominator, making $\frac{16}{40}$ and $\frac{15}{40}$, whose sum is $\frac{31}{40}$.

Ex. 3. What is the sum of $2\frac{3}{4}$, $5\frac{1}{3}$, and $7\frac{5}{6}$? Ans. $15\frac{11}{12}$.

First Method.

$2\frac{3}{4} = \frac{11}{4}$.
$5\frac{1}{3} = \frac{16}{3}$.
$7\frac{5}{6} = \frac{47}{6}$.

$\frac{11}{4} + \frac{16}{3} + \frac{47}{6} = \frac{33}{12} + \frac{64}{12} + \frac{94}{12} = \frac{191}{12} = 15\frac{11}{12}$.

Second Method.

$2\frac{3}{4} = 2\frac{9}{12}$.
$5\frac{1}{3} = 5\frac{4}{12}$.
$7\frac{5}{6} = 7\frac{10}{12}$.

14
$1\frac{11}{12}$

$15\frac{11}{12}$

Explanation.

In the first method, the mixed numbers are reduced to fractions; then these fractions are reduced to a common denominator and added. In the second method, the whole numbers are added by themselves, and the fractions are reduced and added by themselves; then the sums are united

Thus, $\frac{10}{12} + \frac{4}{12} + \frac{9}{12} = \frac{23}{12} = 1\frac{11}{12}$. $14 + 1\frac{11}{12} = 15\frac{11}{12}$.

Rule.—*If the fractions have a common denominator, add their numerators, and write the sum over the denominator.*

If the fractions have different denominators, reduce them first to a common denominator, and write the sum of the new numerators over the common denominator.

Either reduce mixed numbers to fractions and add them as fractions, or find the sums of the integral and fractional portions separately and add these sums.

Note.—To lesson labor, first reduce to lowest terms, and compound fractions to simple fractions.

Examples for Practice.

4. Add $\frac{3}{4}$ and $\frac{5}{8}$. Ans. $1\frac{7}{12}$.
5. What is the sum of $\frac{1}{8} + \frac{1}{9} + \frac{2}{11}$? Ans. $\frac{331}{792}$.
6. What is the sum of $\frac{4}{25} + \frac{6}{25} + \frac{7}{25} + \frac{8}{25}$? Ans. 1.
7. What is the sum of $\frac{9}{73} + \frac{23}{73} + \frac{56}{73} + \frac{19}{73}$? Ans. $1\frac{34}{73}$.
8. Find the sum of $\frac{81}{100} + \frac{63}{100} + \frac{51}{100} + \frac{47}{100}$. Ans. $2\frac{21}{50}$.
9. Find the sum of $\frac{3}{8} + \frac{3}{4} + \frac{1}{12} + \frac{5}{8}$. Ans. $2\frac{1}{24}$.
10. What is the sum of $\frac{3}{4}$ of $\frac{2}{3} + \frac{7}{8}$ of $\frac{5}{6}$? Ans. $1\frac{11}{48}$.
11. What is the sum of $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} + \frac{6}{7} + \frac{7}{8}$? Ans. $5\frac{79}{280}$.
12. Find the sum of $\frac{1}{2}$ of $\frac{2}{3} + \frac{3}{4}$ of $\frac{4}{5} + \frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{8}$. Ans. $1\frac{67}{120}$.

Questions.—116. How do we add fractions? How mixed numbers? What should first be done?

13. What is the sum of $18\frac{1}{2} + 25\frac{3}{4}$? Ans. $44\frac{1}{4}$.
14. What is the sum of $15\frac{3}{8} + 34\frac{5}{12} + 47\frac{9}{16}$? Ans. $97\frac{17}{48}$.
15. Find the sum of $\frac{1}{2} + \frac{1}{7} + \frac{1}{2}$ of $\frac{1}{7}$. Ans. $\frac{5}{7}$.
16. To $\frac{1}{5}$ of $2\frac{1}{2}$ add $\frac{1}{8}$ of $6\frac{1}{4}$. Ans. $1\frac{9}{32}$.
17. To $\frac{3}{4}$ of $\frac{1}{2}$ of 48 add $\frac{5}{8}$ of $\frac{2}{3}$ of 96. Ans. 58.

NOTE.—Do this briefly by a mental process only.

18. To $\frac{1}{2}$ of $\frac{3}{5}$ of $\frac{1}{8}$ of 800 add $\frac{3}{8}$ of $\frac{2}{9}$ of $\frac{9}{10}$ of 1200. Ans. 120
19. To $\frac{2}{7} + \frac{3}{7} + \frac{4}{7}$ add $\frac{5}{9} + \frac{7}{9} + \frac{8}{9}$. Ans. $3\frac{2}{3}$.
20. Add $\frac{3}{11}$ of $\frac{4}{5}$ of $5\frac{1}{2} + 3\frac{3}{4} + \frac{2}{5}$ of $8\frac{7}{11}$. Ans. $6\frac{25}{66}\frac{7}{0}$.

SUBTRACTION OF COMMON FRACTIONS.

Art. 117. To find the difference between two fractions.

Ex. 1. From $\frac{7}{9}$ take $\frac{5}{9}$. Ans. $\frac{2}{9}$.

PROCESS INDICATED. MENTAL PROCESS.

$\frac{7}{9} - \frac{5}{9} = \frac{7-5}{9} = \frac{2}{9}$ Since 5 from 7 leaves 2, it is plain that 5 ninths from 7 ninths leave 2 ninths.

Ex. 2. From $\frac{8}{9}$ take $\frac{3}{5}$. Ans. $\frac{13}{45}$.

PROCESS INDICATED. MENTAL PROCESS.

$\frac{8}{9} - \frac{3}{5} = \frac{40}{45} - \frac{27}{45} = \frac{40-27}{45} = \frac{13}{45}$. Since the things signified by the numbers in subtraction must be alike, (see Art. 31,) it is plain that 3 *fifths* cannot be directly taken from 8 *ninths*. They must first be reduced so as to represent *like parts of a unit*, and this is reducing them to a common denominator, making $\frac{40}{45}$ and $\frac{27}{45}$, whose difference is $\frac{13}{45}$.

Ex. 3. From $4\frac{3}{4}$ take $2\frac{1}{3}$. Ans. $2\frac{5}{12}$.

PROCESS INDICATED.

$4\frac{3}{4} - 2\frac{1}{3} = \frac{19}{4} - \frac{7}{3} = \frac{57}{12} - \frac{28}{12} = \frac{57-28}{12} = \frac{29}{12} = 2\frac{5}{12}$.

SECOND METHOD.

$4\frac{3}{4} = 4\frac{9}{12}$.
$2\frac{1}{3} = 2\frac{4}{12}$.
$2\frac{5}{12}$.

EXPLANATION.

In the first method, the mixed numbers are first reduced to fractions and then subtracted as in Ex. 2. In the second method, the fractional parts are reduced to a common denominator, and the fractions and integers are separately subtracted.

Ex. 4. From 6 take $\frac{4}{7}$. Ans. $5\frac{3}{7}$.

PROCESS INDICATED.

$6 - \frac{4}{7} = \frac{6}{1} - \frac{4}{7} = \frac{42}{7} - \frac{4}{7} = \frac{42-4}{7} = \frac{38}{7} = 5\frac{3}{7}$.

QUESTION.—117. How do we subtract a fraction from a fraction?

Second Method.	Explanation.
6 $\frac{4}{7}$ $5\frac{3}{7}$	In the second method, we say, $\frac{4}{7}$ from 0, impossible: add $\frac{7}{7}$ to the 6, making $6\frac{7}{7}$: then $\frac{4}{7}$ from $\frac{7}{7} = \frac{3}{7}$. Write $\frac{3}{7}$ and add 1 to the lower number, to increase it as much as the 6 was increased. Then 1 from 6 leaves 5.

Ex. 5. From $8\frac{1}{2}$ take $5\frac{2}{3}$. Ans. $2\frac{5}{6}$.

Process Indicated.

$$8\tfrac{1}{2} - 5\tfrac{2}{3} = \tfrac{17}{2} - \tfrac{17}{3} = \tfrac{51}{6} - \tfrac{34}{6} = \tfrac{51-34}{6} = \tfrac{17}{6} = 2\tfrac{5}{6}.$$

Second Method.	Explanation.
$8\frac{1}{2} = 8\frac{3}{6}$ $5\frac{2}{3} = 5\frac{4}{6}$ $2\frac{5}{6}$	In the second method, we say, $\frac{4}{6}$ from $\frac{3}{6}$, impossible: add $\frac{6}{6}$ to $\frac{3}{6}$, making $\frac{9}{6}$. Then $\frac{4}{6}$ from $\frac{9}{6}$ leave $\frac{5}{6}$. Write $\frac{5}{6}$, and carry 1 to 5, making 6. Then 6 from 8 leaves 2.

Ex. 6. From $7\frac{5}{9}$ take $\frac{8}{11}$. Ans. $6\frac{82}{99}$.

Process Indicated.

$$7\tfrac{5}{9} - \tfrac{8}{11} = \tfrac{68}{9} - \tfrac{8}{11} = \tfrac{748}{99} - \tfrac{72}{99} = \tfrac{748-72}{99} = \tfrac{676}{99} = 6\tfrac{82}{99}$$

Second Method.	Explanation.
$7\frac{5}{9} = 7\frac{55}{99}$ $\frac{8}{11} = 0\frac{72}{99}$ $6\frac{82}{99}$	In the second method, we say, $\frac{72}{99}$ from $\frac{55}{99}$, impossible: add $\frac{99}{99}$ to $\frac{55}{99}$, making $\frac{154}{99}$. Then $\frac{72}{99}$ from $\frac{154}{99}$ leave $\frac{82}{99}$. Write $\frac{82}{99}$, and carry 1 to 0, making 1. Then 1 from 7 leaves 6.

Rules.—I. For fractions.

When the fractions have a common denominator, subtract the less numerator from the greater, and write the difference over the denominator.

When the fractions have different denominators, reduce them to a common denominator, then subtract in the manner before stated.

II. For mixed numbers.

Either reduce them to fractions, and subtract as directed for fractions, or,

Write the less number under the greater, and reduce the fractional parts to a common denominator. If the upper fraction is greater than the lower, subtract, and write the difference below: then subtract the lower whole number from the upper. But if the upper number has either no fraction, or one smaller than the lower fraction, add to the upper place or fraction as many

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Questions.—How do we subtract one mixed number from another, or a fraction from a whole or mixed number? What should first be done?

parts, like those of the lower fraction, as make a unit: then subtract the lower fraction, write the difference, and carry 1 *to the lower whole number, which subtract as in whole numbers.*

NOTE.—To lesson labor, first reduce to lowest terms. Compound fractions must be reduced to simple fractions before subtracting.

EXAMPLES FOR PRACTICE.

7. From $\frac{67}{100}$ take $\frac{37}{100}$. Ans. $\frac{3}{10}$.
8. From $\frac{77}{144}$ take $\frac{44}{144}$. Ans. $\frac{1}{4}$.
9. From $\frac{476}{625}$ take $\frac{451}{625}$. Ans. $\frac{1}{25}$.
10. From $\frac{3}{7}$ take $\frac{2}{9}$. Ans. $\frac{13}{63}$.
11. From $\frac{9}{11}$ take $\frac{5}{12}$. Ans. $\frac{53}{132}$.
12. From $\frac{17}{24}$ take $\frac{17}{48}$. Ans. $\frac{17}{48}$.
13. From $\frac{1}{2}$ of $\frac{4}{7}$ take $\frac{1}{3}$ of $\frac{9}{16}$. Ans. $\frac{11}{112}$.
14. From $\frac{3}{8}$ of $\frac{16}{21}$ take $\frac{4}{5}$ of $\frac{15}{44}$. Ans. $\frac{1}{77}$.
15. From $\frac{2}{3}$ take $\frac{1}{3}$ of 2. Ans. 0.
16. What is the difference between $\frac{1}{2}$ of $\frac{3}{8}$ and $\frac{3}{8}$ of $\frac{1}{2}$? Ans. 0.
17. From $\frac{4}{5}$ of $12\frac{1}{2}$ take $\frac{5}{9}$ of $6\frac{1}{4}$. Ans. $6\frac{19}{36}$.

(18.)	(19.)	(20.)	(21.)	(22.)
$14\frac{3}{7}$	$25\frac{4}{5}$	$18\frac{3}{4}$	$37\frac{1}{2}$	$31\frac{1}{4}$
$9\frac{1}{7}$	$6\frac{2}{5}$	$12\frac{1}{2}$	$6\frac{1}{4}$	$6\frac{1}{4}$
$5\frac{2}{7}$	$19\frac{2}{5}$	$6\frac{1}{4}$	$31\frac{1}{4}$	25
(23.)	(24.)	(25.)	(26.)	(27.)
$42\frac{4}{7}$	$16\frac{6}{11}$	$15\frac{2}{3}$	$62\frac{1}{2}$	38
$18\frac{3}{8}$	$5\frac{2}{9}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{11}{15}$
$24\frac{11}{56}$	$11\frac{2}{9}$	$14\frac{11}{12}$	$61\frac{5}{8}$	$37\frac{4}{15}$
(28.)	(29.)	(30.)	(31.)	(32.)
14	105	$91\frac{1}{8}$	$72\frac{3}{5}$	75
$\frac{6}{13}$	$62\frac{1}{2}$	$\frac{9}{10}$	$24\frac{7}{9}$	$18\frac{3}{4}$
$13\frac{7}{13}$	$42\frac{1}{2}$	$90\frac{9}{40}$	$47\frac{37}{45}$	$56\frac{1}{4}$
(33.)	(34.)	(35.)	(36.)	(37.)
$57\frac{1}{9}$	$8\frac{6}{17}$	21	30	$144\frac{4}{19}$
$\frac{4}{5}$	$3\frac{5}{7}$	$4\frac{13}{14}$	$10\frac{3}{7}$	$98\frac{1}{19}$
$56\frac{14}{15}$	$4\frac{76}{119}$	$16\frac{1}{14}$	$19\frac{4}{7}$	$46\frac{3}{19}$

MULTIPLICATION OF COMMON FRACTIONS.

CASE I.

Art. 118. To multiply a fraction by a whole number

Ex. 1. If a yard of calico costs $\frac{1}{8}$ of a dollar, what cost 4 yards? Ans. $\frac{1}{2}$ of a dollar.

FIRST METHOD.

$$\frac{1}{8} \times 4 = \frac{1 \times 4}{8} = \frac{4}{8} = \frac{1}{2}$$

SECOND METHOD.

$$\frac{1}{8} \times 4 = \frac{1}{8 \div 4} = \frac{1}{2}$$

MENTAL PROCESS.

In the first method, we proceed on the principle that multiplying the numerator multiplies the value of the fraction; that is, 4 times 1 eighth is 4 eighths, ($\frac{4}{8}$,) which, in lowest terms, is $\frac{1}{2}$.

In the second method, we proceed on the principle that the value of a fraction is multiplied by dividing its denominator. Four times the value of $\frac{1}{8}$ is the same as the value of 1 divided by $\frac{1}{4}$ of 8.

RULE.—*Multiply the numerator, and write the product over the denominator; or,*

Divide the denominator by the multiplier, when it can be done without a remainder, and write the quotient as a new denominator to the given numerator.

EXAMPLES FOR PRACTICE.

2. Multiply $\frac{5}{8}$ by 4. Ans. $2\frac{1}{2}$.
3. Multiply $\frac{7}{9}$ by 6. Ans. $4\frac{2}{3}$.
4. Multiply $\frac{1}{2}\frac{3}{4}$ by 8. Ans. $4\frac{1}{3}$.
5. Multiply $\frac{1}{3}\frac{1}{6}$ by 18. Ans. $5\frac{1}{2}$.
6. Multiply $\frac{3}{1}\frac{}{6}$ by 16. Ans. 3.
7. Multiply $\frac{2}{3}\frac{1}{7}$ by 9. Ans. $5\frac{4}{7}$.
8. At $\$2\frac{3}{4}$ apiece, what cost 10 hats? Ans. $\$27\frac{1}{2}$.

WRITTEN PROCESS.

$$\begin{array}{r} 2\frac{3}{4} \\ 10 \\ \hline 20 = 10 \times 2 \\ \hline 27\frac{1}{2} \end{array}$$

$$10 \times \tfrac{3}{4} = \tfrac{30}{4} = 7\tfrac{1}{2}$$

MENTAL PROCESS.

Ten hats cost 10 times $\$2\frac{3}{4}$; that is, 10 times \$2 + 10 times $\frac{3}{4}$ of a dollar. 10 times \$2 = \$20, and 10 times $\frac{3}{4}$ of a dollar is $\frac{30}{4}$ of a dollar; that is, $\$7\frac{1}{2}$. $\$20 + \$7\frac{1}{2} = \$27\frac{1}{2}$.

QUESTION.—118. How do we multiply a fraction by a whole number?

9. At $\$1\frac{1}{8}$ per bushel, what cost 40 bushels of wheat?
Ans. \$45.

10. If a man's wages are $\$1\frac{5}{8}$ per day, what are they per working week? Ans. $\$9\frac{3}{4}$.

11. What cost 458 bushels of coal, at $5\frac{3}{4}$ cents per bushel?
Ans. \$26.33,5.

12. At $7\frac{1}{2}$ cents per yard, what cost 43 yards of muslin?
Ans. \$3.22,5.

13. If a candle burns $4\frac{5}{6}$ hours, how long would 15 similar ones last, burning consecutively? Ans. 72 hours 30 min.

Case II.

Art. 119. To multiply a whole number by a fraction.

Ex. 1. Multiply 35 by $\frac{4}{7}$. Ans. 20.

First Method.

$$\begin{array}{r} 35 \\ 4 \\ \hline 7\,)\,140 \\ \hline 20 \end{array}$$

Second Method.

$$\begin{array}{r} 7\,)\,35 \\ \hline 5 \\ 4 \\ \hline 20 \end{array}$$

Mental Process.

Since $\frac{1}{7}$ of 4 times 35, is the same value as 4 times $\frac{1}{7}$ of 35, it is equally proper to multiply the whole number first by the numerator, and divide the product by the denominator; or, first to divide the whole number by the denominator, and multiply the quotient by the numerator. The latter method is preferable, when the division can be performed without a remainder.

Rule.—*Either multiply the whole number by the numerator, and divide the product by the denominator; or,*

Divide the whole number by the denominator, and multiply the quotient by the numerator.

Examples for Practice.

2. Find $\frac{11}{12}$ of 96. Ans. 88.
3. Multiply 625 by $\frac{4}{25}$. Ans. 100.
4. Find $\frac{7}{9}$ of 1206. Ans. 938.

Questions.—119. How do we multiply a whole number by a fraction? How a whole number by a mixed number?

5. Multiply 842 by $\frac{14}{17}$. Ans. $693\frac{7}{17}$.
6. Find $\frac{2}{3}$ of $\frac{7}{8}$ of 871. Ans. $47\frac{3}{3}\frac{7}{7}$.
7. Multiply 1763 by $\frac{8}{15}$. Ans. $940\frac{4}{15}$.
8. Find the product of $18 \times 2\frac{3}{4}$. Ans. $49\frac{1}{2}$.

First Method.

$$\begin{array}{r} 18 \\ 2\frac{3}{4} \\ \hline 36 \quad 4\frac{1}{2} = \frac{1}{4} \text{ of } 18 \\ 3 \times 4\frac{1}{2} = 13\frac{1}{2} = \frac{3}{4} \text{ of } 18 \\ \hline 49\frac{1}{2} \end{array}$$

Second Method.

$18 = \frac{18}{1}$, and $2\frac{3}{4} = \frac{11}{4}$

$\frac{18}{1} \times \frac{11}{4} = \frac{198}{4} = 49\frac{1}{2}$

Mental Process.

The product of 18 by $2\frac{3}{4}$ is equal to the sum of the products by 2 and by $\frac{3}{4}$. Twice 18 is 36, and $\frac{3}{4}$ of 18 is 3 times $\frac{1}{4}$ of 18 $= 13\frac{1}{2}$: $36 + 13\frac{1}{2} = 49\frac{1}{2}$.

9. Multiply 56 by $12\frac{5}{9}$. Ans. $703\frac{1}{9}$.
10. If a cubic foot of brick work weighs 1872 ounces, what is the weight of $9\frac{5}{7}$ cubic feet? Ans. 11cwt. 36lbs. $9\frac{1}{7}$oz.
11. What cost $5\frac{7}{8}$ yards of cloth at \$8 per yard? Ans \$47.
12. What cost $9\frac{3}{5}$ tons of hay at \$12 per ton? Ans. $\$115\frac{1}{5}$.
13. If a family consume $\frac{5}{7}$ of a barrel of flour in a month, how much will they consume in a year? Ans. $8\frac{4}{7}$ bbls.

Case III.

Art. 120. To multiply a fraction by a fraction.

Ex. 1. A farmer had a lot containing $\frac{7}{8}$ of an acre, and sold $\frac{2}{5}$ of it. What part of an acre did he sell? Ans. $\frac{7}{20}$.

Written Process.

$$\frac{2}{5} \text{ of } \frac{7}{8} = \frac{7}{8} \times \frac{2}{5} = \frac{7 \times 2}{8 \times 5} = \frac{14}{40} = \frac{7}{20}$$

By Cancellation.

$$^{4}\,\frac{7}{\cancel{8}} \times \frac{\cancel{2}}{5} = \frac{7}{20}$$

Mental Process.

Here $\frac{1}{5}$ of $\frac{7}{8}$ is to be taken 2 times. But to get a fifth we divide by 5, which is done to a fraction by multiplying its denominator. This produces $\frac{7}{40}$. Now, if $\frac{7}{40}$ are *one-fifth* of $\frac{7}{8}$, two times $\frac{7}{40}$, that is $\frac{14}{40}$, are two fifths of $\frac{7}{8}$. This, in lowest terms, is $\frac{7}{20}$. This is the same process as is used in reducing compound fractions to simple fractions.

RULE.—*Multiply the numerators together for a new numerator, and the denominators together for a new denominator.*

NOTE.—Resort to cancellation whenever it will lessen labor.

EXAMPLES FOR PRACTICE.

2. Find the product of $\frac{1}{2} \times \frac{4}{9} \times \frac{3}{16}$. Ans. $\frac{1}{24}$.
3. Multiply $\frac{24}{25}$ by $\frac{55}{96}$. Ans. $\frac{11}{20}$.
4. Find the product of $\frac{9}{17} \times \frac{51}{81} \times \frac{34}{45}$. Ans. $\frac{34}{135}$.
5. Multiply $\frac{15}{64}$ by $\frac{72}{85}$. Ans. $\frac{27}{136}$.
6. What is the product of $\frac{5}{9} \times \frac{2}{15} \times \frac{9}{11} \times \frac{6}{7}$? Ans. $\frac{4}{77}$.
7. Multiply $\frac{7}{11}$ of $\frac{3}{8}$ by $\frac{8}{9}$ of $\frac{11}{28}$. Ans. $\frac{1}{12}$.
8. Multiply $\frac{2}{3}$ of 8 by $\frac{3}{4}$ of 9. Ans. 36.
9. At the rate of $\frac{2}{3}$ of a mile a minute, how far would a train move in a second? Ans. $\frac{1}{90}$ of a mile.
10. What cost $\frac{3}{4}$ of a pound of tea, at $\frac{2}{3}$ of a dollar per pound? Ans. $\frac{1}{2}$ of a dollar.

CASE IV.

Art. 121. To multiply a mixed number by a mixed number.

Ex. 1. Multiply $7\frac{5}{11}$ by $9\frac{4}{13}$. Ans. $69\frac{5}{13}$.

FIRST METHOD.

$7\frac{5}{11} = \frac{82}{11}$ and $9\frac{4}{13} = \frac{121}{13}$

$\frac{82}{11} \times \frac{121}{13} = \frac{9922}{143} = 69\frac{55}{143} = 69\frac{5}{13}$

EXPLANATION.

In the first method, we reduce the mixed numbers to fractions, and then treat them as in Case III.

SECOND METHOD.

$7\frac{5}{11}$

$9\frac{4}{13}$

$9 \times 7 = 63$

$9 \times \frac{5}{11} = 4\frac{1}{11} = \frac{143}{11}$

$\frac{4}{13} \times 7 = 2\frac{2}{13} = \frac{22}{143}$

$\frac{4}{13} \times \frac{5}{11} = \frac{20}{143} = \frac{20}{143}$

$69\frac{55}{143} = 69\frac{5}{13}$

In the second method, we multiply both parts of $7\frac{5}{11}$ by 9: then by $\frac{4}{13}$, and add the four products. Thus, $9 \times 7 = 63$, and $9 \times \frac{5}{11} = \frac{45}{11} = 4\frac{1}{11}$. Again, $\frac{4}{13} \times 7 = \frac{28}{13} = 2\frac{2}{13}$: also, $\frac{4}{13} \times \frac{5}{11} = \frac{20}{143}$. Then reduce the $\frac{1}{11}$, $\frac{2}{13}$, and $\frac{20}{143}$ to a common denominator, and add them, making $\frac{55}{143}$, or $\frac{5}{13}$. Then add the whole numbers.

RULE.—*Either reduce the mixed numbers to fractions, and multiply as in the case of a fraction by a fraction; or,*

Multiply the whole number and fraction of the multiplicand

QUESTIONS.—120. How do we multiply a fraction by a fraction? How can we sometimes lessen labor?—121. How do we multiply a mixed number by a mixed number?

separately, by the whole number and fraction of the multiplier separately, and unite the products in one sum.

EXAMPLES FOR PRACTICE.

2. What cost $4\frac{1}{2}$lbs. of tea, at $31\frac{1}{4}$ cts. per lb.?
Ans. $\$1.40\frac{5}{8}$.

3. What cost $5\frac{5}{8}$ cords of wood, at $\$4\frac{5}{6}$ per cord?
Ans. $\$27.18\frac{3}{4}$.

4. How many square inches in a slate $10\frac{5}{8}$ inches long and $8\frac{9}{16}$ inches broad? Ans. $90\frac{1}{128}\frac{5}{8}$.

5. How many square rods in a piece of land $75\frac{3}{7}$ rods long and $36\frac{2}{9}$ rods broad? Ans. $2732\frac{4}{2}\frac{}{1}$.

6. How much would $17\frac{1}{2}$ bottles of wine cost, each containing $\frac{2}{9}$ of a gallon, at $\$2\frac{3}{8}$ per gallon? Ans. $\$9\frac{17}{72}$

7. How many cubic feet in a piece of masonry $9\frac{5}{12}$ feet long, $2\frac{3}{4}$ feet thick, and $21\frac{3}{4}$ feet high? Ans. $546\frac{1}{8}$.

8. A cubic foot of ice weighs $58\frac{1}{8}$ lbs. What is the weight of a block of ice $16\frac{1}{3}$ feet long, $9\frac{7}{12}$ feet broad, and 11 inches thick? Ans. 4T. 3cwt. 1qr. $14\frac{1}{1}\frac{1}{1}\frac{4}{5}\frac{7}{2}$lbs.

9. What would be the cost of a piece of cloth $31\frac{7}{8}$yds. long, and $1\frac{3}{8}$yd. broad, at $11\frac{1}{2}$cts. per sq. yd.? Ans. $\$5.04\frac{3}{1}\frac{}{2}\frac{}{8}$.

DIVISION OF COMMON FRACTIONS.

CASE I.

Art. 122. To divide a fraction by a whole number.

Ex. 1. Divide $\frac{6}{11}$ by 2. Ans. $\frac{3}{11}$.

METHODS INDICATED.

FIRST. $\frac{6}{11} \div 2 = \frac{6 \div 2}{11} = \frac{3}{11}$

SECOND. $\frac{6}{11} \div 2 = \frac{6}{11 \times 2} = \frac{6}{22} = \frac{3}{11}$

THIRD. $\frac{6}{11} \div 2 = \frac{6}{11} \times \frac{1}{2} = \frac{6}{22} = \frac{3}{11}$

EXPLANATION.

In the first method, we divide the numerator, because that divides the value of the fraction. (See Art. 103.) In the second method, we multiply the denominator, because that divides the value of the fraction. (See Art. 103.) In the third method, we reason that $\frac{6}{11}$ divided by 2 is the same as $\frac{1}{2}$ of $\frac{6}{11}$, which is a compound fraction, to be reduced as in Art. 113.

RULE.—*Divide the numerator by the whole number, and write the quotient over the denominator.*

QUESTION.—122. How do we divide a fraction by a whole number?

Or, *Multiply the denominator by the whole number, and write the product under the numerator.*

NOTE 1.—The former method is preferable, when the whole number is contained in the numerator without a remainder.

NOTE 2.—Resort to cancellation when it will lessen labor.

EXAMPLES FOR PRACTICE.

2. Divide $\frac{8}{17}$ by 4. Ans. $\frac{2}{17}$.
3. Divide $\frac{25}{27}$ by 5. Ans. $\frac{5}{27}$.
4. Divide $\frac{11}{12}$ by 6. Ans. $\frac{11}{72}$.
5. Divide $\frac{88}{103}$ by 11. Ans. $\frac{8}{103}$.
6. Divide $\frac{9}{19}$ by 6. Ans. $\frac{3}{38}$.
7. Divide $\frac{18}{35}$ by 12. Ans. $\frac{3}{70}$.
8. Divide $\frac{24}{47}$ by 16. Ans. $\frac{3}{94}$.
9. Divide $\frac{144}{155}$ by 60. Ans. $\frac{12}{775}$.
10. Divide $\frac{178}{184}\frac{28}{5}$ by 36. Ans. $\frac{48}{1845}$.
11. Divide $\frac{132}{227}\frac{20}{7}$ by 9. Ans. $\frac{440}{68831}$.
12. If a cask of molasses, containing 75 gallons, costs $\$33\frac{3}{4}$, what is the cost of 1 gallon? Ans. $\frac{9}{20}$ of a dollar.

PROCESS. $33\frac{3}{4} = {}^{135}\!/_{4}$. $\quad {}^{135}\!/_{4} \div 75 = \frac{135}{300} = \frac{9}{20}$.

13. If a dozen dinner-plates cost $62\frac{1}{2}$ cents, what is the cost of 1 plate? Ans. $5\frac{5}{24}$ cents.
14. If a gross of steel pens cost $37\frac{1}{2}$ cents, what is the cost of 1 pen? Ans. $\frac{25}{96}$ of a cent.
15. If a set of half a dozen knives and forks cost $\$4\frac{1}{2}$, what is the average cost per piece? Ans. $\frac{3}{4}$ of a dollar.
16. Divide $\$16\frac{4}{7}$ equally among four men.

Ans. Each has $\$4\frac{1}{7}$.

PROCESS. $4)16\frac{4}{7}$ or $16\frac{4}{7} \div 4 = \frac{116}{7} \div 4 = \frac{29}{7} = 4\frac{1}{7}$.

17. Divide $\$35\frac{15}{16}$ equally among 5 men. Ans. Each has $\$7\frac{3}{16}$.

CASE II.

Art. 123. To divide a whole number by a fraction.

Ex. 1. Divide 10 by $\frac{2}{5}$. Ans. 25.

PROCESSES INDICATED.	MENTAL PROCESS.
FIRST. $10 \div \frac{2}{5} = \frac{10}{1} \div \frac{2}{5} = \frac{50}{5} \div \frac{2}{5} = 25$	Since divisor and dividend must mean *the same things*, (see Art. 43,) it is necessary, if we divide 10 by $\frac{2}{5}$, first to reduce 10 to fifths making 50 fifths. Then 2 fifths are contained in 50 fifths 25 times
SECOND. $\frac{10}{1} \div \frac{2}{5} = \frac{10}{1} \times \frac{5}{2} = \frac{50}{2} = 25$	

Now, 10 is thus reduced and divided by simply inverting the divisor $\frac{2}{5}$, making $\frac{5}{2}$, and multiplying 10 by it, making $\frac{5}{2}0 = 25$. Or we may reason as follows:—If we divide 10 by the numerator 2 only, we get a result *five times too small*, because it is not required to divide by the *whole* of two, but by a *fifth* of 2. Therefore, we should multiply that result by 5, the denominator. This also is attained by inverting the $\frac{2}{5}$, and multiplying 10 by it.

RULE.— *Write the numerator for the denominator, and the denominator for the numerator, and multiply the whole number by the resulting fraction.*

EXAMPLES FOR PRACTICE.

2. Divide 36 by $\frac{6}{11}$. Ans. 66.
3. Divide 42 by $\frac{28}{33}$. Ans. $49\frac{1}{2}$.
4. Divide 63 by $\frac{15}{22}$. Ans. $92\frac{2}{5}$.
5. Divide 75 by $8\frac{1}{3}$. Ans. 9.

PROCESS. $75 \div 8\frac{1}{3} = 75 \div \frac{25}{3} = \overset{3}{\cancel{75}} \times \frac{3}{\cancel{25}} = 9.$

6. At $4\frac{1}{2}$ cents per lb., how much rice can be bought for 90 cents? Ans. 20lbs.
7. At $\$3\frac{1}{4}$ per hat, how many hats will \$104 buy? Ans. 32.
8. If one pound of sugar costs $12\frac{1}{2}$ cents, how many pounds can be bought for \$5.50? Ans. 44lbs.

Art. 124. The *reciprocal* of a number is the quotient arising from dividing 1 by that number. Thus, the reciprocal of 5 is $\frac{1}{5}$; of 9 it is $\frac{1}{9}$, &c.

To form the reciprocal of a fraction.

Ex. 1. What is the reciprocal of $\frac{3}{5}$? Ans. $\frac{5}{3}$.

DEMONSTRATION.

$1 \div \frac{3}{5} = 1 \times \frac{5}{3} = \frac{5}{3}$ Hence the following

RULE.—*Interchange the terms of the fraction.*

EXAMPLES FOR PRACTICE.

2. What is the reciprocal of $\frac{4}{9}$? of $\frac{3}{8}$? of $\frac{6}{11}$? of $\frac{8}{13}$?
3. What is the reciprocal of $\frac{7}{15}$? of $\frac{6}{7}$? of $\frac{11}{12}$? of $\frac{15}{16}$? of $\frac{9}{17}$?
4. What is the reciprocal of $\frac{12}{19}$? of $\frac{14}{23}$? of $\frac{17}{25}$? of $\frac{19}{36}$ of $\frac{21}{37}$?

QUESTIONS.—123. How do we divide a whole number by a fraction?—124. What is the reciprocal of a number? How do we form the reciprocal of a fraction?

Case III.

Art. 125. To divide a fraction by a fraction.

Ex. 1. How many times $\frac{2}{11}$ in $\frac{19}{20}$? Ans. $5\frac{9}{40}$.

Process Indicated.

$$\frac{19}{20} \div \frac{2}{11} = \frac{19}{20} \times \frac{11}{2} = \frac{209}{40} = 5\frac{9}{40}$$

Mental Process.

If we divide $\frac{19}{20}$ by 2, we obtain $\frac{19}{40}$. But $\frac{19}{40}$ must be *eleven times too small*, because it is not required to divide by 2, but by *one eleventh* of 2. Hence, we must multiply $\frac{19}{40}$ by 11, making $\frac{19 \times 11}{40} = \frac{209}{40} = 5\frac{9}{40}$.

Ex. 2. Divide $\frac{1}{2}$ of $\frac{4}{9}$ of $\frac{8}{15}$ by $\frac{4}{5}$ of $\frac{8}{9}$. Ans. $\frac{1}{6}$.

Process Indicated.

$$\frac{1}{2} \times \frac{4}{9} \times \frac{8}{15} \div \frac{4}{5} \times \frac{8}{9} = \frac{1}{2} \times \frac{\cancel{4}}{\cancel{9}} \times \frac{\cancel{8}}{\cancel{15}} \times \frac{\cancel{5}}{\cancel{4}} \times \frac{\cancel{9}}{\cancel{8}} = \frac{1}{6}.$$

3

Explanation.

Since the *value* of a compound fraction can be expressed in a simple fraction, it is plain that its members can have their terms interchanged and used as multipliers, as in Example 1. Then the dividend and divisor become one compound fraction, to be reduced to a simple fraction in the usual way.

Rule.—*Multiply the dividend by the reciprocal of the divisor.*

Note 1.—To divide one mixed number by another, first reduce them to fractions, and proceed as in the rule.

Note 2.—Resort to cancellation, when it will lessen labor.

Examples for Practice.

3. Divide $\frac{7}{9}$ by $\frac{6}{7}$. Ans. $\frac{49}{54}$.
4. Divide $\frac{12}{17}$ by $\frac{4}{5}$. Ans. $\frac{15}{17}$.
5. Divide $\frac{9}{20}$ by $\frac{12}{13}$. Ans. $\frac{39}{80}$.
6. Divide $\frac{3}{7}$ by $\frac{5}{7}$. Ans. $\frac{3}{5}$.
7. Divide $\frac{8}{11}$ by $\frac{7}{22}$. Ans. $2\frac{2}{7}$.
8. Divide $\frac{9}{10}$ by $\frac{2}{7}$. Ans. $3\frac{3}{20}$.
9. Divide $\frac{1}{3}$ of $\frac{6}{7}$ by $\frac{5}{7}$ of $\frac{12}{13}$. Ans. $\frac{13}{30}$
10. Divide $\frac{3}{5}$ of $2\frac{1}{2}$ by $\frac{4}{5}$ of 9. Ans. $\frac{5}{24}$.
11. Divide $\frac{4}{5}$ of $\frac{15}{16}$ of $\frac{12}{13}$ by $\frac{5}{26}$ of $\frac{18}{19}$. Ans. $3\frac{4}{5}$.
12. Divide $11\frac{1}{2}$ by $2\frac{1}{4}$. Ans. $5\frac{1}{9}$.
13. Divide $31\frac{1}{4}$ by $6\frac{1}{4}$. Ans. 5.
14. If $\frac{2}{3}$ of a barrel of flour cost $5\frac{1}{2}$ dollars, what will 1 barrel cost? Ans. $8.25.

Questions.—125. How do we divide a fraction by a fraction? How do we divide a mixed number by a mixed number?

15 If $\frac{3}{4}$ of a yard of cloth cost \$7, what will 1 yard cost? Ans. $\$9\frac{1}{3}$.

16. If a man average $23\frac{3}{4}$ miles' travel per day, how long will he be in going $134\frac{4}{5}$ miles? Ans. $5\frac{3}{4}\frac{2}{7}\frac{1}{5}$ days.

17. At $\$2\frac{3}{8}$ per pair, how many pairs of shoes can be bought for $\$35\frac{5}{8}$? Ans. 15 pairs.

18. If $\frac{5}{6}$ of an acre of land cost $\$43\frac{1}{2}$, what will 1 acre cost? Ans. $\$52\frac{1}{5}$.

COMPLEX FRACTIONS.

Art. 126. To find the value of a complex fraction.

Ex. 1. What is the value of $\dfrac{\frac{2}{3}}{2\frac{1}{2}}$? Ans. $\frac{4}{15}$.

PROCESS INDICATED.

$$\frac{\frac{2}{3}}{2\frac{1}{2}} = \frac{2}{3} \div 2\tfrac{1}{2} = \frac{2}{3} \div \frac{5}{2} = \frac{2}{3} \times \frac{2}{5} = \frac{4}{15}$$

MENTAL PROCESS.

Since a fraction expresses the division of its numerator by its denominator, the complex fraction $\dfrac{\frac{2}{3}}{2\frac{1}{2}}$ expresses the quotient of $\frac{2}{3} \div 2\frac{1}{2}$. This reduces to $\frac{4}{15}$, as shown in the process.

RULE.—*Reduce the expression in each term of the complex fraction to a simple fraction if necessary: then divide the numerator of the complex fraction by its denominator.*

EXAMPLES FOR PRACTICE.

2. What is the value of $\dfrac{15}{\frac{3}{7}}$? Ans. 35.

3. What is the value of $\dfrac{\frac{3}{7}}{15}$? Ans. $\frac{1}{35}$.

4. What is the value of $\dfrac{6\frac{1}{8}}{7\frac{5}{9}}$? Ans. $\frac{441}{544}$.

5. What is the value of $\dfrac{\frac{1}{2} + \frac{2}{3}}{\frac{4}{5}}$? Ans. $1\frac{11}{24}$.

6. What is the value of $\dfrac{\frac{5}{9} - \frac{1}{4}}{\frac{7}{8} - \frac{2}{3}}$? Ans. $1\frac{7}{15}$.

QUESTION.—126. How do we find the value of a complex fraction?

7. What is the value of $\dfrac{\frac{4}{5} \times \frac{11}{12}}{\frac{2}{7} \times \frac{14}{15}}$? Ans. $2\frac{3}{4}$.

8. What is the value of $\dfrac{\frac{6}{7} \div \frac{1}{2}}{\frac{5}{8} \div \frac{2}{5}}$? Ans. $1\frac{17}{175}$.

9. Simplify $\dfrac{(\frac{1}{8} \times \frac{3}{7}) - (\frac{1}{9} \times \frac{4}{11}) + 5}{\frac{5}{12} - (\frac{3}{4} \times \frac{2}{15}) + 7}$. Ans. $\frac{138965}{202818}$.

10. Simplify $\dfrac{(\frac{3}{4} \times 5\frac{1}{2}) + (\frac{8}{9} \times 6\frac{3}{5})}{5\frac{3}{4} \div (\frac{2}{3} \times 6\frac{7}{10})}$. Ans. $7\frac{78833}{103350}$.

DENOMINATE FRACTIONS.

Art. 127. A DENOMINATE FRACTION is a fraction which expresses one or more parts of a measuring unit. Thus, $\frac{3}{4}$ of an hour, $\frac{2}{3}$ of an ounce, and $\frac{5}{8}$ of a yard are denominate fractions.

REDUCTION OF DENOMINATE FRACTIONS.

Art. 128. REDUCTION of a denominate fraction is converting a fraction expressing parts of a certain measuring unit into an equivalent expression in other measuring units.

CASE I.

Art. 129. To reduce a fraction to an equivalent fraction in a lower denomination.

Ex. 1. What part of a dime is $\frac{1}{20}$ of a dollar? Ans. $\frac{1}{2}$.

PROCESS INDICATED.

$\frac{1}{20}$ of \$1 $= \frac{1}{20}$ of 10 dimes $= \frac{1}{20}$ of $\frac{10}{1} = \frac{10}{20} = \frac{1}{2}$

MENTAL PROCESS.

Since 10 dimes make 1 dollar, $\frac{1}{20}$ of a dollar must be $\frac{1}{20}$ of 10 dimes; that is, $\frac{10}{20}$ of 1 dime; that is, $\frac{1}{2}$ of a dime.

Ex. 2. What part of a farthing is $\frac{1}{1000}$ of a pound? Ans. $\frac{24}{25}$.

FIRST PROCESS INDICATED.

$\frac{1}{\cancel{1000}^{\cancel{600}\,25}} \times \frac{\cancel{20}}{1} \times \frac{\cancel{12}^{6}}{1} \times \frac{4}{1} = \frac{24}{25}$

SECOND PROCESS INDICATED.

$\frac{1}{1000} \times \frac{960}{1} = \frac{96}{100} = \frac{24}{25}$

MENTAL PROCESS.

$\frac{1}{1000}$ of £1 $= \frac{1}{1000}$ of 20s., $= \frac{20}{1000}$ of 1s. $\frac{20}{1000}$ of 1s. $= \frac{20}{1000}$ of 12d. $= \frac{240}{1000}$ of 1d. $\frac{240}{1000}$ of 1d. $= \frac{240}{1000}$ of 4qrs. $= \frac{960}{1000}$ of 1qr. It is plain that all these operations can be represented in one, and shortened by cancellation. In the second process we say at once that $\frac{1}{1000}$ of £1 $= \frac{1}{1000}$ of 960qrs., since 960 qrs. make £1.

RULE.—*Multiply the fraction by the numbers that reduce its denomination to the denomination required.*

EXAMPLES FOR PRACTICE.

3. Reduce $\frac{1}{1260}$ of a £ to the fraction of a penny. Ans. $\frac{4}{21}$.
4. What part of a foot is $\frac{2}{11985}$ of a mile? Ans. $\frac{704}{999}$.
5. Express $\frac{1}{91800}$ of an acre in terms of a sq. ft. Ans. $\frac{12}{25}\frac{2}{5}\frac{1}{5}$.
6. Express $\frac{3}{186624}$ of a cu. yd. in terms of a cu. in. Ans. $\frac{3}{4}$.
7. What part of a grain is $\frac{1}{172800}$ of a pound Troy? Ans. $\frac{1}{3}$.
8. What part of a ℈ is $\frac{3}{5}\frac{3}{3}$ of an ℥? Ans. $\frac{7}{8}\frac{2}{3}$.
9. Reduce $\frac{1}{36000}$ of a ton to terms of an ounce. Ans. $\frac{8}{9}$.
10. What part of a pint is $\frac{1}{12}$ of a gallon? Ans. $\frac{2}{3}$.
11. What part of a f℥ is $\frac{1}{2}\frac{1}{5}\frac{1}{6}$ of a pint? Ans. $\frac{1}{2}$.
12. What part of a minute of arc is $\frac{1}{30000}$ of a circumference? Ans. $\frac{1}{2}\frac{8}{5}$.
13. What part of a minute is $\frac{1}{21600}$ of a day? Ans. $\frac{2}{3}$.
14. What part of a pint is $\frac{1}{112}$ of a bushel? Ans. $\frac{4}{7}$.

CASE II.

Art. 130. To reduce a fraction to an equivalent fraction of a higher denomination.

Ex. 1. What part of a dollar is $\frac{1}{2}$ of a dime? Ans. $\frac{1}{20}$.

PROCESS INDICATED.

$\frac{1}{2}$ of a dime $= \frac{1}{2}$ of $\frac{1}{10}$ of a dollar $= \frac{1}{20}$ of a dollar.

Ex. 2. What part of a £ is $\frac{24}{25}$ of a farthing? Ans. $\frac{1}{1000}$.

FIRST PROCESS INDICATED.

$$\frac{\overset{2}{\cancel{24}}}{25} \times \frac{1}{\underset{2}{\cancel{4}}} \times \frac{1}{\cancel{12}} \times \frac{1}{20} = \frac{1}{1000}.$$

SECOND PROCESS INDICATED.

$$\frac{24}{25} \times \frac{1}{\underset{40}{\cancel{960}}} = \frac{1}{1000}.$$

MENTAL PROCESS.

$\frac{24}{25}$ of 1 qr. $= \frac{24}{25}$ of $\frac{1}{4}$ of a penny, that is $\frac{24}{100}$ of a penny. $\frac{24}{100}$ of a penny $= \frac{24}{1000}$ of $\frac{1}{12}$ of 1s. $= \frac{24}{12000}$ of 1s. $= \frac{24}{12000}$ of 1s. $= \frac{24}{12000}$ of $\frac{1}{20}$ of £1 $= \frac{24}{240000}$ of £1 $= \frac{1}{10000}$ of £. It is plain that all these operations can be represented in one, and shortened by cancellation. In the second process we say at once, $\frac{24}{25}$ of a farthing $= \frac{24}{25}$ of $\frac{1}{960}$ of a £, since 960 farthings make a £.

QUESTIONS.—127. What is a denominate fraction?—128. What is reduction of a denominate fraction?—129. How do we reduce a fraction to terms of a lower denomination?

RULE.—*Divide the fraction by the numbers that reduce its denomination to the denomination required.*

EXAMPLES FOR PRACTICE.

3. What part of a mile is $\frac{3}{4}$ of a yard? Ans. $\frac{3}{7040}$.
4. What part of a chain is $\frac{198}{25}$ of an inch? Ans. $\frac{1}{100}$.
5. What part of a square mile is $\frac{1}{2}$ of a rod square? Ans. $\frac{1}{4096000}$.
6. What part of a square foot is $\frac{1}{2}$ of a foot square? Ans. $\frac{1}{4}$.
7. What part of $\frac{3}{4}$ of a square yard is $\frac{3}{4}$ of a yard square? Ans. $\frac{3}{4}$.
8. What part of a cubic foot is $\frac{1}{2}$ of a foot cube? Ans. $\frac{1}{8}$.
9. What part of a gallon is $\frac{3}{4}$ of a pint? Ans. $\frac{3}{32}$.
10. What part of a ton is $\frac{5}{8}$ of a pound? Ans. $\frac{1}{32000}$.
11. What part of a week is $\frac{2}{3}$ of an hour? Ans. $\frac{1}{252}$.
12. What part of a circumference is $\frac{4}{5}$ of a minute? Ans. $\frac{1}{27000}$.

CASE III.

Art. 131. To find the value of a denominate fraction.

Ex. 1. What is the value of $\frac{19}{20}$ of a lb. avoirdupois? Ans. 15oz. $3\frac{1}{5}$dr.

WRITTEN PROCESS.

$$\begin{array}{r} 19 \\ 16 \\ \hline 114 \\ 19 \\ \hline 20)304(15 \\ 20 \\ \hline 104 \\ 100 \\ \hline 4 \\ 16 \\ \hline 20)64(3\frac{1}{5} \\ 60 \\ \hline \frac{4}{20} = \frac{1}{5} \end{array}$$

MENTAL PROCESS.

$\frac{19}{20}$ of a lb. av. $= \frac{19}{20}$ of 16oz. $= \frac{304}{20}$ of 1oz. $=$ 15oz. and $\frac{4}{20}$ of an oz. Since 15oz. is already a whole number, we have only to find in the same manner the value of $\frac{4}{20}$ of an oz. $\frac{4}{20}$ of 1oz. $= \frac{4}{20}$ of 16dr. $= \frac{64}{20}$ of 1dr. $= 3$dr. and $\frac{4}{20}$ of a dr. Now, there is no lower denomination than drams: hence we cannot find the value of $\frac{4}{20}$ of a dram, but must reduce it to lowest terms and annex it to the whole number.

QUESTION.—130. How do we reduce a fraction to a fraction of a higher denomination?

RULE.—*Reduce the fraction to the first denomination in which it will be an improper fraction; then find the value of this improper fraction. If the result is a mixed number, reduce its fractional part in the same manner, and so on till the complete value is obtained.*

EXAMPLES FOR PRACTICE.

2. What is the value of $\frac{9}{10}$ of a yd.? Ans. 3qr. $2\frac{2}{5}$na.
3. What is the value of $\frac{4}{7}$ of an acre?
Ans. 2R. 11P. 116ft. $97\frac{5}{7}$in.
4. What is the value of $\frac{5}{9}$ of a cwt.?
Ans. 55lbs. 8oz. $14\frac{2}{9}$dr.
5. What is the value of $\frac{7}{22}$ of $365\frac{1}{4}$ days?
Ans. 116da. 5hr. 10min. $54\frac{6}{11}$sec.
6. What is the value of $\frac{5}{11}$ of a circumference?
Ans. 163° 38′ $10\frac{10}{11}$″.
7. What is the value of $\frac{4}{9}$ of a mile?
Ans. 3fur. 22rd. 3ft. 8in.
8. What is the value of $\frac{2}{3}$ of a bushel?
Ans. 2pk. 5qt. $0\frac{2}{3}$pt.
9. What is the value of $\frac{5}{7}$ of a gallon?
Ans. 2qt. 1pt. $2\frac{6}{7}$gi.
10. What is the value of $\frac{3}{5}$ of a ton? Ans. 12cwt.

Art. 132. To find the fraction that one denominate number is of another.

Ex. 1. What part of a week are 1hr. 10min. 30sec.?
Ans. $\frac{47}{6720}$.

PROCESS INDICATED.

$$\frac{\text{1hr. 10min. 30sec.}}{\text{7 days.}} = \frac{4230\text{sec.}}{604800\text{sec.}} = \frac{141}{20160} = \frac{47}{6720}$$

MENTAL PROCESS.

Since things compared must be of the same kind, we reduce both quantities to seconds, the lowest denomination in either of them. Then 1 hour 10min. 30sec. is that part of 7 days which the seconds in the former are of the seconds in the latter.

RULE.—*Reduce both numbers to the same denomination: then write that which is to be the part, for a numerator, and the other for a denominator.*

QUESTIONS.—131. How do we find the value of a denominate fraction?—132. How do we find the fraction that one denominate number is of another?

NOTE.—It is generally most convenient to select the lowest-mentioned denomination as that to which both should be reduced, and to reduce the fraction to lowest terms after it is constructed.

EXAMPLES FOR PRACTICE.

2. What part of \$5 are 75 cents? Ans. $\frac{3}{20}$.
3. What part of 8 miles are 180rds. 9ft.? Ans. $\frac{99}{1408}$.
4. What part of 4mi. 37ch. are 69ch.? Ans. $\frac{23}{119}$.
5. What part of $27\frac{1}{2}$ yds. are 5yds. 2ft. 9in.? Ans. $\frac{71}{330}$
6. What part of 10 feet square are 10 square feet?
Ans. $\frac{1}{10}$.
7. What part of a cord of wood is a load 8 feet long, $3\frac{1}{2}$ feet wide, and 2 feet 9in. high? Ans. $\frac{77}{128}$.
8. Having a barrel containing 40gal. of alcohol, I sold 7gal 3qt. 1pt. What part of the whole remained in the barrel?
Ans. $\frac{257}{320}$.
9. A druggist bought 5lbs of rhubarb, and used in prescriptions 2℔ 9ʒ 6ʒ 2ϑ. What part of it did he use?
Ans. $\frac{116}{275}$.
10. What is the least number of pound-weights of iron that will balance a whole number of pounds of gold?
Ans. 144.

ADDITION OF DENOMINATE FRACTIONS.

Art. 133. To find the sum of two or more denominate fractions.

Ex. 1. What is the sum of $\frac{5}{7}$ of a day, and $\frac{3}{11}$ of an hour?
Ans. 17hr. 24min. $56\frac{8}{77}$sec.

FIRST METHOD.

$\frac{5}{7}$ of a day = 17hr. 8min. $34\frac{2}{7}$sec.
$\frac{3}{11}$ of an hour = 16 " $21\frac{9}{11}$ "
Sum, 17hr. 24min. $56\frac{8}{77}$sec. Ans.

REMARK.

On performing the processes here indicated by Art. 131, the learner will need no further explanation.

SECOND METHOD.

$\frac{3}{11}$ of an hour = $\frac{3}{11}$ of $\frac{1}{24}$ of a day = $\frac{1}{88}$ of a day.
$\frac{5}{7}$ of a day + $\frac{1}{88}$ of a day = $\frac{447}{616}$ of a day = 17hr. 24min. $56\frac{8}{77}$sec.

QUESTION.—133. How do we find the sum of two or more denominate fractions?

RULE.—*Find the value of each fraction separately, and then add these values as in compound addition;* or,

Reduce the fractions to the same denomination, then add them, and find the value of their sum.

EXAMPLES FOR PRACTICE.

2. What is the sum of $\frac{3}{11}$ of a circumference, and $\frac{4}{7}$ of a degree? Ans. 98° 45′ $11\frac{5}{7}\frac{3}{7}$″.

3. To $\frac{4}{13}$ of a ton add $\frac{5}{13}$ of a cwt., and $\frac{6}{13}$ of a lb.
Ans. 6cwt. 54lb. 4oz. $14\frac{10}{13}$dr.

4. Add together $\frac{1}{3}$ of an acre, $\frac{1}{5}$ of a sq. rd., and $\frac{1}{7}$ of a sq. ft.
Ans. 53sq. rd. 145sq. ft. $49\frac{3}{5}$sq. in.

5. Find the sum of $\frac{6}{7}$ of a bushel, $\frac{2}{3}$ of a peck, and $\frac{4}{5}$ of a qt.
Ans. 1bu. 1qt. $1\frac{13}{105}$pt.

6. To get to the school-house, John walks $1\frac{3}{7}$ miles, James $2\frac{2}{9}$ miles, and Mary $\frac{1}{7}$ of a mile. What is the average of the distances walked by the three?
Ans. 1mi. 84rd. 10ft. $9\frac{19}{21}$in.

SUBTRACTION OF DENOMINATE FRACTIONS.

Art. 134. To subtract one denominate fraction from another.

Ex. 1. From $\frac{3}{7}$ of a £ take $\frac{2}{9}$ of a shilling.
Ans. 8s. 4d. $0\frac{16}{21}$qrs.

FIRST METHOD.

$\frac{3}{7}$£ = 8s. 6d. $3\frac{3}{7}$qr.
$\frac{2}{9}$s. = 2 $2\frac{2}{3}$
Diff. 8s. 4d. $0\frac{16}{21}$qr. Ans.

REMARK.

On performing the processes here indicated, by Art. 131, the learner will need no further explanation.

SECOND METHOD.

$\frac{2}{9}$s. = $\frac{2}{9} \times \frac{1}{20} = \frac{1}{90}$£
$\frac{3}{7} - \frac{1}{90} = \frac{263}{630}$£ = 8s. 4d. $0\frac{16}{21}$qr. Ans.

RULE.—*Find the value of each fraction separately, and then subtract one value from the other, by compound subtraction.*

Or, *Reduce the fractions to the same denomination, then subtract one from the other, and find the value of the resulting fraction.*

QUESTION.—134. How do we subtract one denominate fraction from another?

EXAMPLES FOR PRACTICE.

2. From $\frac{5}{11}$ of a mile, take $\frac{9}{10}$ of a rod.
Ans. 144rd. 9ft. $1\frac{4}{5}$ in.

3 From $\frac{7}{9}$ of a sq. mile, take $\frac{11}{13}$ of an acre.
Ans. 496A. 3R. 29P. 16ft. $41\frac{7}{13}$in.

4. From $\frac{4}{5}$ of a bushel, take $\frac{2}{3}$ of a peck.
Ans. 2pk. 4qt. $0\frac{8}{15}$pt.

5. From a hogshead capable of holding 123 gallons, but which was two-thirds full of vinegar, $27\frac{3}{8}$ gallons were sold: how much remained in the cask? Ans. 54gal. 2qt. 1pt.

6. A person, of $55\frac{1}{4}$ years of age, had spent $\frac{1}{3}$ of his life before he began to earn any thing: of the remainder of his life he had spent $23\frac{1}{7}$ years in other ways than labor. What time was he engaged in earning, reckoning $365\frac{1}{4}$ days as a year?
Ans. 13yr. 252da. 4hr. 42min. $51\frac{3}{7}$sec.

DUODECIMALS.

Art. 135. DUODECIMALS are those fractions of a linear, square, or cubic foot, which are expressed in descending denominations, in each of which a unit is one-twelfth of the next higher.

TABLE.

12 primes (′)	make	1 foot (ft.)
12 seconds (″)	"	1 prime.
12 thirds (‴)	"	1 second.
12 fourths (⁗)	"	1 third.
&c.	&c.	&c. &c.

The marks ′, ″, ‴, &c., denoting the denominations, are called *indices*.

From the table it appears that

1 prime	$= \frac{1}{12}$ of a	linear,	square, or cubic foot.
1 second $= \frac{1}{12}$ of $\frac{1}{12} = \frac{1}{144}$	"	"	"
1 third $= \frac{1}{12}$ of $\frac{1}{144} = \frac{1}{1728}$	"	"	"
1 fourth $= \frac{1}{12}$ of $\frac{1}{1728} = \frac{1}{20736}$	"	"	"
&c.	&c.	&c.	&c.

QUESTIONS.—135. What are duodecimals? Repeat the table. What are the indices?

Art. 136. To find the value of a duodecimal expression.

Ex. 1. What is the value of 6ft. 5′ 7″ 8‴ linear measure?

Ans. $6\frac{2}{4}\frac{0}{3}\frac{3}{2}$ linear feet.

Written Process.

6ft. $= 6$

$5' = \frac{5}{12} = \frac{720}{1728}$

$7'' = \frac{7}{144} = \frac{84}{1728}$

$8''' = \frac{8}{1728} = \frac{8}{1728}$

$6\frac{812}{1728} = 6\frac{2}{4}\frac{0}{3}\frac{3}{2}$

Rule.—*Express the primes, seconds, thirds, &c. as fractions, then add them, and annex their value to the number of feet.*

Examples for Practice.

2. What is the value of 9sq. ft. 3′ 11″ 10‴?

Ans. $9\frac{2}{8}\frac{8}{6}\frac{7}{4}$sq. ft.

3. What is the value of 25cu. ft. 4′ 6″ 2‴ 9⁗?

Ans. $25\frac{2}{6}\frac{6}{9}\frac{0}{1}\frac{3}{2}$cu. ft.

Art. 137. Duodecimal expressions may be added; or subtracted, and multiplied or divided by a whole number, like compound numbers.

Note.—Examples of these processes have been already given in Compound Numbers.

Art. 138. To find the denomination of the product of one duodecimal number by another.

When one duodecimal number, expressing *length*, is multiplied by another, the product expresses *surface*. Thus, 1ft. linear $\times$ 1′ linear $=$ 1′ square measure; that is, a surface 1ft. long, and 1 inch broad, which is equal to $\frac{1}{12}$ of a square foot.

Again, 1′ $\times$ 1′ linear $=$ 1 inch $\times$ 1 inch $=$ 1 square inch; $= \frac{1}{12} \times \frac{1}{12} = \frac{1}{144}$ of a square foot $=$ 1″ square measure.

Again, 1′ $\times$ 1″ linear $= \frac{1}{12} \times \frac{1}{144} = \frac{1}{1728}$ of a square foot; $=$ 1‴ square measure.

Also, 1″ $\times$ 1″ linear $= \frac{1}{144} \times \frac{1}{144} = \frac{1}{20736}$ of a square foot; $=$ 1⁗ square measure.

&c. &c. &c. &c.

When one duodecimal number, expressing *surface*, is multi-

Questions.—136. How do we find the value of a duodecimal expression?—137. How are duodecimals added, subtracted, and multiplied or divided by a whole number?—138. How do we express the denomination of a product?

plied by another expressing *length*, the product expresses *volume* or *cubical capacity*.

Thus, 1ft. sq. $\times$ 1′ linear = 1′ cubic; that is, $\frac{1}{12}$ of a cubic foot.

Again, 1′ sq. $\times$ 1′ linear = $\frac{1}{12}$ of a sq. ft. $\times$ $\frac{1}{12}$ of a linear foot = $\frac{1}{144}$ of a cubic foot = 1″ cubic.

&c. &c. &c.

From these illustrations we perceive that

Feet multiplied by *feet*, produce *feet*.

Feet multiplied by *another denomination*, produce *that denomination*.

Primes, multiplied by *primes*, produce *seconds;* by *seconds*, produce *thirds;* by *thirds*, produce *fourths*, &c.

Seconds, multiplied by *seconds*, produce *fourths;* by *thirds*, produce *fifths;* &c. Hence the

RULE.—*The denomination of a product is expressed by the sum of the indices of its factors.*

Art. 139. To multiply one duodecimal expression by another.

Ex. 1. Multiply 4ft. 3′ linear by 2ft. 5′ linear.

Ans. 10sq. ft. 3′ 3″.

WRITTEN PROCESS.

4ft.	3′	
2ft.	5′	
1	9′	3″
8	6′	
10ft.	3′	3″

EXPLANATION.

First 5′ $\times$ 3′ = 15″ = 1′ 3″. Write 3″ and carry the 1′ to the next product. Then 5′ $\times$ 4ft. = 20′; add the 1′ *carried*, making 21′, which = 1ft. 9′; write the whole. Then 2ft. $\times$ 3′ = 6′; write the 6′ under the primes. Then 2ft. $\times$ 4ft. = 8ft.; write the whole. Now add these products, as in compound addition.

RULE.—*Write the denominations of the multiplier under like denominations of the multiplicand.*

Beginning at the right hand, multiply in order the number in each denomination of the multiplicand, by the number in each denomination of the multiplier, considering and reducing each partial product as a denomination expressed by the sum of the indices of its factors.

Add the partial products as in Compound Addition.

QUESTION.—139. How do we multiply one duodecimal expression by another?

Examples for Practice.

2. Multiply 7ft. 9′ by 6ft. 8′. Ans. 51sq. ft. 8′.
3. Multiply 9sq. ft. 11′ by 5ft. 10′. Ans. 57cu. ft. 10′ 2″.
4. What is the area of one side of a board 15ft. 4′ long, and 1ft. 6′ broad? Ans. 23sq. ft.
5. How much will it cost to paint a surface 50ft. 6′ long, and 8ft. 9′ broad, at 2 cents per sq. ft.? Ans. $8.83,7+.
6. How much wood is there in a pile 25ft. 8′ long, 5ft. 5′ high, and 3ft. 4′ wide? Ans. 3 cords, 79cu. ft. 5′ 1″ 4‴.
7. How much plastering in a room 15ft. 5′ long, 12ft. 7′ wide, and 9ft. 11′ high, there being 2 doors, each 3ft. 11′ by 7ft. 8′; a mantel-piece, 5ft. 3′ long, by 4ft. 11′ high; 2 windows, each 3ft. 10′ by 6ft. 2′; and a wash-board, 9 inches high?
Ans. 583sq. ft. 11′ 11″.
8. How many cubic feet of earth were removed in digging a cellar 40ft. 9′ long, 22ft. 6′ wide, and 5ft. 5′ deep?
Ans. 4966cu. ft. 4′ 10″ 6‴.

Art. 140. To divide one duodecimal expression by another.

Ex. 1. A certain window contains 10sq. ft. 3′ 3″, and its breadth is 2ft. 5′. What is its length? Ans. 4ft. 3′.

Written Process.

2ft. 5′) 10ft. 3′ 3″ (4ft. 3
9 8
7 3
7 3

Mental Process.

Since surface is the product of length by breadth, if we divide surface by length, which is one factor, we obtain breadth, the other factor. In this written process, we first divide 10ft. by 2ft. 5′. It will often be proper to reduce both divisor and that part of the dividend divided to the same terms, that we may at once approximate to the true quotient term. Thus 2ft. 5′ = 29′, and 10ft. = 120′. 120′ ÷ 29′ = 4. Write 4 in the quotient, and multiply 2ft. 5′ by it, producing 9ft. 8′. Subtract, and bring down 3″. Then 2 in 7′, 3 times; 3′ × 2ft. 5′ = 7′ 3″. Or we can reduce 7′ 3″ to seconds, making 87″; then 29′ in 87″ gives 3′, which multiplied into 2ft. 5′ = 7′ 3″.

Second Method.

2ft. 5′ = 29′ and 10ft. 3′ 3″ = 1479″
1479″ ÷ 29′ = 51′ = 4ft. 3′

Explanation.

In this process we reduce both numbers; and then the quotient to higher terms.

Question.—140. How do we divide one duodecimal expression by another?

RULE.—*Find how many times the divisor is contained in the least left-hand portion of the dividend, that will contain it, and write the number for the first term of the quotient. Then multiply the divisor by this quotient, and subtract the terms of the product from the corresponding terms of the dividend. To the remainder annex the next term of the dividend, and divide as before. Do thus till the division is complete.*

Or, *Reduce divisor and dividend each to the lowest denomination in it, then divide, and reduce the quotient to higher denominations.*

EXAMPLES FOR PRACTICE.

2. Divide 51ft. 8′ by 6ft. 8′. Ans. 7ft. 9′.

3. Divide 57cu. ft. 10′ 2″ by 5ft. 10′. Ans. 9sq. ft. 11′.

4. The area of a board is 23 square feet, and its breadth is 1ft. 6′: what is its length? Ans. 15ft. 4′.

5. A wood pile, containing 3 cords 79cu. ft. 5′ 1″ 4‴, is 5ft. 5′ high, and 3ft. 4′ broad. What is its length?

Ans. 25ft. 8′.

MISCELLANEOUS EXERCISES IN COMMON FRACTIONS.

1. How many persons must share 12oz. of confectionery so that each receives $\frac{1}{8}$ of a pound? Ans. 6 persons.

2. A piece of cloth was made up into garments as follows:—One took $2\frac{3}{4}$ yds., another $3\frac{1}{8}$ yds., another $\frac{5}{6}$ of a yd., another $4\frac{1}{2}$ yds., another $8\frac{1}{3}$ yds., another $4\frac{1}{3}$ yds., and another $6\frac{5}{8}$ yds. What was the length of the piece, the average quantity of cloth for a garment, and the average cost per garment, at the rate of $\$6\frac{3}{4}$ per yard? Ans. 31yds. 1in. whole length; $4\frac{19}{25}$yd. av.; $\$29.91,9+$ av. cost.

3. F. May was 1 of 5 heirs who shared equally $862\frac{1}{4}$ acres of land; and from his share he sold to S. North $12\frac{1}{2}$ acres, and to H. Black $\frac{3}{4}$ of an acre, for a house-lot. How much land had he left? Ans. 159A. 32P.

4. How many times can a box, $1\frac{1}{4}$ft. long, $\frac{7}{8}$ of a ft. wide, and $\frac{1}{3}$ of a ft. deep, be emptied into a bin $2\frac{1}{2}$ft. long, $1\frac{3}{4}$ft. wide, and $1\frac{5}{6}$ft. deep? Ans. 8 times.

5. What cost $\frac{2}{5}$ of a ton of sugar, at $\$8\frac{3}{4}$ per cwt.?

Ans. $70.

6. C. Wood bought flour, and sold it at $\frac{1}{4}$ of a dollar per bbl. more than he gave, thereby gaining $180. How many barrels did he buy? Ans. 720 barrels.

7. From a bin of wheat, 12ft. 4′ long, 5ft. 6′ broad, and 4ft 10′ deep, the grain was sold 1ft. 9′ from the top. How many bushels remained? Ans. $168\frac{7}{10}\frac{2}{7}\frac{7}{5}\frac{2}{21}$ bushels.

8. What part of an imperial gallon is a wine gallon U.S.?
Ans. $\frac{1}{1}\frac{1}{3}\frac{5}{8}\frac{5}{6}\frac{0}{3}\frac{0}{7}$.

9. What part of an imperial bushel is a U.S. bushel?
Ans. $\frac{5}{5}\frac{3}{5}\frac{7}{4}\frac{6}{5}\frac{0}{4}\frac{5}{8}$.

10. What number multiplied by $9\frac{5}{7}$ produces 34?
Ans. $3\frac{1}{2}$

11. How much "richer" is John, who has $88\frac{1}{2}$ cents, than James, who has $18\frac{3}{4}$ cents? Ans. $4\frac{1}{2}\frac{8}{5}$ times.

12. How much advantage is there, in respect to weight, in transporting gold U.S. coin instead of half-dollars?
Ans. $14\frac{3}{4}\frac{8}{3}$.

13. A copper-mining company, having their property represented by 6000 shares, voted to represent it by 20000 shares. What part of an old share was a new one? Ans. $\frac{3}{10}$.

14. If a lot has a front of 100 feet, how far back at right angles must it extend to contain an acre? Ans. 435ft. $7\frac{1}{5}$in.

15. N. Dayton and S. Ives shared equally a hogshead of sugar, containing 1625lbs. Then Ives shared his part equally with two others, then sold $\frac{1}{2}$ of what was left, at $6\frac{1}{2}$ cents a pound. What did he receive? Ans. $8.80,$2\frac{1}{12}$.

16. What is the value of 45 cu. ft. 9′ 8″ 7‴?
Ans. 45 cu. ft. 1399 cu. in.

17. From a piece of cloth containing $37\frac{3}{4}$yd., there were cut at different times $9\frac{7}{8}$yd., $6\frac{1}{2}$yd., $3\frac{5}{8}$yd., and $7\frac{2}{3}$yd. How much remained? Ans. $10\frac{1}{12}$yd.

18. At $3\frac{3}{4}$ miles per hour, how long will it take to walk $17\frac{1}{3}$ miles? Ans. 4hr. 37min. 20sec.

19. How old is G. Ames compared with Wm. Graham, the former having lived 30yr. 8mo. 12da., and the latter 36yr. 6mo.?
Ans. $\frac{3}{3}\frac{0}{6}\frac{7}{5}$ as old.

20. How many inch boards can be sawed from the first $21\frac{1}{4}$ inches of the breadth of a square log, allowing $\frac{1}{4}$ of an inch for the kerf? Ans. 17.

21. What is the difference between the value of $\frac{7}{11}$ of a ton $+ \frac{1}{2}\frac{7}{2}$ of a cwt. and that of $\frac{3}{8}$ of a ton $- \frac{4}{5}$ of a cwt.?
Ans. 6cwt. 3qr. 5lb.

22. Bought a piece of cloth for $96, and, it getting damaged, I sold it for $\frac{5}{8}$ of the cost, thereby losing $2 per yd. How many yards were in the piece? Ans. 18 yards.

CHAPTER XII.

DECIMAL FRACTIONS.

Art. 141. DECIMAL Fractions are those fractions which express the division of the unit by *ten*, or by the product of two or more *tens*. Thus, $1 \div 10 = \frac{1}{10}$; $1 \div 10 \times 10 = \frac{1}{100}$; $1 \div 10 \times 10 \times 10 = \frac{1}{1000}$; &c.

It is manifest that

$\frac{1}{10} = \frac{1}{10}$ of 1: $\frac{1}{100} = \frac{1}{10}$ of $\frac{1}{10}$: $\frac{1}{1000} = \frac{1}{10}$ of $\frac{1}{100}$: in which we see that a fraction expresses ten times less value at every annexing of a cipher to its denominator. So does a figure in the Arabic Notation, every time it is written one figure's place toward the right. Therefore it is possible to dispense with the denominator of a decimal fraction, and to express its value *by the place of the numerator*, as follows:

900 ÷ 10 =	90.	
90 ÷ 10 =	9.	
9 ÷ 10 =	.9	$= \frac{9}{10}$
.9 ÷ 10 =	.09	$= \frac{9}{100}$
.09 ÷ 10 =	.009	$= \frac{9}{1000}$
009 ÷ 10 =	.0009	$= \frac{9}{10000}$
&c.	&c.	&c. &c.

Hence the numerators of decimal fractions are usually written as the *continuation* of a whole number to the right, and separated from the whole number by a point called *a decimal point* or *separatrix*. Such a form of fraction is called *a decimal*.

NUMERATION TABLE OF DECIMAL FRACTIONS.

Decimal point.	Tenths.	Hundredths.	Thousandths.	Ten thousandths.	Hundred thousandths.	Millionths.	Ten millionths.	Hundred millionths.	
.8									is read 8 tenths.
.7	8								" 78 hundredths.
.6	7	8							" 678 thousandths.
.5	6	7	8						" 5678 ten thousandths.
.4	5	6	7	8					" 45678 hundred thousandths.
.3	4	5	6	7	8				" 345678 millionths.
.2	3	4	5	6	7	8			" 2345678 ten millionths.
.1	2	3	4	5	6	7	8		" 12345678 hundred millionths.

QUESTIONS.—141. What are decimal fractions? How are they usually written? Why? What is the decimal point? Name the fractional places from tenths toward the right.

Art. 142. Since a figure in the first place to the right of units expresses *tenths*, and in the second place *tenths of tenths*, or *hundredths*, &c., it follows that a figure expresses ten times less value every time it is placed at the right of one more figure. Therefore *ciphers placed between the separatrix and any figure of a decimal fraction affect its value.*

Since $\frac{2}{10} = \frac{20}{100} = \frac{200}{1000}$, &c., it follows that .2 = .20 = .200. Since annexing a cipher to a decimal fraction multiplies its numerator by 10, and at the same time so increases its number of places that it expresses 10 times its former denominator, it follows that *ciphers on the right of a decimal fraction do not affect its value.* (See Art. 103.)

Art. 143. To read decimal fractions.

RULE.—*Reckon from the decimal point and find the name of the place of the last right-hand figure: then read the fraction as a whole number of that name.*

ILLUSTRATION.—5.042 is not read 5 *and* 4 *hundredths and* 2 *thousandths;* but, 5 *and* 42 *thousandths.*

Pronounce, or write in words, the following numbers.

1. .6	6. .06084	11. .0050067
2. .35	7. 9.10025	12. 3.10050062
3. .07	8. 35.024061	13. .008010435
4. .409	9. 4.000019	14. 6.310062093
5. .0048	10. 1.101202	15. .587443651

Art. 144. To write decimal fractions.

RULE.—*Write the decimal as a whole number; then, if necessary, prefix as many ciphers as will make its right-hand figure occupy the place whose name is expressed by the fraction. Place the decimal point at the left of this arrangement.*

ILLUSTRATION.—To write *four hundred and five ten thousandths,* we first write 405; then, on reckoning, we see that the 5 must be in the *fourth* place from the decimal point to express *ten thousandths.* Hence we prefix a cipher, making .0405.

Write in figures the following numbers.

1. Sixteen, and thirty-five hundredths.

QUESTIONS.—142. Do ciphers written between the decimal point and a figure affect the value of the figure? Show it. Does a cipher annexed affect the value of a decimal? Prove it.—143. How do we read decimal fractions?—144. How do we write decimal fractions?

2. Eight, and eight tenths.
3. Twelve, and four thousandths.
4. One hundred five ten-thousandths.
5. One, and five ten-thousandths.
6. Fourteen hundred-thousandths.
7. Six hundred seventy-two hundred-thousandths.
8. Fifteen thousand twenty-four millionths.
9. Fifteen thousand, *and* twenty-four millionths.
10. Nine hundred five ten-millionths.
11. Nine hundred *and* five ten-millionths
12. Six thousand six hundred-millionths.
13. Six thousand, and six thousandths.
14. One-hundred-one thousand one-hundred-one millionths.
15. One hundred and one thousand, and one-hundred-one millionths.

Art. 145. In United States Money the dollars form the integer of the number, and the cents and mills the decimal portion: because the dimes are *tenths* of a dollar, the cents *tenths of a dime*, and the mills *tenths of a cent.*

Hence a number in United States Money can be expressed decimally. Also figures on the right of mills can be expressed as decimals of a mill, because their decimal notation corresponds with that of United States Money.

Ex. 1. $2.09 = Two dollars and 9 hundredths of a dollar.
Ex. 2. $1.045 = One dollar and 45 thousandths of a dollar.
Ex. 3. $9.2584 = Nine dollars, 25 cents, 8 mills, and 4 tenths of a mill, or $9 and 2584 ten-thousandths of a dollar.

Ex. 4. $4.0126
Ex. 5. $0.1875
Ex. 6. $8.10
Ex. 7. $2.01625
Ex. 8. $9.0995432
Ex. 9. $7.00125.

ADDITION OF DECIMALS.

Art. 146. To find the sum of two or more decimals.

Ex. 1. What is the sum of 64.012, 3.98, 281.00365, .308, and .00309? Ans. 349.30674.

Questions.—145. How can United States Money be expressed? Why? How can figures on the right of mills be expressed? Why?

WRITTEN PROCESS.

64.012
3.98
281.00365
.308
.00309

349.30674

MENTAL PROCESS.

Since the figures to be added must express the same parts of a unit, the decimal points of the numbers are arranged in a column, so as to cause the like denominations in the different numbers to form separate columns. The sum of the right column is $\frac{14}{100000}$, which equals $\frac{10}{100000} + \frac{4}{100000}$. Write the 4 and carry the 1. So add and carry through all the columns, as in whole numbers. The sum of the tenths is 13, which equals 1 *unit* and 3 *tenths*. Write the 3 tenths, and carry the 1 unit to the column of units. Hence *no change is necessary in continuing the addition from the decimal to the integer.*

RULE.— *Write the numbers so that figures of the same denomination shall form a column.*

Add as in simple numbers, and put the decimal point of the result so that there shall be as many decimal places as equal the greatest number of decimal places in any of the numbers added

PROOF.—To detect error, carefully review the work.

EXAMPLES FOR PRACTICE.

2. What is the sum of 5.031, .0297, .06, .9, and 18.098 ?
Ans. 24.1187.

3. What is the sum of 143.143, 6.06, 17.0017, 8.00206, and .000259 ? Ans. 174.207019.

4. What is the sum of 48.1, .0481, 4.81, .00481, .481, and 481 ? Ans. 534.44391.

5. What is the sum of .5692, .05692, 0.005692, 5.692, 56.92, 569.2, and 5692 ? Ans. 6324.443812.

6. Add three hundred twenty-eight and eight thousandths; thirty-two, and eight thousand eight ten-thousandths; three thousand two hundred eighty, and eight hundredths; and thirty-two thousand eight hundred with the decimal eight tenths.
Ans. 36441.6888.

7. Add sixty-three with the decimal ninety-five millionths; two hundred twenty-four, with the decimal eighty-four hundred-thousandths; seven with the decimal seventy-three ten-thousandths; forty-five with the decimal sixty-two thousandths; nine with the decimal eighty-eight hundredths; and nine tenths.
Ans. 349.850235.

8. Add 1001 hundred-thousandths; 45 million; 45 millionths; 17 thousand; 17 thousandths; 8 hundred; 8 hundredths; 9 tens; 9 tenths; and 5. Ans. 45017896.007055

QUESTION.—146. How do we find the sum of two or more decimals?

SUBTRACTION OF DECIMALS.

Art. 147. To subtract one decimal from another.

Ex. 1. From 13.25 take 8.375. Ans. 4.875.

WRITTEN PROCESS.

13.25
8.375
4.875

MENTAL PROCESS.

Since the denomination of the subtrahend must be like that of the minuend, the decimal point of the subtrahend is placed under that of the minuend, so as to cause tenths to be under tenths, hundredths under hundredths, &c. Then say, 5 from 0 impossible; add 10. Then 5 from 10 leaves 5. Write 5 and carry 1 to 7, making 8. Continue thus through the whole.

RULE.—*Write each figure of the less number under the figure or place of the same denomination in the greater.*

Subtract as in simple numbers, and put the decimal point of the result as in addition of decimals.

PROOF.—To detect error, proceed as in proving subtraction of simple numbers.

EXAMPLES FOR PRACTICE.

(2.)	(3.)	(4.)	(5.)
28.015	52.163	6.08	345.607
9.19	8.00846	5.7931	44.5947
18.825	44.15454	.2869	301.0123

6. From 1 take .0181. Ans. .9819.
7. From 181 take .0001. Ans. 180.9999.
8. From 1 take .1. Ans. .9.
9. From 10 take .01. Ans. 9.99.
10. From 300 take .003. Ans. 299.997.
11. From 4567 take .7654. Ans. 4566.2346.
12. From two hundred seventy-nine with the decimal three hundred fifty-eight millionths, take eighty-eight with the decimal twenty-five thousand seven hundred sixty-nine millionths. Ans. 190.974589.
13. From one thousand seven hundred and seven, with the decimal five thousand and nine hundred-millionths, take nine hundred and forty-nine, with the decimal five thousand and nine hundred-thousandths. Ans. 757.94996009.

QUESTION.—147. How do we subtract one decimal from another?

MULTIPLICATION OF DECIMALS.

Art. 148. To multiply a decimal by a decimal or an integer.

Ex. 1. Multiply .75 by .3. Ans. .225.

Written Process.	Mental Process.
.75 .3 .225	Since in common fractions $\frac{75}{100} \times \frac{3}{10} = \frac{225}{1000}$, which is equal to .225 in decimals, it is plain that in decimal fractions $.75 \times .3 = .225$.

Ex. 2. Multiply 1.03 by .004. Ans. .00412.

Written Process.	Demonstration.
1.03 .004 .00412	$1.03 = \frac{100}{100} + \frac{3}{100} = \frac{103}{100}$ $\frac{103}{100} \times \frac{4}{1000} = \frac{412}{100000} = .00412$

Ex. 3. Multiply 7.28 by 12. Ans. 87.36.

Demonstration.—$7.28 \times 12 = \frac{728}{100} \times 12 = \frac{8736}{100} = 87.36$.

Ex. 4. Multiply 6.5129 by 10. By 100.
Ans. 65.129 and 651.29.

Demonstration.—$6.5129 \times 10 = \frac{65129}{10000} \times 10 = \frac{651290}{10000} = 65.1290 = 65.129$. $6.5129 \times 100 = \frac{65129}{10000} \times 100 = \frac{6512900}{10000} = 651.2900 = 651.29$. (See Note 2.)

Rule.—*Multiply as in simple numbers, and point off, if possible, from the right of the product, as many figures for decimals, as there are decimal places in both factors. When there are not so many figures in the product, prefix ciphers to supply the deficiency.*

Note 1.—After fixing the decimal point, it is best to erase the right-hand ciphers of the decimal part of the product, and the left-hand ciphers of its integral part, since they do not affect its value.

Note 2.—To multiply by 1 with a cipher or ciphers annexed, remove the decimal point of the multiplicand as many places toward the right, as there are ciphers in the multiplier. If there are not enough figures in the number for this, annex ciphers to supply the deficiency.

Proof.—To detect error, proceed as in proving multiplication of simple numbers.

Questions.—148. How do we multiply a decimal by a decimal or an integer? How by 1 with ciphers annexed? What should be done with the right-han ciphers of the product?

EXAMPLES FOR PRACTICE.

(5.)	(6.)	(7.)	(8.)
8.62	90.608	12.98175	492.875
4	5	4	80
34.48	453.04	51.927	39430.

(9.)	(10.)	(11.)	(12.)
492.875	.0125	.0125	40.4
.08	.8	80	.5
39.43	.1	1.	20.2

13. Multiply 18.96 by 2.5. Ans. 47.4.
14. Multiply 8.00634 by .057. Ans. .45636138.
15. Multiply 1800 by .18. Ans. 324.
16. Multiply .00235 by .0068. Ans. .00001598.
17. Multiply 9.01029 by .0078. Ans. .070280262.
18. Multiply .0062853 by 1000. Ans. 6.2853.
19. Multiply .1462935 by 10000.
20. Multiply .001 by 1000.
21. Multiply .87 by 10000. Ans. 8700.
22. Multiply .123 by 100000.
23. Multiply .000079 by .000036. Ans. .000000002844.
24. Multiply $45.625 by .75. Ans. $34.21875.
25. How many dollars are 479 thousandths of $75625.12,5? Ans. $36224.43,4,875.

26. C. Rich, worth $185725, is taxed $\frac{3}{10}$ of a mill on a dollar, for the poor. How much is his poor-tax? Ans. $55.71,7,5.

27. B. Poore, worth $275, is taxed $7\frac{5}{10}$ mills on a dollar, for school-purposes. How much is his school-tax? Ans. $2.06,2,5.

28. What cost 15.75lbs. of raisins, at $0.18,75 per lb.? Ans. $2.95,3,125.

Art. 149. Since those figures of a decimal fraction, which are beyond the eighth or tenth decimal place, express a very small value, it is customary to neglect them in products. For most purposes, a decimal of four or five places is sufficiently accurate. If, therefore, the factors are decimals of many places, and a few only are required in the product, it is desirable to omit all computation whose results do not appear in the product

CONTRACTION IN MULTIPLICATION OF DECIMALS.

Art. 150. To multiply one decimal by another, so as to produce a required number of decimal places, without unnecessary labor.

Ex. 1. Multiply 4.57243 by 2.9618, retaining four decimal places in the product. Ans. 13.5426.

Common Process.		Short Process.	
4.57243		4.5724	3
2.9618		2.	9618
36	57944	91448	6
45	7243	41151	8
2743	458	2743	4
41151	87	45	7
91448	6	36	5
13.5426	23174	13.5426	

Explanation. By inspecting these processes, we see that the partial products are, *as far as they go*, in the short process, the same as in the common, only their order is reversed. The short method is explained by the

Rule.—*Draw a line so that as many decimal places of the multiplicand shall be on the left of the line as there are decimal places required in the product. If there are not so many decimal places in the multiplicand, supply the deficiency with ciphers.*

Write the multiplier under the multiplicand, so that its place of units shall be next left of the line.

Beginning at the left of the multiplier, form the partial products by starting with that figure of the multiplicand which is as many places from the line on one side as the figure by which you multiply is on the other side.

After adding in the figure that must be carried from the product of the figure in the next lower place of the multiplicand, write the first figure of each partial product on the right of the line, and the other figures in order on the left.

Add the partial products; reject the units in the sum of the column on the right of the line, but carry its tens; and point off the required number of decimal places from the figures on the left of it.

Examples for Practice.

2. Find the product of 4.57243 by 222.9618, true to four decimals. Ans. 1019.4772.

Questions.—149. How many decimal places are usually retained in products? Why? What is desirable in multiplying decimals?—150. How do we produce a required number of decimal places without unnecessary l[illegible]or?

WRITTEN PROCESS

4.5724	300
222.	9618
9144860	0
914486	0
91448	6
41151	8
2743	4
45	7
36	5

1019.4772

EXPLANATION.

In this case, to carry out the rule, we make, with ciphers, as many decimal places in the multiplicand as there are integral figures in the multiplier.

3. What is the product of 8.00634 by .057, true to three places of decimals? Ans. .456.

4. What is the product of .03687 by 83.649, true to five places of decimals? Ans. 3.08413.

5. What is the product of .04683 by 210.735, true to two places of decimals? Ans. 9.86.

6. What is the product of 11.23674 by 112.3674, true to three places of decimals? Ans. 1262.643.

DIVISION OF DECIMALS.

Art. 151. To divide a decimal by a decimal, a decimal by an integer, or an integer by a decimal.

Ex. 1. Divide 17.28 by 1.2. Ans. 14.4.

WRITTEN PROCESS.

1.2)17.28
14.4

MENTAL PROCESS.

Since, in common fractions, $\frac{1728}{100} \div \frac{12}{10} = \frac{1728}{100} \times \frac{10}{12} = \frac{17280}{1200} = 14\frac{480}{1200} = 14\frac{4}{10} = 14.4$, it is plain that, in decimal fractions, $17.28 \div 1.2 = 14.4$.

Ex. 2. Divide .576 by 9.6. Ans. .06.

WRITTEN PROCESS.

9.6).576(.06
576

DEMONSTRATION.

$\frac{576}{1000} \div \frac{96}{10} = \frac{576}{1000} \times \frac{10}{96} = \frac{5760}{96000} = \frac{6}{100} = .06.$

Ex. 3. Divide 492.12 by .12. Ans. 4101.

WRITTEN PROCESS.

.12)492.12
4101

DEMONSTRATION.

$\frac{49212}{100} \div \frac{12}{100} = 49212 \div 12 = 4101.$

Ex. 4. Divide 246 by .06. Ans. 4100.

WRITTEN PROCESS.

$$.06)246.00$$
$$4100$$

DEMONSTRATION.

$$246 \div \tfrac{6}{100} = 246 \times \tfrac{100}{6} = \tfrac{24600}{6} = 4100.$$

Ex. 5. Divide 34.571 by .06. Ans. 576.183+.

WRITTEN PROCESS.

$$.06)34.57100$$
$$576.183+$$

REMARK. This process is like that in Ex. 4, except that, by annexing ciphers, new decimal places are made in the dividend, in order to continue the division, when there is a remainder.

Ex. 6. Divide 624.25 by 100. Ans. 6.2425.

DEMONSTRATION.—$\tfrac{62425}{100} \div 100 = \tfrac{62425}{10000} = 6.2425.$ (See Note 3.)

RULE.—*Divide as in simple numbers, and point off, if possible, as many figures from the right of the quotient, for decimals, as the decimal places of the dividend exceed those of the divisor. When there are not so many figures in the quotient, prefix ciphers to supply the deficiency.*

NOTE 1.—When there are not so many decimal places in the dividend as in the divisor, make them as many by annexing ciphers. The quotient is then a whole number.

NOTE 2.—When there is a remainder, to approximate the true answer, annex ciphers as decimals to the dividend, and continue the division. If a perfect quotient is not thus obtained, annex + to it, to signify that the quotient is not complete.

NOTE 3.—To divide by 1 with a cipher or ciphers annexed, remove the decimal point of the dividend as many places toward the left, as there are ciphers in the divisor. If there are not enough figures in the number for this, prefix ciphers to supply the deficiency.

PROOF.—To detect error, proceed as in proving division of simple numbers.

EXAMPLES FOR PRACTICE.

7. Divide 34.48 by 4. Ans. 8.62
8. Divide 72.108 by 9. Ans. 8.012.
9. Divide 299.04 by 48. Ans. 6.23.
10. Divide 14.402 by 3.8. Ans. 3.79.
11. Divide 478.3212 by 37.8. Ans. 12.654.

QUESTIONS.—151. What is the rule for division of decimals? When there are not so many decimals in dividend as divisor? When there is a remainder? When the divisor is 1 with a cipher or ciphers annexed?

12. Divide .070280262 by .0078. Ans. 9.01029.
13. Divide 6.51168 by 77.52. Ans. .084.
14. Divide .017325 by 6.93. Ans. .0025.
15. Divide .00001598 by .0068. Ans .00235.
16. Divide 396.267 by 5.743. Ans. 69.
17. Divide 294 by 6.125. Ans. 48.
18. Divide 312.9 by 84. Ans. 3.725.
19. Divide 52.0061 by 3.7. Ans. 14.0557+.
20. Divide 98.3209 by 5.4. Ans. 18.207574+.
21. Divide 1000 by .001. Ans. 1000000.
22. Divide .001 by 1000. Ans. .000001.
23. Divide 75631.21 by 10000.
24. Divide 840.39 by 100000.
25. Divide twelve hundred by four millionths.

Ans. 300000000.

26. If a man pays $151.25 tax, when the tax is $5\frac{5}{10}$ mills on a dollar, what amount of his property is taxed?

Ans. $27500.

CONTRACTION IN DIVISION OF DECIMALS.

Art. 152. To divide one decimal by another, so as to produce a required number of decimal places, without unnecessary labor.

Ex. 1. Divide 19.40846225918 by 5.6142495, retaining three decimal places in the quotient. Ans. 3.457.

Common Process.

```
5.6142495 ) 19.40846225918 ( 3.457
            16 8427485
            ----------
             2 56571375
             2 24569980
             ----------
               320013959
               280712475
               ---------
                39301484 1
                39299746 5
                ----------
                     1737 6
```

Short Process.

```
5.614 ) 19.408 ( 3.457
          16 843
          ------
           2 565
           2 245
           -----
             320
             281
             ---
              39
              39
```

Question.—152. How do we contract division of decimals?

RULE.—*To find the first quotient figure, divide by as many of the left-hand figures of the divisor, as equal the number of quotient figures required. If there are not so many figures in the divisor, annex decimal ciphers to supply the deficiency.*

Annex no new figure to any remainder, but, at each new division, use the divisor next preceding it, except its right-hand figure.

To the right-hand figure of the product of such divisor by a quotient figure, add the figure to be carried from the product of the rejected figures; also add 1, *if the other figure of this product is* 5 *or more. Thus continue till no figure is left for a divisor: then point off the quotient as required.*

EXAMPLES FOR PRACTICE.

2. Divide 133.056032983 by 9.1836, to five decimal places. Ans. 14.48843.
3. Divide 49.3142536 by 31.42, to four decimal places. Ans. 1.5695
4. Divide 84.56 by 8.567, to three decimal places. Ans. 9.870

REDUCTION OF DECIMALS.

CASE I.

Art. 153. To reduce a decimal to a common fraction.

Ex. 1. Reduce .0625 to a common fraction.

PROCESS INDICATED. $.0625 = \frac{625}{10000} = \frac{1}{16}$ Ans.

RULE.—*Erase the decimal point, and express the decimal as a whole number: then write the denominator under it and reduce the fraction, thus made, to lowest terms.*

EXAMPLES FOR PRACTICE.

2. Express .5 as a common fraction in lowest terms. Ans. $\frac{1}{2}$
3. What is the value of .75? Ans. $\frac{3}{4}$
4. What is the value of .625? Ans. $\frac{5}{8}$
5. What part of a dollar is \$0.87,5? Ans. $\frac{7}{8}$
6. Reduce .415 to a common fraction. Ans. $\frac{83}{200}$
7. Reduce .21875 to a common fraction. Ans. $\frac{7}{32}$
8. Reduce .4453125 to a common fraction. Ans. $\frac{57}{128}$

QUESTION.—153. How do we reduce a decimal to a common fraction?

NOTE.—To estimate *approximately* the value of a decimal, take *th first two or three figures* and reduce them as above.

Thus, .50123 is nearly $\frac{1}{2}$; .74991 is nearly $\frac{3}{4}$, &c.

CASE II.

Art. 154. To reduce a common fraction to a decimal

Ex. 1. Reduce $\frac{7}{8}$ to a decimal. Ans. .875

WRITTEN PROCESS.

8) 7.000

.875

MENTAL PROCESS.

The fraction $\frac{7}{8}$ expresses the division of 7 by 8. To effect this, we proceed as in division of decimals, annexing ciphers as decimals, and continuing the division till a perfect or sufficient quotient is obtained.

RULE.—*Annex ciphers as decimals to the numerator, and divide this result by the denominator.*

EXAMPLES FOR PRACTICE.

2. Reduce $\frac{9}{16}$ to a decimal. Ans. .5625.
3. Reduce $\frac{21}{32}$ to a decimal. Ans. .65625.
4. Reduce $\frac{43}{200}$ to a decimal. Ans. .0215.
5. Reduce $\frac{5}{256}$ to a decimal. Ans. .01953125.
6. Reduce $\frac{7}{17}$ to a decimal. Ans. .4117647058+.
7. Reduce $\frac{16}{21}$ to a decimal. Ans. .761904+.

Art. 155. A *circulating, or periodical decimal* is a decimal whose figures are repetitions of the same figure or figures.

A *repetend* is the figure, or set of figures, whose repetitions compose the periodical decimal.

Periodical decimals are denoted by a *dot* over the figure of the repetend, if single, or over the first and last figures, if more than one. Thus, .3333+ is denoted by $.\dot{3}$; .727272+, is denoted by $.\dot{7}\dot{2}$; and .001001+ is denoted by $.\dot{0}0\dot{1}$.

Ex. 1. Express $\frac{10}{37}$ as a decimal. Ans. $.\dot{2}7\dot{0}$.
2. Express $\frac{14}{75}$ as a decimal. Ans. $.18\dot{6}$.
3. Express $\frac{8}{21}$ as a decimal. Ans. $.\dot{3}80952\dot{8}$.

A *mixed repetend* is a decimal whose first portion is not a repetend, and is followed by a repetend. Thus $.18\dot{6}$ is a mixed repetend, being equal to .18666666+.

QUESTIONS.—154. How do we reduce a common fraction to a decimal?—155. What is a circulating or periodical decimal? A repetend? How are periodical decimals denoted? What is a mixed repetend?

A repetend can be expressed as a common fraction, by writing for a denominator as many *nines* as the repetend has figures.

Ex. 1. $.\dot{3} = \frac{3}{9} = \frac{1}{3}$.

2. $.\dot{6} = \frac{6}{9} = \frac{2}{3}$.

3. $.1\dot{3}5 = \frac{135}{999} = \frac{5}{37}$.

4. $.\dot{7}61904 = \frac{761904}{999999} = \frac{16}{21}$.

5. $.18\dot{6} = \frac{18\frac{6}{9}}{100} = \frac{168}{900} = \frac{14}{75}$.

6. $.02\dot{2}\dot{7} = \frac{2\frac{27}{99}}{100} = \frac{225}{9900} = \frac{1}{44}$.

7. $.4\dot{7}4\dot{5} = \frac{47\frac{45}{99}}{100} = \frac{47\frac{5}{11}}{100} = \frac{522}{1100} = \frac{261}{550}$.

8. $.016\dot{9}94266\dot{6} = \frac{16\frac{9942666}{9999999}}{1000} = \frac{83}{4884}$.

Case III.

Art. 156. To reduce a denominate number to the decimal of a higher denomination.

Ex. 1. What decimal of a pound Troy are 144gr.?

Ans. .025.

First Process Indicated.

144grs. $= \frac{144}{5760}$ of a lb. Troy $= .025$

Explanation.

This is the process in Art. 132 extended to a decimal by Art. 154.

Second Process.

24)144

20)6.

12).3

.025

Mental Process.

Since 1gr. $= \frac{1}{24}$ of a pwt. 144grs. $= \frac{144}{24}$ of a pwt. $=$ 6pwt. Since 1pwt. is $\frac{1}{20}$ of an oz., 6 pwts. $= \frac{6}{20}$ of an oz. $= .3$oz. Since 1oz. is $\frac{1}{12}$ of a lb., .3oz. is $.3 \times \frac{1}{12}$ of a lb. $= .025$lb.

Ex. 2. What decimal of a £ are 7s. 9d. 3qr.?

Ans. .390625.

Question —155. How can a repetend be expressed as a common fraction?

First Process Indicated.

7s. 9d. 3qr. = 375qrs. = $\frac{375}{960}$ of a £ = .390625 of a £.

Second Process.

4	3.
12	9.75
20	7.8125
	.390625

Explanation.

In this process, we arrange the numbers vertically, placing that of the lowest denomination at the top. Then we reason and divide as in Ex. 1.

Rule.—*If the denominate number is simple, divide it by that number of its kind, which makes one of the required denomination: if it is compound, reduce it to its lowest denomination, and then divide as aforesaid; or,*

Divide the lowest denomination by that number which makes one of the next higher. Write the quotient as a decimal to the given number of this next higher denomination. So continue till the result is of the denomination required.

Note.—One compound number may be divided by another, by first reducing them to a mixed decimal of the same denomination. Thus, (£65 19s. 11d. 1qr.) ÷ (£9 8s. 6d. 3qr.) = £65.996875 ÷ £9.428125. See Art. 91, Ex. 1.

Examples for Practice.

3. Reduce 13cwt. 3qr. 15lb. to the decimal of a ton. Ans. .695.

4. Reduce 24rd. to the decimal of a league. Ans. .025.

5. Reduce 3fur. 32rd. 8ft. 3in. to the decimal of a mile. Ans. .4765625.

6. Reduce 3R. 24P. 136 sq. ft. 36 sq. in. to the decimal of an acre. Ans. .903127869+.

Case IV.

Art. 157. To find the value of a denominate decimal in whole numbers.

Ex. 1. What is the value of .390625 of a £? Ans. 7s. 9d. 3qr.

Question.—156. How do we reduce a denominate number to the decimal of a higher denomination?

WRITTEN PROCESS.

390625
20

7.812500 s.
12

9.7500 d.
4

3.00 qr.

MENTAL PROCESS.

Since 20s. make £1, .390625 of £1 = .390625 of 20s. = 7.8125 shillings. Since 12d. make 1s., .8125 of a shilling = .8125 of 12d. = 9.75d. Since 4qrs. make 1 penny, .75 of 1 penny = .75 of 4qrs. = 3qrs. Hence the answer is 7s. 9d. 3qr.

RULE.—*Multiply the given decimal by that number of the next lower denomination which makes one of the given denomination. Multiply the decimal part of this product in the same manner, and so continue till the operation is complete. The integers, on the left of the successive products, form the answer.*

EXAMPLES FOR PRACTICE.

2. What is the value of .0432 of a mile?
Ans. 13rd. 13ft. $7\frac{9}{125}$in.

3. What is the value of .7913 of an acre?
Ans. 3R. 6P. 165 sq. ft. $76\frac{4}{144}$ in.

4. What is the value of .9035 of a cu. yd.?
Ans. 24 cu. ft. $681\frac{87}{125}$ cu. in.

5. What is the value of .6977 of a mile?
Ans. 55ch. 3p. $6\frac{3}{5}$li.

6. What is the value of .6789 of a lb. Troy?
Ans. 8oz. 2pwt. 22.464grs.

7. What is the value of .35721 of a ℔ Apothecaries' Weight? Ans. 4℥ 2℥ 0℈ 17.5296grs.

8. What is the value of .093507 of a ton?
Ans. 1cwt. 3qr. 12lb. 3.584dr.

9. What is the value of .57069 of a bbl. of 40 gallons?
Ans. 22gal. 3qt. 2.4832gi.

10. What is the value of .99 of a bushel?
Ans. 3pk. 7qt. 1.36pt.

11. What is the quantity of land in .376 of a township 6 miles square? Ans. 13 sq. mi. 343A. 6P. 108 sq. ft. 129.6 sq. in.

12. What is the value of .1875 of a common year?
Ans. 68da. 10hr. 30min.

QUESTION.—157. How do we find the value of a denominate decimal in whole numbers?

MISCELLANEOUS EXERCISES IN DECIMALS.

1. What is the product of 3.14159265 by .31830989 true to three decimal places? Ans. .999.

2. What cost 7yd. 3qr. 3na., at $9.75 per yd.?
Ans. $77.39+.

3. What cost 45A. 1R. 25P. at $31.50 per acre?
Ans. $1430.29,6+.

4. What cost 12 cords 48 feet of wood, at $5.37½ per cord?
Ans. $66.51,5⅝.

5. Sold a pile of wood 35ft. 6′ long, 5ft. 3′ high, and 4ft. 9′ wide, for $4.31¼ per cord. How much did I receive for it?
Ans. $29.82,6+.

6. At 31¼cts. per bushel, how many bushels of potatoes can be bought for $18.59⅜? Ans. 59bu. 2pk.

7. What is the sum of .4 of a mile, .9 of a furlong, and .5 of a rod? Ans. 4fur. 4rd. 8ft. 3in.

8. From .82 of a day take .48 of an hour.
Ans. 19hr. 12min.

9. What is the value of .75 of 365¼ days?
Ans. 273da. 22hr. 30min.

10. At $21820.25 per mile, find the cost of 32m. 6 fur. 36rd. of railroad, carrying the product to cents only.
Ans. $717067.96.

11. Divide decimally 20mi. 3fur. 27rd. 10ft. 6in. by 1mi. 5fur. 25rd. 10ft. 6in. Ans. 12.

12. What is the value of an endless decimal, all nines?
Ans. 1.

13. What is the value of the decimal .499, the nines being continued without end? Ans. ½.

14. Indicate and perform the following:—From the sum of five and six hundredths, plus nine and four-tenths, take three and seven thousandths, and divide the remainder by one and three-tenths. Ans. 8.81.

15. What is the quotient of six with the decimal eighteen thousand nine hundred and forty-five millionths, divided by eight with the decimal four thousand and sixteen ten-thousandths, true to four places of decimals? Ans. .7164.

16. What is the average of five hundred with the decimal sixty-five ten-thousandths; forty-eight with the decimal eight hundred and four thousandths; and seventeen with the decimal eight hundredths? Ans. 188.63016.

CHAPTER XIII.

ANALYSIS.

Art. 158. ANALYSIS, in Arithmetic, is a method of solving questions by separately computing each of their conditions. It is often called *Practice.*

To analyze a question, is to produce its answer, by reasoning from one condition to another, till every condition has been separately considered.

An *aliquot part* of a quantity is any quantity which is contained in it an exact number of times.

ANALYSIS BY ALIQUOT PARTS.

Art. 159. Analysis by aliquot parts is a method of obtaining the product of two numbers, by computing separately the product of one number by that part of the other which expresses the units of comparison, and by those parts which are aliquot parts of such unit.

Ex. 1. What cost 46yd. of linen at $43\frac{3}{4}$cts. per yd.?

Ans. $\$20.12\frac{1}{2}$.

	WRITTEN PROCESS.			MENTAL PROCESS.
25cts. $= \frac{1}{4}$	46	= cost at $1		At $1 per yd., the cost of 46yd. would be $46. Since 25cts. is $\frac{1}{4}$ of $1, the cost of 46yd. at 25cts. would be $\frac{1}{4}$ of the cost at $1; that is, $11.50. Since $12\frac{1}{2}$cts. is $\frac{1}{2}$ of 25cts., the cost at $12\frac{1}{2}$cts. would be $\frac{1}{2}$ of the cost at 25cts.; that is, $5.75. Since $6\frac{1}{4}$cts. is $\frac{1}{2}$ of $12\frac{1}{2}$cts., the cost at $6\frac{1}{4}$cts. would be $\frac{1}{2}$ of the cost at $12\frac{1}{2}$cts; that is, $\$2.87\frac{1}{2}$. Since 25cts. + $12\frac{1}{2}$cts. + $6\frac{1}{4}$cts. = $43\frac{3}{4}$cts., the cost at $43\frac{3}{4}$cts. must be the sum of the costs at 25cts., $12\frac{1}{2}$cts., $6\frac{1}{4}$cts.; that is, $\$20.12\frac{1}{2}$.
$12\frac{1}{2} = \frac{1}{2}$	11.50	= " "	25cts.	
$6\frac{1}{4} = \frac{1}{2}$	5.75	= " "	$12\frac{1}{2}$cts.	
	$2.87\frac{1}{2}$	= " "	$6\frac{1}{4}$cts.	
	$\$20.12\frac{1}{2}$	" "	$43\frac{3}{4}$cts.	

Ex. 2. What cost 24 reams of paper at 15s. 9d. per ream?

Ans. £18 18s.

QUESTIONS.—158. What is analysis? What is analyzing a question? What is an aliquot part of a quantity?

WRITTEN PROCESS.

10s. = £½	24		= cost at £1				
5s. = ½	12£		= " "		10s.		
6d. = $\frac{1}{10}$	6£		= " "		5s.		
3d. = $\frac{1}{2}$		12s.	= " "			6d.	
		6s.	= " "			3d.	
	£18	18s.	= " "		15s.	9d.	

Ex. 3. At the rate of 3mi. 6fur. 36rd. per hour, how far will a person walk in 5hr. 40min. 45sec.?

Ans. 21mi. 7fur. 19.45rd.

WRITTEN PROCESS.

30min. = ½hr.	3mi.	6fur.	36rd.	= dist. in 1hr.					
			5						
	19	2	20	= "		5hr.	min.	sec.	
10min. = ⅓	1	7	18	= "			30		
30sec. = $\frac{1}{20}$		5	6	= "			10		
15sec. = ½			10.3	= "				30	
			5.15	= "				15	
	21mi.	7fur.	19.45rd.	"		5hr.	40min.	45sec.	

RULE.—*If a part of the multiplier is one or more units of comparison, multiply by such part, and to the product add those aliquot parts of the multiplicand which the other parts of the multiplier are of its unit.*

If no part of the multiplier is the unit of comparison, find the value of the multiplicand at the rate of such unit, and take those aliquot parts of this value which the given parts of the multiplier are of its unit.

NOTE 1.—It is best so to take the first aliquot part of the multiplicand, as to allow every subsequent part to be determined from its immediate predecessor.

NOTE 2.—This method of solving questions is much practiced in ordinary business, and is valuable for the mental exercise it affords, as well as for its frequent brevity of calculation.

For some of the following examples, consult the

QUESTIONS.—159. What is analysis by aliquot parts? How do we analyze by aliquot parts? What is the best method of taking aliquot parts?

Table of Aliquot Parts of a Dollar.

4cts. $= \frac{1}{25}$ of a dollar.	$12\frac{1}{2}$cts. $= \frac{1}{8}$ of a dollar.
5cts. $= \frac{1}{20}$ of a dollar.	$16\frac{2}{3}$cts. $= \frac{1}{6}$ of a dollar.
$6\frac{1}{4}$cts. $= \frac{1}{16}$ of a dollar.	20cts. $= \frac{1}{5}$ of a dollar.
$8\frac{1}{3}$cts. $= \frac{1}{12}$ of a dollar.	25cts. $= \frac{1}{4}$ of a dollar.
10cts. $= \frac{1}{10}$ of a dollar.	50cts. $= \frac{1}{2}$ of a dollar.

Examples for Practice.

What would be the cost of

4. 250 boxes blacking, at 4cts. per box? Ans. $10.
5. 375 bottles ink, at 5cts. per bottle? Ans. $18.75.
6. 148yd. muslin, at $6\frac{1}{4}$cts. per yd.? Ans. $9.25.
7. 165lb. sugar, at $8\frac{1}{3}$cts. per lb.? Ans. $13.75.
8. 96 gross buttons, at 4s. per gross? Ans. £19 4s.
9. 178 bottles wine, at 5s. per bottle? Ans. £44 10s.
10. 75bbl. salt, at $87\frac{1}{2}$cts. per bbl.? Ans. 65.62\frac{1}{2}$.
11. 525bbl. flour, at 3.62\frac{1}{2}$ per bbl.? Ans. 1903.12\frac{1}{2}$.
12. 142bu. apples, at $31\frac{1}{4}$cts. per bu.? Ans. 44.37\frac{1}{2}$.
13. 84bbl. flour, at £1 2s. 6d. per bbl.? Ans. £94 10s.
14. 80A. of land, at 15.37\frac{1}{2}$ per A.? Ans. $1230.
15. 19yd. merino, at $66\frac{2}{3}$cts. per yd.? Ans. 12.66\frac{2}{3}$.
16. 22yd. silk, at $1.16\frac{2}{3}$ per yd.? Ans. 25.66\frac{2}{3}$.
17. 43bu. rye, at $56\frac{1}{4}$cts. per bu.? Ans. 24.18\frac{3}{4}$.
18. 45lb. candy, at $18\frac{3}{4}$cts. per lb.? Ans. 8.43\frac{3}{4}$.
19. 68da. labor, at $1.33\frac{1}{3}$ per day? Ans. 90.66\frac{2}{3}$.
20. 347bu. wheat, at $93\frac{3}{4}$cts. per bu.? Ans. 325.31\frac{1}{4}$.
21. 5doz. tumblers, at $1.75 per doz.? Ans. $8.75.
22. 40A. 3R. 25P. land, at $22.50 per A.? Ans. $920.39+.
23. 5yd. 3qr. cloth, at $5.75 per yd.? Ans. 33.06\frac{1}{4}$.
24. 30A. 2R. 20P., at £5 4s. 6d. per A.?
Ans. £160 0s. 3d. 3qr.
25. 8bu. 1pk. 5qt. clover-seed, at 3.37\frac{1}{2}$ per bu.?
Ans. $28.37,1+.
26. 20lb. 12oz. butter, at $18\frac{3}{4}$cts. per lb.? Ans. $3.89+.
27. 40gal. 3qt. 1pt. wine, at $2.75 per gal.?
Ans. $112.40,6+.
28. How many cubic inches in 12gal. 3qt. 1pt. 2 gi. of wine?
Ans. $2988\frac{9}{16}$ cu. in.
29. How many cubic inches in 36gal. 1qt. 1pt. of beer?
Ans. $10257\frac{3}{4}$ cu. in.

30. If the moon's average daily motion is 13° 10′ 35″, how much of her orbit does she go through in 10da. 11hr. 12min. 13sec.? Ans. 137° 54′ 53.47″+.

GENERAL ANALYSIS.

Art. 160. In general analysis, proceed by the following

RULE.—*From the quantity depending upon a given number of units or parts, infer the quantity depending upon one of those units or parts: and then, if necessary, from this quantity infer the quantity depending upon the required number.*

QUESTIONS TO BE ANALYZED.

1. If 1yd. of cloth costs \$3.75, what will $\frac{3}{5}$ of a yd. cost? Ans. \$2.25.

ANALYSIS.—If 1 yd., that is, $\frac{5}{5}$ of a yd., cost \$3.75, $\frac{1}{5}$ of a yd. will cost $\frac{1}{5}$ of \$3.75, that is, \$0.75; and $\frac{3}{5}$ of a yd. will cost 3 times \$0.75, that is, \$2.25.

2. At \$24.50 per ton, what cost $\frac{5}{8}$ of a ton of hay? Ans. \$15.31¼.

3. At the rate of 28mi. 7fur. 35rd. per day, how far would a person walk in $\frac{6}{7}$ of a day? Ans. 24mi. 6fur. 30rd.

4. James Spencer is $1\frac{1}{2}$ as old as his brother John, whose age is 36yr. 6mo. 12da.: how old is James? Ans. 33yr. 5mo. 26da.

5. I am to receive $\frac{7}{9}$ of the crop of oats in a certain field. What is my share, if the crop is 324bu. 3pk. 3qt.? Ans. 252bu. 2pk. 5qt.

6. At the rate of \$53247 per mile, what cost $\frac{5}{6}$ of a mile of railroad? Ans. \$44372.50.

7. If $\frac{3}{5}$ of a yd. of cloth cost \$2.25, what cost 1 yd? Ans. \$3.75.

ANALYSIS.—If $\frac{3}{5}$ of a yd. cost \$2.25, $\frac{1}{5}$ of a yd. will cost $\frac{1}{3}$ of \$2.25, that is, \$0.75; and $\frac{5}{5}$ of a yd., that is, 1yd., will cost 5 times \$0.75, that is, \$3.75.

8. If $\frac{5}{8}$ of a ton of hay cost \$15.31¼, what is the price per ton? Ans. \$24.50.

9. If a person walks 24mi. 6fur. 30rd. in $\frac{6}{7}$ of a day, how far, at that rate, would he walk in a day? Ans. 28mi. 7fur. 35rd.

QUESTION.—160. What is the general method of analysis by inference?

10 If 13oz. of indigo sell for $1.17, what is the price per lb.?
Ans. $1.44.

11. If 11oz of gold-dust sell for $193.60, what is the cost of 1lb.? Ans. $211.20.

12. Anson Rand inherited $\frac{5}{9}$ of his father's estate, and sold his claim to $\frac{1}{2}$ of his share for $6250. At that rate, what was the value of the estate? Ans. $22500.

13. If $\frac{5}{12}$ of a person's income are $312.50, how much are $\frac{1}{8}$ of it? Ans. $656.25.

Analysis.—If $\frac{5}{12}$ of his income are $312.50, $\frac{1}{12}$ is $\frac{1}{5}$ of $312.50, that is, $62.50; and $\frac{12}{12}$, equal to the whole, are 12 times $62.50, that is, $750. Now, if his income is $750, $\frac{1}{8}$ of his income is $\frac{1}{8}$ of $750, that is, $93.75; and $\frac{7}{8}$ are 7 times $93.75, that is, $656.25.

14. If $\frac{3}{4}$ of a ton of bar-iron cost $48.30, what cost $\frac{6}{7}$ of a ton?
Ans. $55.20.

15. If $\frac{7}{10}$ of a box of soap cost $3.85, what cost $\frac{8}{11}$ of it?
Ans. $4.

16. If $\frac{5}{7}$ of a cask of syrup are 96gal. 1qt., how much are $\frac{7}{11}$ of it? Ans. 85gal. 3qt.

17. S. Payne sold $\frac{3}{10}$ of a lot to J. Peck, for the purpose of a house-lot, for $187.50; afterward, $\frac{2}{3}$ of the remainder, for a barn-yard, at the same rate. What did the barn-yard cost?
Ans. $291.66⅔.

18. David Holt, spending on a journey $\frac{2}{3}$ of the money which he had at starting, had $48.50 left, and determined to keep $\frac{3}{16}$ of the original sum till his journey was ended. How much had he yet to spend? Ans. $25.76,5⅝.

19. If 12lb. of tea cost $9, what will 5lb. cost? Ans. $3.75.

Analysis.—If 12lb. cost $9, 1lb. will cost $\frac{1}{12}$ of $9, that is, $0.75; and if 1lb. costs $0.75, 5lb. will cost 5 times $0.75, that is, $3.75.

20. If 4 horses consume 14bu. of oats per week, how many bushels will 10 horses consume in the same time? Ans. 35.

21. Paid $187.50 for 30yd. of broadcloth, and sold 13yd. to Wm. Brooks, at the same price. How much did I receive?
Ans. $81.25.

22. If a gross of bottles of porter costs £3 6s., what will 20doz. cost? Ans. £5 10s.

23. If 19 thousand feet of boards cost $622.25, what will 25 thousand feet cost? Ans. $818.75.

24. If 10 cu. ft. pure water weigh 625lb., what will 12½ cu ft. weigh? Ans. 781lb. 4oz.

ANALYSIS.—If 10 cu. ft. weigh 625lb., 1 cu. ft. will weigh $\frac{1}{10}$ of 625lb., that is, 62½lb. In 12½ cu. ft. there are $\frac{25}{2}$ cu. ft., and if 1 cu. ft. weighs 62½lb., ½ of a cu. ft. will weigh ½ of 62½lb., that is, 31¼lb.; and $\frac{25}{2}$ will weigh 25 times 31¼lb., that is, 781¼lb.; equal to 781lb. 4oz.

25. If 25bbl. of flour cost $118.75, what will 5½bbl. cost?
Ans. $26.12½.

26. If 7 weeks' board cost $22.75, what will board for 15 weeks 4 days cost? Ans. $50.60,7+.

27. If 24A. 3R. of land cost $350, what will 8A. 2R. 10P. cost? Ans. $121.08,5+.

28. If 12 thousand brick weigh 25 T. 4cwt., how much will 35¼ thousand weigh? Ans. 74 T. 50lb.

29. If 11⅔ sq. yd. of paving cost $7.70, what would it cost to pave a surface 20¼ft. long and 8ft. broad? Ans. $11.88.

30. If 5½bbl. of flour cost $26.12½, what will 25bbl. cost?
Ans. $118.75.

ANALYSIS.—In 5½bbl. there are $\frac{11}{2}$bbl. If $\frac{11}{2}$bbl. cost $26.12,5, ½bbl. will cost $\frac{1}{11}$ of $26.12,5, that is, $2.37,5; and $\frac{2}{2}$bbl., that is, 1 bbl., will cost 2 times $2.37,5, that is, $4.75; and 25bbl. will cost 25 times $4.75, that is, $118.75.

31. If 8¼doz. slates cost $28.87½, what will a gross cost?
Ans. $42.

32. If 5⅝bu. clover-seed cost $20.25, what cost 9bu.?
Ans. $32.40.

33. If 9⅓ cu. yd. masonry cost $42, what cost 63 cu. yd.?
Ans. $283.50.

34. If 8¼ tons of soda cost $825, what will 11 tons cost?
Ans. $1100.

35. If 35½ thousand brick weigh 74 tons 50lb., what will 8 thousand weigh? Ans. 16 tons 16cwt.

36. If 7⅘ tons of iron cost $663, what will 8¼ tons cost?
Ans. $701.25.

ANALYSIS.—In 7⅘ tons there are $\frac{39}{5}$ of a ton. If $\frac{39}{5}$ of a ton cost $663, ⅕ of a ton will cost $\frac{1}{39}$ of $663, that is, $17; and ⅘ of a ton, that is, 1 ton, will cost 5 times $17, that is, $85. In 8¼ tons there are $\frac{33}{4}$ of a ton. If 1 ton cost $85, ¼ of a ton will cost ¼ of $85, that is, $21.25; and $\frac{33}{4}$ will cost 33 times $21.25, that is, $701.25.

37. At $393.75 for 18¾A. land, what would 30⅐A. cost?
Ans. $633.

38. At $390 for 5⅕ months, what should I earn in 9⅕ months? Ans. $687.50.

39. If I pay $3.75 for the use of $100, 7½ months, what should I pay for its use $10\frac{1}{10}$ months? Ans. $5.05.

40. If I pay $8.75 for flooring $19\frac{4}{9}$ sq. yd., what should be paid for flooring a room 15½ feet long and 14 broad?

Ans. $10.85.

41. If 5 horses eat a certain quantity of oats in 9 days, how long will 15 horses be in eating the same? Ans. 3 days.

ANALYSIS.—If 5 horses eat the oats in 9 days, it will take 1 horse 5 times 9 days, that is, 45 days, to eat them; and it will take 15 horses $\frac{1}{15}$ of 45 days, that is, 3 days, to eat them.

42. If 4 compositors set 400 pages of type in 25 days, how long would 10 compositors be in doing the same?

Ans. 10 days.

43. If 6 cows consume the grass of a field in 4 weeks, how long would 16 cows be in doing the same? Ans. 1½ weeks.

44. A certain vessel, having provision for 9 persons 11 weeks, rescued 30 persons from shipwreck. How long should the provisions last with regular rations? Ans. $2\frac{7}{13}$ weeks.

45. If 7 bricklayers lay the brick of a certain building in 4 weeks 2 days, how long would it take 4 bricklayers to do the same? Ans. 7 weeks 3½ days.

46. If the harvesting of a farmer would occupy 10 hands 23 days, and he can find only 8 hands, how long will it take to gather his harvest? Ans. $28\frac{3}{4}$ days.

47. If 11 men pay $165 for 5 weeks' board, how much would it cost 9 men for 7 weeks' board? Ans. $189.

ANALYSIS.—If 5 weeks' board of 11 men cost $165, 1 week's board of 11 men costs $\frac{1}{5}$ of $165, that is, $33. If 11 men pay $33 for 1 week's board, 1 man should pay $\frac{1}{11}$ of $33, that is, $3, and 9 men should pay 9 times $3, that is, $27. If 9 men pay $27 for 1 week's board, they should pay 7 times $27, that is, $189, for 7 weeks' board.

48. If a single pair of millstones grind 750bbl. flour in 5 weeks, how many bbl. would 4 pairs grind in 9 weeks?

Ans. 5400bbl.

49. If 40 looms weave 14400yd. cloth in 4 weeks, how many yd. would 150 looms weave in 50 weeks? Ans. 675000yd.

50. If $100 gain $6 in 12 months, how many dollars would $500 gain in 4 months? Ans. $10.

51. If the 4 cent loaf weighs 9oz. when flour is $8 a bbl., how many oz. should the 6 cent loaf weigh when flour is $6 a bbl.?

Ans. 18oz.

CHAPTER XIV.

RATIO. PROPORTION. VARIATION.

RATIO.

Art. 161. RATIO is the relation of difference or quotient between two numbers, and expresses the comparison of one number with another.

This comparison is of two kinds,—viz.,

1st, By subtraction, to find *how much* one number *exceeds* another.

2d, By division, to find *how many times* one number *contains* another.

In the comparison of two numbers by subtraction, the difference is called the *arithmetical ratio.*

In the comparison of two numbers by division, the quotient is called the *geometrical ratio.*

NOTE.—The word *ratio*, by itself, always signifies geometrical ratio. The word *difference* is now mostly used for arithmetical ratio.

ILLUSTRATIONS OF RATIO.

1. What relation has 3 to 6? Ans. It is $\frac{3}{6}$ or $\frac{1}{2}$ as much.
2. What relation has 6 to 3? Ans. It is $\frac{6}{3}$ or twice as much.
3. What is 2, compared with 8? Ans. It is $\frac{2}{8}$ or $\frac{1}{4}$ as much.
4. What is 8, compared with 2? Ans. It is $\frac{8}{2}$ or 4 times as much.

Art. 162. The *terms* of a ratio are the two numbers which are compared. The *antecedent* is the first term of a ratio; the *consequent* is the second term.

Ratio is represented to the eye in two ways,—viz.

FIRST, By a fraction whose numerator is the antecedent, and whose denominator is the consequent;* and,

* This is the English method. The French usually write the consequent for the numerator, and the antecedent for the denominator.

QUESTIONS.—161. What is ratio? What does it express? What are the kinds of comparison? What is arithmetical ratio? Geometrical ratio?—162. What are the terms of a ratio? The antecedent? The consequent? How is ratio written? What is the French method?

SECOND. By placing a colon between the antecedent and consequent.

Thus, the ratio of 5 to 9 is either written $\frac{5}{9}$, or 5 : 9, and is read "5 is to 9."

Art. 163. The terms of a ratio must be of the same denomination. There can be no ratio between *yards* and *gallons*, between *pounds* and *weeks*. There can be no ratio *directly* between 3 *minutes* and 6 *hours*, since it could not be $\frac{3}{6}$; but reducing 6 hours to minutes, there is a ratio between 3 *minute* and 360 *minutes*.

Art. 164. Since the ratio of two numbers is the *quotient* of one divided by the other, and is expressed by a *fraction*, whose numerator is the antecedent, and whose denominator is the consequent, the following propositions must be true.

1st. *Multiplying the antecedent, or dividing the consequent, multiplies the value of the ratio.* (See Art. 103.)

2d. *Dividing the antecedent, or multiplying the consequent, divides the value of the ratio.* (See Art. 103.)

3d. *Multiplying or dividing both terms by the same number does not affect the value of the ratio.* (See Art. 103.)

Art. 165. In reference to the number of their terms, ratios are *simple* or *compound*.

A *simple ratio* is a ratio having only one antecedent and one consequent; as 3 : 5, or $\frac{3}{5}$.

A *compound ratio* is a ratio whose antecedent expresses the product of the antecedents of two or more ratios, and whose consequent expresses the product of the consequents of the same ratios: as $3 \times 4 : 5 \times 7$, or $\frac{3 \times 4}{5 \times 7}$.

EXAMPLES FOR PRACTICE.

1. What is the ratio of 7 to 9? 8 to 12? 10 to 14? 6 to 10?
2. What is the ratio of 16 to 4? 14 to 7? 15 to 3? 18 to 3?
3. What is the ratio of 14 to 4? 16 to 5? 8 to 3? 12 to 7?
4. Is the ratio of 6 to 9 greater or less than that of 9 to 12?
   Ans. Less. (Prove it.)
5. Is the ratio of 8 to 14 greater or less than that of 8 to 16?
   Ans. Greater. (Prove it.)

QUESTIONS.—163. What must the terms of a ratio be?—164. What propositions are true of ratios? Why?—165. How are ratios classified? What is a simple ratio? What is a compound ratio?

6. What is the difference between the ratio of $3\frac{1}{2}$ to $5\frac{1}{4}$, and that of $6\frac{1}{4}$ to $7\frac{1}{2}$? Ans. English $\frac{1}{6}$; French $\frac{3}{10}$.

7. Compare the ratio of 5 to 11 with that of 35 to 77.
Ans. They are equal.

8. Compare the ratio of 9 to 4 with that of $13\frac{1}{2}$ to 6.
Ans. They are equal.

9. What is the ratio of 80 rods to 1 mile? Ans. $\frac{1}{4}$.

10. What is the ratio compounded of 5 to 6, and 12 to 15?
Ans. $\frac{2}{3}$.

Art. 166. One thing varies *directly* as another, when it increases proportionally as the other increases, and decreases proportionally as the other decreases. Thus, the *cost* of articles varies directly as their *quantity*, if the price is fixed; and, in general, the quantity of *effect* varies directly as the quantity of *cause*.

One thing varies *inversely* as another, when it increases proportionally as the other decreases, and decreases proportionally as the other increases. Thus, the *time* occupied in traveling a certain distance varies inversely as the velocity; that is, with *twice* a given velocity, the distance would be traveled in *half* the time; with *three times* the velocity, in *one-third* the time, &c.

PROPORTION.

Art. 167. A PROPORTION is a statement of the equality of ratios whose terms are fully expressed. Four numbers are in proportion when the ratio of the first to the second is the same as that of the third to the fourth.

The equality of one ratio to another is represented in three ways,—viz.:

FIRST. As the equality of one fraction to another; as, $\frac{4}{8} = \frac{3}{6}$.

SECOND. By representing the ratios with colons, and placing the sign of equality between them; as, $4 : 8 = 3 : 6$.

THIRD. By a double colon between the ratios; as, $4 : 8 :: 3 : 6$.

NOTE.—The last is the common method, and the proportion given is read "4 is to 8 as 3 is to 6."

Art. 168. The numbers which form a proportion are called *proportionals*, or *terms of the proportion*.

QUESTIONS.—166. When does one thing vary directly as another? When does one thing vary inversely as another?—167. What is a proportion? When are four numbers in proportion? How is a proportion written?—168. What are the terms of a proportion?

The *extremes* of a proportion are its *first* and *fourth* terms.

The *means* of a proportion are its *second* and *third* terms.

Thus, in the proportion 3 : 4 :: 6 : 8, the *extremes* are 3 and 8, and the *means* are 4 and 6.

Art. 169. In reference to the number of terms in their ratios, proportions composed of two ratios are *simple* or *compound*.

A *simple proportion* is a statement of the equality of *two simple ratios* whose terms are fully expressed; as, 3 : 9 :: 6 : 18.

A *compound proportion* is a statement of the equality of a *simple* to a *compound ratio*, of both which the terms are fully expressed; as, $3 \times 4 : 5 \times 7 :: 24 : 70$.

Art. 170. *If, in a proportion, both terms of any ratio are multiplied or divided by the same number, the resulting numbers are proportional with the remaining numbers.*

Thus, the proportion 4 : 8 :: 6 : 12 may become

By multiplication, 8 : 16 :: 6 : 12, or 4 : 8 :: 12 : 24, &c.

By division, 2 : 4 :: 6 : 12, or 4 : 8 :: 3 : 6, &c.

Art. 171. *If, in a proportion, both antecedents or both consequents be multiplied or divided by the same number, the resulting numbers will be proportional with the remaining numbers.*

Thus, the proportion 3 : 9 :: 6 : 18 may become

By multiplication, 6 : 9 :: 12 : 18, or 3 : 18 :: 6 : 36, &c.

By division, 1 : 9 :: 2 : 18, or 3 : 1 :: 6 : 2, &c.

Art. 172. *The product of the extremes of a proportion is equal to the product of its means.*

Illustration.—The proportion 2 : 3 :: 8 : 12 may be written thus: $\frac{2}{3} = \frac{8}{12}$. Now, multiplying both ratios by 12, we have $\frac{2 \times 12}{3} = 8$; and, multiplying this result by 3, we have $2 \times 12 = 3 \times 8$.

Art. 173. *If three terms of a proportion are given, the fourth may be found, if it is an extreme, by dividing the product of the means by the given extreme; or, if it is a mean, by dividing the product of the extremes by the given mean.*

Questions.—168. What are the extremes of a proportion? The means?—169. How are proportions composed of two ratios classified? What is a simple proportion? A compound proportion?—170. What effect has multiplying or dividing both terms of a ratio by the same number?—171. What is the effect of multiplying or dividing both antecedents or both consequents by the same number?—172. What is the product of the extremes?—173. How can a fourth proportional be found when three are given?

ILLUSTRATIONS.

1st. 2 : 4 :: 6 : what? Ans. $\frac{4 \times 6}{2} = 12.$

2d. 2 : 4 :: what? : 12 Ans. $\frac{2 \times 12}{4} = 6.$

3d. 2 : what? :: 6 : 12 Ans. $\frac{2 \times 12}{6} = 4.$

4th. What? : 4 :: 6 : 12 Ans. $\frac{4 \times 6}{12} = 2.$

REMARK.

This results from the fact stated in Art. 172, since a *product*, and one *factor*, are given to find the *other*.

SIMPLE PROPORTION.

Art. 174. Simple Proportion is employed to find the fourth term of a proportion, when three terms are given.

Art. 175. To state and solve questions in Simple Proportion.

Ex. 1. If 56lb. of tea cost $42, what will 12lb. cost?

Ans. $9.

WRITTEN PROCESS.

56lb. : 12lb. :: $42 : How much?

12

56) 504 ($9. Ans.
504

BY CANCELLATION.

56 : 12 :: 42
4 : 12 :: 3
1 : 3 :: 3
3 × 3 = 9 Ans.

MENTAL PROCESS.

The ratio of 56lb. to 12lb. must be the same as that of the *cost* of 56lb. to the *cost* of 12lb. If we make the first ratio with the numbers expressing *quantity* of tea, the second ratio must be made with the numbers expressing *cost*. The unknown cost of 12lb. must be less than the known cost of 56lb., because 12lb. are less than 56lb. If, therefore, we make the greater cost the *antecedent* of the second ratio, then the greater quantity of tea must be the antecedent of the first ratio, and the smaller the consequent. But, if we make the greater cost the *consequent* of the second ratio, then the greater quantity of tea must be the consequent of the first ratio, and the smaller the antecedent. We are not limited to any particular place in the proportion, in which to put the unknown

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QUESTIONS.—174. In what is Simple Proportion employed?—175. How do we state and solve questions in Simple Proportion? What reduction is necessary?

quantity, (see Art. 173;) but the *fourth* or last place is usually assigned to it. Then find the product of the means, and divide it by the given extreme. From Arts. 170 and 171 we see that any factor common to the solitary term of the defective ratio, and *either of the other two terms*, can be canceled out without destroying proportionality of terms. Thus, 14 from 56 and 42 gives 4 : 12 : : 3; and 4 from 4 and 12 gives 1 : 3 : : 3.

Ex. 2. A pasture will sustain 3 cows 5 weeks. How long will it sustain 4 cows? Ans. $3\frac{3}{4}$ weeks.

COMMON WRITTEN PROCESS.

cows. cows. weeks.
4 : 3 : : 5 : What time?

$$\begin{array}{r} 3 \\ \hline 4)15 \end{array}$$

$3\frac{3}{4}$ weeks. Ans.

MENTAL PROCESS.

Since the pasture will sustain 4 cows less time than it will 3 cows, the less number of cows should be one consequent if the answer is assigned to the place of the other; or should be one antecedent if the answer is assigned to the place of the other. In any arrangement, find the required term by Art. 173.

OTHER ARRANGEMENTS OF TERMS.

What time? : 5 weeks : : 3 cows : 4 cows.
5 weeks : What time? : : 4 cows : 3 cows.
3 cows : 4 cows : : What time? : 5 weeks.

RULE.— *Write, for the third term, that number which is of the same kind as the number sought.*

Of the other two numbers, write the greater for the second term, when the answer should be greater than the third term; otherwise write the less for the second term.

Multiply the second and third terms together, and divide the product by the first term.

NOTE 1.—Before thus multiplying and dividing, reduce the first and second terms to the same denomination, if they are of different denominations.

NOTE 2.—After arranging the terms, before proceeding with the multiplication and division, it is advisable to cancel all factors common to the first term and either or both of the other two terms.

NOTE 3.—If the third term is compound, it is sometimes best to reduce it to the lowest denomination before proceeding. The answer, being in that denomination, should then be reduced to higher denominations.

QUESTIONS.—175. How can we cancel? What is this rule called?

NOTE 4.—The required term *need* not be the fourth, since we can arrange the given terms so as to make the required term occupy any other place in the proportion. In such a case, we find the required term by Art. 173.

NOTE 5.—The learner should solve the following questions by both *analysis* and *proportion*.

NOTE 6.—This rule is often called the "*Single Rule of Three.*"

EXAMPLES FOR PRACTICE.

3. If 16yd. cloth cost $56, what cost 12yd.? Ans. $42.

4. If $42 purchase 12yd. cloth, how many yd. will $56 purchase? Ans. 16yd.

5. If 4 horses can transport the stone for a building from the quarry in 30 days, how many horses must be employed to do it in 12 days? Ans. 10 horses.

6. If 40bu. oats supply my horses 24 days, how long will 60bu. supply them? Ans. 36 days.

7. If 60bu. oats supply my 4 horses 36 days, and I buy 2 more horses, how long will the oats supply the whole? Ans. 24 days.

8. If a certain quantity of food suffices for 9 persons 7 days, how long will it suffice for 6 persons? Ans. $10\frac{1}{2}$ days.

9. I am offered 3doz. of eggs for 25 cents, and conclude to take 8doz. at the same rate. How much must I pay? Ans. $66\frac{2}{3}$ cents.

10. If 34gal. of molasses cost $13.60, what is the cost of 17gal.? Ans. $6.80.

11. If $5\frac{1}{2}$yd. of cloth cost $30.25, how many yd. can be bought for $57.75? Ans. $10\frac{1}{2}$yd.

12. If 2 bricklayers can lay 36 thousand brick in 6 days, how long would it take 8 bricklayers to do the same? Ans. $1\frac{1}{2}$ days.

13. At the rate of 20 miles in 30 minutes, how far would a railroad-train go in $13\frac{1}{2}$ hours? Ans. 540 miles.

14. If by laboring 12 hours a day a man can dig a well in 15 days, how long would it take him if he labored only 10 hours a day? Ans. 18 days

15. If 75 men build a bridge in $1\frac{1}{2}$ years, how many men would it take to build it in 6 months? Ans. 225 men.

16. If 7bu. of wheat will purchase 2bbl. of flour, how many bu. must be given for 16bbl. of flour? Ans. 56bu.

17. If a certain spring delivers 42gal. 2qt. in 1 hour 20 minutes, how long will it be in delivering 127gal. 2qt.?
Ans. 4 hours.

18. If the 5 cent loaf weighs $\frac{3}{4}$ of a lb. when flour is $7 a bbl., how much should it weigh when flour is $6 a bbl.?
Ans. 14oz.

19. A wood-chopper, 5ft. 10in. high, found that the shadow of a certain tree, on level ground, was 121ft. long, while his own shadow measured 8ft. 9in. How high was the tree?
Ans. 80ft. 8in.

20. I borrowed of a friend $40 for 2 months; he afterward borrowed $20 of me: how long should he keep it to equalize the favors? Ans. 4 months.

21. If it takes 22½yd. carpeting, $\frac{3}{4}$ of a yd. wide, to carpet a certain room, how many yd., 1¼yd. wide, will it take to do the same? Ans. 13½yd.

22. If 4 men could mow a certain field in 15 hours, and 2 more men join them, how long will it take the whole?
Ans. 10 hours.

23. If a steamer, whose speed is 15 miles per hour, is sent in pursuit of another whose speed is 12 miles per hour, and which has had 10½ hours' start, how long before the first overtakes the second? Ans. 1 day 18 hours.

24. If a watch gains 3½min. a day, how long before its time will be 20min. past 2 o'clock, at noon? Ans. 40 days.

25. If the fore-wheel of a carriage is 8ft. 6in. in circumference, and turns 3600 times in a certain journey, how many times will the hind-wheel, which is 10ft. 7½in. in circumference, turn in the same journey? Ans. 2880 times.

26. If with a false peck-measure, containing only 7qt. 1½pt., a merchant sells you clover-seed, how much do you get in buying 5½ false bu., and how much are you cheated?
Ans. 1st, 5bu. 1pk. 2qt. 1pt.; 2d, 5qt. 1pt.

27. If an incorrect two-foot rule, actually 2ft. 1½ tenths of an in. long, makes a wall appear to be 40ft. long, what is the actual length of the wall? Ans. 40ft. 3in.

28. Supposing the earth a perfect sphere, what would be its circumference if 1° 15′ of latitude measured 86m. 101$\frac{2}{3}$rd.?
Ans. 24859m. 140$\frac{4}{6}$rd.

29. If a person's pulse beats 5625 times in 1hr. 15m., how many times a minute does it beat? Ans. 75 times.

30. A sick person's pulse beat 253 times in 2min. 12sec. what was the rate per minute? Ans. 115.

Art. 176. To resolve a number into parts having a given ratio.

Ex. 1. Resolve 72 into two parts, of which the first shall be to the second as 4 to 5. Ans. 32 and 40.

Analysis.—If the parts may be represented in ratio by 4 and 5, the whole may be represented by $4 + 5$, that is, 9; and one part must be $\frac{4}{9}$ of the whole, and the other part $\frac{5}{9}$ of the whole. One-ninth of 72 is 8; four-ninths are 4 times 8, that is, 32; and five-ninths are 5 times 8, that is, 40.

Process by Proportion.

$4 + 5 : 4 :: 72 : 32$ the less part.

$4 + 5 : 5 :: 72 : 40$ the greater part.

Mental Process.

As $4 + 5$, or 9, which *represents* the number, is to 4, which *represents* the less part of the number, so is 72, the *actual whole*, to 32, the *actual less part;* and as 9, which *represents* the number, is to 5, which *represents* the greater part of the number, so is 72, the *actual* whole, to 40, the *actual* greater part. Or, after getting 32, the less part, subtract it from 72, to find 40, the greater part.

Rule.—*Divide the given number by the sum of the terms of the ratio, and multiply the quotient by each term; or,*

As the sum of the terms of the ratio is to one of them, so is the number to be resolved to its corresponding part.

Examples for Practice.

2. Resolve 96 into parts, which are as 7 to 9.
Ans. 42 and 54.

3. What numbers, composing 84, are as 3, 4, and 5?
Ans. 21; 28; 35.

4. What numbers, composing 132, are as $2\frac{1}{2}$, $5\frac{1}{2}$, and $8\frac{1}{2}$?
Ans. 20; 44; 68.

5. What numbers, composing 154, are as $1\frac{1}{4}$, $2\frac{1}{4}$, $3\frac{1}{4}$, and $4\frac{1}{4}$?
Ans. $17\frac{1}{2}$; $31\frac{1}{2}$; $45\frac{1}{2}$; $59\frac{1}{2}$.

6. Two men, 180 miles apart, travel toward each other, one going 5 miles while the other goes 4. What is the distance traveled by each? Ans. One 80; the other 100.

7. Four heirs, aged respectively 15, 17, 19, and 21 years, share a legacy of $25200, according to their ages: what were their shares? Ans. $5250; $5950; $6650; and $7350.

Question.—176. How do we resolve a number into parts having a given ratio?

COMPOUND PROPORTION.

Art. 177. Compound Proportion is employed when the solution of a question by simple proportion requires more than one statement.

When the answer depends upon *several pairs* of conditions, these conditions may be arranged so as to make the first ratio of a statement *compound.*

Art. 178. To state and solve questions in Compound Proportion.

Ex. 1. If 8 men dig, in 10 days, a ditch 50ft. long, 10ft. wide, and 5ft. deep, in how many days will 18 men dig a ditch 95ft. long, 12ft. wide, and 8ft. deep? Ans. $16\frac{16}{75}$ days.

Common Written Process.	By Cancellation.
18 men : 8 men : : 10 days.	3 18 : 8 : : 10
50ft. l. : 95ft. l.	5 10 50 : 95 19
10ft. w. : 12ft. w.	10 : 12 2
5ft. d. : 8ft. d.	5 : 8
45000 : 72960 : : 10 : $16\frac{16}{75}$	75 :)1216
	$16\frac{16}{75}$

Mental Process.—Since the answer must express days, we may put the given 10 days as the third term, that is, as the antecedent of the unfinished ratio. Now, separately considering each pair of the remaining conditions, since the demanded 18 men will do the piece of work in less days than the stated 8 men, make 8, the less number of men, a consequent, and 18, the greater number, an antecedent of the first ratio. Again, since the demanded 95ft. of length will require more days than the stated 50ft. of length, make 95, the greater, a consequent, and 50, the less, an antecedent in the first ratio. Again, since the demanded 12ft. of width will require more days than the stated 10ft. of width, make 12, the greater, a consequent, and 10, the less, an antecedent in the first ratio. Lastly, since the demanded 8ft. of depth will require more days than the stated 5ft. of depth, make 8, the greater, a consequent, and 5, the less, an antecedent in the first ratio. The first ratio, thus constructed, is *compound,* (see Art. 165,) and, by multiplying the factors in each term, can be made a simple ratio. But it frequently lessens labor, first to cancel every factor common to any one of the first antecedent terms, and any of the other terms, as in simple proportion; and then multiply together the factors remaining in the means, and divide the product by the product of the factors remaining in the first extreme.

Questions.—177. When is Compound Proportion employed? How can several pairs of conditions be arranged in one statement?

The following form illustrates a method of statement and solution preferred by many:—

As the stated effect, $50 \times 10 \times 5$, viz., cubical contents,	:	is to the demanded effect, $95 \times 12 \times 8$, viz., cubical contents,	::	so is the stated cause, 8 men working 10 days,	:	to the demanded cause, 18 men working how many days?	

Making the product of the means equal to the product of the extremes,

$(95 \times 12 \times 8) \times 8 \times 10 = (50 \times 10 \times 5) \times 18 \times$ Ans.; and, canceling, we have $64 \times 19 = 75 \times$ Ans. Therefore the answer is $\frac{64 \times 19}{75} = \frac{1216}{75} = 16\frac{16}{75}$ days.

RULE.—*Write for the third term that number which is of the same kind as the number sought.*

Of the remaining numbers, arrange those of the same kind in ratios, as in simple proportion.

Multiply together all the numbers in the means, and divide the result by the product of the numbers in the first extreme.

NOTE 1.—To lessen labor, cancel, if possible, as in Simple Proportion.

NOTE 2.—This rule treats the numerical conditions of a question as *abstract* numbers, affecting the answer by their *ratio:* since the product of *men* into *cubical contents*, &c., that is, *of one concrete number by another, is absurd.*

NOTE 3.—The learner should solve the following questions by Analysis, Simple Proportion, and Compound Proportion.

NOTE 4.—This rule is often called the "*Double Rule of Three.*"

EXAMPLES FOR PRACTICE.

2. If 20 men, working 10 hours a day, can build a wall 25ft. long, 24ft. high, and 12ft. thick, in 6 days, how many hours per day must 10 men work, to build a wall 50ft. long, 12ft. high, and 18ft. thick, in 18 days? Ans. 10 hours per day.

3. If 10 persons eat \$50 worth of flour in 8 months, when flour is \$6 a barrel, how many dollars worth will 15 persons eat in 12 months, when flour is \$5 a barrel? Ans. \$93.75.

4. If 25 men, in 10 days, receive \$375 wages, how much would 15 men receive in 30 days, at the same rate?

Ans. \$675.

QUESTIONS.—178. How do we state and solve questions in Compound Proportion? How does this rule treat the numbers in questions? What is the rule often called?

5. If a certain company pays 150 hands $187.50 wages every Saturday evening, what must it pay at a time if it employs 30 more hands at the same rate, and pays every 4 weeks?

Ans. $900.

6. If a marble block, 2ft. long, 1ft. broad, and 6in. thick, weighs 170lb., what is the weight of a slab 8ft. long, 20in. broad, and 4in. thick? Ans. $755\frac{5}{9}$ lb.

7. If 7lb. wool make 8yd. of cloth 3 quarters wide, how many pounds will it take to make 36yd. 6 quarters wide?

Ans. 63 lb.

8. If 3 bricklayers can lay 54 thousand brick in 6 days, working 10 hours a day, how many can 9 bricklayers lay in 24 days, working 11 hours a day? Ans. 712800.

9. If a pasture of 14 acres can sustain 6 horses 5 months, how long would 12 acres of that land sustain 8 horses?

Ans. $3\frac{3}{14}$ months.

10. If 3 compositors, in 54 days, working 12 hours a day, can compose 10 sheets of 32 pages each, 44 lines in a page, and 52 letters in a line, in how many days, working 10 hours a day, can 6 compositors compose 32 sheets of 12 pages in a sheet, 45 lines in a page, and 50 letters in a line? Ans. $38\frac{67}{286}$ days.

11. If $800 gain $32 in 8 months, how much would $100 gain in 12 months? Ans. $6.

12. If $100 gain $6 in 12 months, how long will it take $800 to gain $32? Ans. 8 months.

13. If the transportation of 8cwt. 3qr. 175 miles costs $35.75, what will be the cost of transporting 9cwt. 25lb. 285 miles, at the same rate? Ans. $61.54,8+.

14. If a certain cask by being emptied 1092 times into a cistern, 35ft. long, $10\frac{1}{2}$ft. broad, and 13ft. deep, fills it, how many such caskfuls will fill a cistern 16ft. long, 7ft. broad, and 15ft. deep? Ans. 384.

15. If 112 measures of wheat fill a bin 7ft. long, 5ft. wide, and 4ft. deep, how deep must a bin, 18ft. long by $3\frac{3}{4}$ft. wide, be made in order to contain 240 such measures? Ans. $4\frac{4}{9}$ft.

16. If it requires 4500 brick 8in. long by 4in. wide to pave, with their broad sides, a yard 50ft. long by 20ft. broad, how many brick will it take, when laid on their long edges, 2in. thick, to pave a place 40ft. long and 10ft. broad? Ans. 3600.

CHAPTER XV.

PARTNERSHIP. BANKRUPTCY.

PARTNERSHIP.

Art. 178. PARTNERSHIP is the association of two or more persons in business, with an agreement to share the profits or losses.

Persons thus associated are called *partners.*

A business association is styled a *Company, Firm,* and sometimes a *House.*

Capital, sometimes called *Stock,* is the property employed in business.

A *dividend* is that property, or value, which is shared by the partners.

An *assessment* is a demand made upon each partner to meet his share of a loss, or to furnish his share of the money used in doing business.

The *liabilities* of a company are its debts.

CASE I.

Art. 179. To find each partner's share of profit, or liability, when their times of engagement are equal.

Ex. 1. W. T. Moorhead and J. Brown form a partnership with a capital of $6500. Moorhead furnishes $4000, and Brown $2500. At the end of a year, they divide a gain of $2678; what is each man's share?

Ans. Moorhead $1648; Brown $1030.

QUESTIONS.—178. What is partnership? Who are partners? What is a business association styled? What is capital? What is a dividend? What is an assessment?—179. How do we find each partner's share, when their times of engagement are equal?

WRITTEN PROCESS.

6500)2678.000(.412
26000

7800
6500

13000
13000

.412 × 2500 = $1030, Brown's share.
.412 × 4000 = $1648, Moorhead's share.

MENTAL PROCESS.

If $6500 gain $2678, one dollar gains $\frac{5}{6500}$ of $2678; that is, $0.412; and $2500 gain 2500 times $0.412; that is, $1030; and $4000 gain 4000 times $0.412; that is, $1648.

BY PROPORTION.

$6500 : $2500 :: 2678 : 1030
$6500 : $4000 :: 2678 : 1648

RULE.—*Divide the sum to be shared by the whole capital, to find the share for one dollar, or unit of money; then multiply this quotient by the capital of each partner, to find his share.*

Or, *As the whole stock is to any partner's stock, so is the whole sum to be shared to his share of it.*

PROOF.—The sum of the shares should equal the dividend or assessment.

EXAMPLES FOR PRACTICE.

2. Elliot & Rose began business with $12000, of which Elliot furnished $7500, and Rose the rest. At the end of a year they share $3472; what was each man's share?

Ans. E. $2170; R. $1302.

3. At the end of the second year they dissolve partnership, sharing a loss of $864 besides their capital. How much was assessed upon each? Ans. E. $540; R. $324.

4. Agnew, Fitch & Gould associated to purchase a steamboat for $24500; A. furnishing $10250, F. $8750, and G. the rest. When the "running season" ended, they had gained $14000, and they dissolved partnership, selling the boat for $16000. What was each man's share of the dividend?

Ans. A. 12551.02\frac{2}{7}$; F. $10714.28,5$\frac{5}{7}$; G. 6734.69\frac{4}{9}$.

5. The firm of Abel, Bunn, Carter & Day, grocers, divided a gain of £583 10s. 9d. A. had furnished £178 18s. 8d., B. £236 15s. 8d., C. £493 18s. 8d., and D. £213 17s. 6d. What was the share of each? Ans. A. £92 18s. 8¼d.; B. £122 19s. 7½d.; C. £256 10s. 9½d.; D. £111 1s. 7¾d.

6. A mining company, whose property was represented by 10000 shares, assessed upon themselves $35000 to carry on the business. How much did M pay, who owned 45 shares?

Ans. $157.50.

Case II.

Art. 180. To find each partner's share of profit or liability, when their times of engagement are not equal.

Ex. 1. The firm of Emmet & Langdon lost $4240, whereupon they dissolved partnership. E. had furnished $8000 for 16 months, and L. $7000 for 12 months. How must they share the loss? Ans. E. $2560; L $1680.

Written Process.

$$8000 \times 16 = 128000$$
$$7000 \times 12 = 84000$$

$$212000 \overline{)} 4240.00 \,(\, .02$$
$$424000$$

$$.02 \times 128000 = \$2560, \text{ E.'s share.}$$
$$.02 \times 84000 = \$1680, \text{ L.'s share.}$$

Mental Process.

$8000 used 16 months is the same amount of use as would be that of 16 times $8000 for 1 month; that is, $128000 for 1 month; and $7000 used 12 months is the same use as 12 times $7000 for 1 month; that is, $84000 for 1 month. Both these uses amount to $212000 for 1 month. Now, if $212000 bear $4240 loss, $1 must bear $\frac{1}{212000}$ of $4240; that is, $0.02; and $128000 must bear 128000 times $0.02; that is, $2560; and, 84000 must bear 84000 times $0.02; that is, $1680.

By Proportion.

$$212000 : 128000 :: \$4240 : \$2560$$
$$212000 : 84000 :: \$4240 : \$1680$$

Rule.—*Reduce all the times to the same denomination of time. Multiply each partner's stock by its time of engagement, add the products, and divide the amount to be shared by their sum. By the quotient multiply each partner's product to find his share.*

Or, *As the sum of the partners' products is to any partner's product, so is the sum to be shared, to his share of it.*

Examples for Practice.

2. The firm of Adams & Sumner began business with $15500, of which A. furnished $9500, and S. the rest. At the end of

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Question.—180. How do we find each partner's share of profit or liability, when their times of engagement are not equal?

7 months they took in Leach as a partner, who put $8000 more into the concern. At the end of 9 months more they dissolved partnership, and divided $5475. What was the share of each?

Ans. A. $2600.62½; S. $1642.50; L. $1231.87½.

3. A, B, C, and D, hired a pasture together, in which A pastured 3 cows 15 weeks, B 5 cows 11 weeks, C 2 cows 13 weeks, and D 4 cows 12 weeks. They paid a rent of $90. What was the share of each?

Ans. A $23.27,5+; B $28.44,8+; C $13.44,8+; D $24.82,7.

4. Baker, Law & Perry did business as follows:—B. at first paid in $4000, and 6 months later $2000 more; L. at first paid in $3250, and 9 months later $1500 more; P. at first paid in $4500, and 6 months later drew out $2000. At the end of 15 months they dissolved partnership, and divided $7500. What was the share of each?

Ans. B. $3157.89,4+; L. $2338.05,6+; P. $2004.04,8+.

5. Horton & Ingram, failing in business, dissolved partnership, and paid their liabilities, amounting to $8750. H. had had $12000 in the concern 1 year, and $4000 6 months. I. had had $6000 in the concern 1 year, and $3000 4 months. How much was assessed on each? Ans. H. $5833.33⅓; I. $2916.66⅔.

6. In a certain school the whole attendance was equal to that of 1 pupil 1950 days. A had sent 2 pupils 75 days, B 3 pupils 72 days, C 2 pupils 70 days, and 1 pupil 85 days. How much of $105, the whole cost of the school, should be assessed upon each? Ans. A 8.07\frac{9}{13}$; B 11.63\frac{1}{13}$; C 12.11\frac{7}{13}$.

BANKRUPTCY.

Art. 181. BANKRUPTCY, or *insolvency*, is inability, through lack of property, to pay debts.

Debtors are persons who owe. *Creditors* are persons to whom debts are owed.

The *assets* of a debtor are such portions of his property as can be appropriated to paying his debts.

Net proceeds are the value remaining after all necessary deductions.

An *assignee* is a person selected to take charge of the assets of a debtor and to apply them to the payment of the creditors.

Art. 182. To find each creditor's share of the assets of a bankrupt.

QUESTIONS.—181. What is insolvency? Who are debtors? Who are creditors? What are assets? What are net proceeds? Who is an assignee?

Ex. 1. The assignee of F sells F's property for $6275.19. F owes A $3175.27, B $4550, and C $2873.15. The assignee charges $250 for his trouble. How much do A, B, and C receive? Ans. A $1805.14; B $2586.67; C $1633.38.

WRITTEN PROCESS.

$6275.19 — $250 = $6025.19 = net proceeds.
$3175.27 = A's claim.
4550. = B's claim.
2873.15 = C's claim.

10598.42 = total claims.
6025.19 ÷ 10598.42 = $0.5685 = div'd on $1.
$0.5685 × 3175.27 = $1805.14+ A's share.
$0.5685 × 4550 = $2586.67+ B's share.
$0.5685 × 2873.15 = $1633.38+ C's share.

BY PROPORTION.

$10598.42 : $3175.27 :: $6025.19 : $1805.14.
&c. &c. &c.
the number of dollars in his claim.

MENTAL PROCESS.

Deduct assignee's charges, $250, from assets, to find net proceeds, $6025.19. Now, if $10598.42 claims receive $6025.19, *one dollar* will receive $\frac{6025.19}{10598.42}$ of $6025.19, that is, $0.5685; and each man's claim will receive as many times $0.5685 as the number of dollars in his claim.

RULE.—*Divide the net proceeds by the sum of the liabilities and multiply the quotient by the claim of each creditor.*

Or, *As the sum of the claims is to any creditor's claim, so are the net proceeds to his share of them.*

EXAMPLES FOR PRACTICE.

2 If A fails for $25000, and the net proceeds of his assets are $6250, how much does B get, whose claim is $3000?
Ans. $750.

3. G assigned to H for the benefit of his creditors. He owed A $2340, B $1750, C $3260; his effects netted $4924.50. How much can he pay on a dollar, and how much will each creditor get? Ans. 67cts. on $1; A $1567.80; B $1172.50; C $2184.20.

4. Knapp & Wilson's liabilities were $63500, assets $55566, and the expenses of settling $1591. How much did they pay on a dollar, and what was E. Lynch's share, whose claim was $9562.25? Ans. 85cts. on $1; $8127.91¼.

QUESTION.—182. How do we find each creditor's share of the assets of a bankrupt?

CHAPTER XVI.

PERCENTAGE AND ITS PRINCIPAL APPLICATIONS.

PERCENTAGE.

Art. 183. PERCENTAGE treats of the expression and computation of numbers as *hundredths*.

A certain *percentage* or *per cent.* of a quantity is so many *hundredths* of that quantity. Thus, 6 per cent. of a gallon is 6 hundredths of a gallon; 8 per cent. of a person's property is 8 hundredths of it, &c.

NOTE.—*Per cent.* is a contraction of the Latin *per centum*, and signifies *by the hundred.*

Rate per cent. is the number which is compared with one hundred, and expresses the *number* of hundredths.

NOTE.—Percentage is a *quantity*, but rate per cent. is a *number* or *expression* of hundredths.

Art. 184. Since one hundred hundredths of any quantity are equal to the quantity itself,

FIRST, *One hundred per cent. of a quantity is that quantity.* Thus, 100 per cent. of 8 bushels is 8 bushels.

SECONDLY, *Less than one hundred per cent. of a quantity is less than that quantity.* Thus, 5 per cent. of 20lb. is $\frac{5}{100}$ of 20lb., that is, 1lb.; and 1 per cent. of a dollar is 1 cent.

THIRDLY, *Less than one per cent. of a quantity is less than one-hundredth of that quantity.* Thus, *one-half per cent.* of a dollar is $\frac{1}{2}$ of $\frac{1}{100}$ of a dollar, that is, $\frac{1}{2}$ of 1 cent.

FOURTHLY, *More than one hundred per cent. of a quantity is more than that quantity.* Thus, 105 per cent. of a dollar is 1 dollar and $\frac{5}{100}$ of a dollar, that is, $1.05; 200 per cent. of a quantity is *twice* that quantity; 300 per cent. of a quantity, *three times* that quantity, &c.

QUESTIONS.—183. Of what does percentage treat? What is a certain percentage of a quantity? What does per cent. signify? What is rate per cent.? —184. How much is 100 per cent. of a quantity? Less than 100 per cent. of a quantity? Less than 1 per cent. of a quantity? More than 100 per cent. of a quantity?

Art. 185. Rate per cent. is usually written, when possible, as a *decimal.*

ILLUSTRATIONS.

4 per cent.	is written	.04	¼ per cent.	is written	.0025
5 per cent.	"	.05	100 per cent.	"	1.
9½ per cent.	"	.095	106 per cent.	"	1.06
1 per cent.	"	.01	125 per cent.	"	1.25
½ per cent.	"	.00½ or .005	300 per cent.	"	3.
⅛ per cent.	"	.00⅛ or .00125	5⅓ per cent.	"	.05⅓

In business, the sign % is often written instead of the words "per cent." Thus, 8 per cent. is written 8%.

CASE I.

Art. 186. To find any percentage of a number or quantity.

Ex. 1. W. T. Moorhead bought a farm for $7550, and sold it for 8 per cent. more than he gave. How much did he gain?

Ans. $604.

WRITTEN PROCESS.

$7550
.08
———
$604.00

BY PROPORTION.

100 : 8 :: $7550 : $604

Or,

1 : .08 :: $7550 : $604

RULE.—*Multiply the number by the per cent. expressed as a decimal.*

Or, *As* 1 *is to the rate, so is the given number to the required percentage of it.*

EXAMPLES FOR PRACTICE.

2. How much is 1% of $250? Ans. $2.50.
3. How much is 2% of $64? Ans. $1.28.
4. How much is 5% of $257.80? Ans. $12.89.
5. How much is 3¼% of $528.40? Ans. $17.17,3.
6. How much is 4⅓% of $726.63? Ans. $31.48,73.
7. How much is 6¾% of $48.32? Ans. $3.26,16.
8. How many are 25% of 1000 men? Ans. 250 men.

QUESTIONS.—185. How is rate per cent. written? What sign signifies per cent. in business?—186. How do we find a percentage of a quantity?

9. If I buy a house for $800, and sell it at a gain of 100%, how much do I get for it? Ans. $1600.

10. G. Scott bought a share in a mine for $42, and sold it so as to gain 250%. How much did he receive? Ans. $147.

11. S. Barr bought 32bu. of apples, and lost $16\frac{2}{3}\%$ by decay. How many remained for use? Ans. 26bu. $2\frac{2}{3}$pk.

12. A friend invested $3200 for me, and charged me $\frac{3}{8}\%$ for his trouble. How much was his due? Ans. $12.

Case II.

Art. 187. To find what per cent. one quantity is of another.

Ex. 1. Of a school of 70 pupils, 14 study algebra. What per cent. of the whole are they? Ans. 20%.

Written Process.

$14 = \frac{14}{70}$ of $70 = \frac{1}{5}$ of $70 = .20$ of 70

Or,

$\frac{14}{70} \times 100 = \frac{1400}{70} = 20$

By Proportion.

70 : 14 :: 100 : 20

Or,

70 : 14 :: 1 : .20

Mental Process.

One hundredth of 70 $= \frac{70}{100}$; and 14 is as many hundredths of 70 as $\frac{70}{100}$ are contained times in 14. Now, $14 \div \frac{70}{100} = 14 \times \frac{100}{70} = \frac{1400}{70} = 20$ hundredths; that is, 20 per cent.

Rule.—*Multiply that number which is called the per cent. by* 100, *and divide the product by that number of which it is called a per cent.*

Or, *As the quantity of which a percentage is sought is to the quantity which is that percentage, so is* 1 *to the rate required.*

Examples for Practice.

2. What per cent. of $16 is $2? Ans. $12\frac{1}{2}\%$.
3. What per cent. of 5 gallons is 1qt.? Ans. 5%.
4. What per cent. of 3 tons is 50lb.? Ans. $\frac{5}{6}$ of 1 per cent.
5. What per cent. of 6yd. 2qr. is 8 per cent. of 25yd.? Ans. $30\frac{10}{13}$ per cent.
6. What per cent. is given to the miller for toll, if he takes a half-peck from every bushel? Ans. $12\frac{1}{2}\%$.

Question.—187. How do we find what per cent. one number is of another?

7. What per cent. of the weight of U.S. silver coin is the weight of copper in them? Ans. 10 per cent.

Case III.

Art. 188. To find of what quantity a given quantity is a given per cent.

Ex 1. Of what number of pounds are 16lb. five per cent.? Ans. 320lbs.

Written Process.

5 : 100 :: 16lb. : 320lb.; Or, $16 \times \frac{100}{5} = 320$

Or,

.05 : 1 :: 16lb. : 320lb

Mental Process.

If 16lb. are 5 hundredths of a number of pounds, that number must be as many times 16lb. as 5 is contained in 100; that is, 20 times 16lb., equal to 320lb.

Rule.—*Multiply the given number by* 100, *and divide the product by the given rate per cent.*

Or, *As the given rate is to* 1, *so is the given quantity to the required quantity.*

Examples for Practice.

2. Of what is $84 12 per cent? Ans. $700.

3. Of how many trees are 15 trees 15 per cent.? Ans. 100 trees.

4. A hogshead of molasses leaked 10 gallons, which was $6\frac{2}{3}\%$ of its contents. How much remained in the hhd.? Ans. 140 gallons.

5. An army lost $9\frac{1}{2}$ per cent. of its men by having 380 killed in a battle. How large was the army after battle? Ans. 3620 men.

Case IV.

Art. 189. To find the quantity on which percentage is reckoned.

Ex. 1. I paid a teamster $3 for a load of coal, in which price he included 25 per cent. as his charge for delivery. What did the coal cost him? Ans. $2.40.

Question.—188. How do we find of what quantity a given quantity is a given per cent?

WRITTEN PROCESS.

125 per cent. : 100 per cent. :: $3 : $2.40

Or,

1.25 : 1 :: $3 : $2.40

MENTAL PROCESS.

Since $3 includes the cost and 25 per cent. of it beside, $3 is 125 per cent., that is, $\frac{125}{100}$ of the cost. Now, as 125 is to 100, on which it is reckoned, so is $3 to the quantity of which it is reckoned 125 per cent.

Ex. 2. A regiment lost 10 per cent. of its men in a battle, and had 720 men left. How many had it at first?

Ans. 800 men.

WRITTEN PROCESS.

100 — 10 = 90

90 : 100 :: 720 : 800

Or,

.9 : 1 :: 720 : 800

MENTAL PROCESS.

If 10 per cent. are killed, 90 per cent. remain. Now, as 90 is to 100, so are 720 men to the number of which they are 90 per cent.

RULE.—*Divide the given number by* 1 *increased by the rate per cent., or by* 1 *diminished by the rate per cent., as the case requires.*

EXAMPLES FOR PRACTICE.

3. A certain city, of 24000 inhabitants, has increased 20 per cent. in five years. What was its population 5 years ago?

Ans. 20000.

4. A certain political candidate for re-election obtained 9877 votes, being 15 per cent. less than he had at first. How many had he at first? Ans. 11620.

Art. 190. The principal commercial applications of percentage are Taxes, Insurance, Commission, Profit and Loss, Interest, Discount, and Stocks.

ASSESSMENT OF TAXES.

Art. 191. A TAX is a sum of money required from individuals by Government for public purposes.

Taxes are either *direct* or *indirect*.

A *direct* tax is a tax assessed directly upon the property or persons of taxable individuals.

QUESTIONS.—189. How do we find the quantity on which percentage is reckoned?—190. What are the principal commercial applications of percentage?—191. What is a tax? What is a direct tax?

An *indirect* tax is a tax on articles of consumption in their transit from one person to another.

Direct taxes are either *poll-taxes* or *property-taxes*.

A *poll-tax*, sometimes called a *capitation-tax*, is a tax imposed equally upon taxable persons, without regard to the amount of their property.

NOTE.—A *poll*, in law, is a taxable person.

A *property-tax* is a tax imposed at a given rate upon the value of property.

Property is either *real estate* or *personal property*.

Real estate is fixed property, such as *lands*, *houses*, &c.

Personal property includes all kinds of property not fixed, such as *furniture*, *vehicles*, *live stock*, *money*, &c.

An *inventory* is a list of articles of property.

DIRECT TAXES.

Art. 192. To assess a direct property-tax.

Ex. 1. A certain town voted to raise \$8000 by a poll-tax of 25 cents, and a property-tax. The number of taxables was 848, and the property of the town was assessed at \$311520. What was the tax on one dollar? Ans. $2\frac{1}{2}$ cents.

WRITTEN PROCESS.

$848 \times \$0.25 = \$212$

$\$8000 - \$212 = \$7788$

$311520)7788.000(.025$

623040

1557600

1557600

EXPLANATION.

At 25 cents apiece, 848 taxables would pay \$212, leaving \$7788 to be paid by a property-tax. Now, if \$311520 pay \$7788 tax, \$1 will pay $\frac{1}{311520}$ of \$7788, that is, \$0.025.

RULE.—*From the sum to be raised subtract the amount to be raised by poll-tax. Divide the remainder by the assessed value of the property to be taxed; the quotient is the tax on one dollar, or unit of money.*

EXAMPLES FOR PRACTICE.

2. Taxables, 596; poll-tax, 50 cents; amount to be raised, \$1250; property assessed at \$76160. What is the tax on \$1? Ans. $1\frac{1}{4}$ cents.

QUESTIONS.—191. What is an indirect tax? What is a poll-tax? What is a poll? What is a property-tax? What is real estate? What is personal property? What is an inventory?—192. How are direct property-taxes assessed?

3. Taxables, 200; poll-tax, 25 cents; amount to be raised, $3550; property assessed at $700000. What is the tax on $1?
Ans. 5 mills.

4. Taxables, 4500; tax needed, $50000; collectors allowed 5%; 5% allowed for lost taxes; poll-tax, $1. How much should be assessed upon the property of the inhabitants?
Ans. $50901.662+.

Art. 193. To find the amount of a person's tax.

RULE.—*Multiply the assessed value of his property by the tax on one dollar, or unit of money, and to the product add his poll-tax, when there is any.*

EXAMPLES FOR PRACTICE.

1. Poll-tax, 25 cents; tax on a dollar, 2½ cents; how much is Henry Evan's tax, whose property is assessed at $18500, and who pays for 2 polls? Ans. $463.

2. Poll-tax, 50 cents; tax on a dollar, 1¼ cents; how much is Lewis Mann's tax, whose real estate is assessed at $22500, and personal property at $15250, and who pays for 1 poll?
Ans. $472.37½.

3. Poll-tax, 25 cents; tax on a dollar, 5 mills; Edward Hay is a non-resident, but has property in the town assessed at $9875. How much is his tax? Ans. $49.37½.

4. Poll-tax, 5s.; tax on one pound, 2d.; how much is John Bright's tax, whose inventory amounts to £15625 10s. 6d., and who pays for 4 polls? Ans. £131 4s. 3d. 0.2qr.

DUTIES, OR INDIRECT TAXES.

Art. 194. DUTIES are taxes imposed by government upon merchandise in transportation.

Customs are duties imposed upon merchandise imported or exported.

Excise is an inland duty imposed upon articles manufactured, sold, or consumed.

A *tariff* is a list of duties.

Revenue is the income derived from duties and other sources.

A *custom-house* is a house or office where is performed the government business concerning imports, exports, and duties.

QUESTIONS.—193. How do we find the amount of a person's tax?—194. What are duties? What are customs? What is excise? What is a tariff? What is revenue? What is a custom-house?

An *invoice*, or a *manifest*, is a written account of the articles of merchandise transported.

Duties on imported goods are either *specific* or *ad valorem*.

A *specific* duty is a duty imposed upon a definite quantity of an article, without reference to its value.

An *ad valorem** duty is a duty defined as a certain per cent. of the cost of an article.

SPECIFIC DUTIES.

Art. 195. In estimating specific duties, certain deductions, or allowances, are sometimes made for the weight of that which contains the merchandise, or for waste in transportation.

Gross weight is the weight of the goods with the thing which contains them.

Net weight is the weight remaining when the proper deductions from the gross weight have been made.

Breakage is a deduction for loss by breaking.

Leakage is a deduction for waste, allowed on liquors in casks, when taxed by the gallon.

Tare is an allowance for the weight of the thing containing the goods.

Draft, or Tret, is an allowance for refuse matter, or for waste in the weight of goods caused by handling them.

Art. 196. To find the specific duty on a given quantity of merchandise.

Ex. 1. What is the duty on 35 cases of indigo, each weighing 195lb.; tare being 15%; duty 5cts. per lb.? Ans. $290.06¼.

WRITTEN PROCESS.

195lb. × 35 = 6825lb. gross wt.
.85 = 100 per cent. less tare.

34125
54600

5801.25 lb. net wt.
.05 rate of duty.

$290.0625 = total duty.

* The words *ad valorem* are Latin, meaning *according to the value.*

QUESTIONS.—194. What is an invoice or manifest? What is a specific duty? What is an ad valorem duty?—195. What is gross weight? What is net weight? What is breakage? What is leakage? What is tare? What is draft or tret —196. How do we find the specific duty on a given quantity of merchandise?

RULE.—*First make all the required deductions from the given quantity, then multiply the remainder by the duty on a unit of the same.*

EXAMPLES FOR PRACTICE.

2. What is the duty on 100 hogsheads of molasses, each containing, per invoice, 175 gallons; leakage being 2%, and duty being 10 cents per gallon? Ans. $1715.

3. What is the duty on 25 boxes of sugar, each weighing 600lb.; tare being 10%; duty 2 cents per lb.? Ans. $270.

4. What is the duty on 1000 dozen of bottles of porter, allowing a dozen to contain $2\frac{3}{4}$ gallons, and 10% for breakage, the duty being 10 cents per gallon? Ans. $247.50.

AD VALOREM DUTIES.

Art. 197. To find the ad valorem duty on a given quantity of merchandise.

RULE.—*From the quantity stated in the invoice deduct the actual weight of that which contains the goods, and the actual waste or damage of them; then find that percentage of the invoiced value of the remainder which is mentioned in the tariff for that article.*

SELECTIONS FROM THE U.S. TARIFF OF 1857.

AD VALOREM.

30 per cent.	24 per cent.	15 per cent.	8 per cent.
Almonds.	Ale.	Alum.	Camphor.
Brandy.	Carpeting.	Arrow-root.	Currants.
Cut Glass.	Cutlery.	Beeswax.	Dates.
Cigars.	Earthen-ware.	Chocolate.	Figs.
Snuff.	Hemp.	Gunpowder.	Furs.
Wines.	Ink.	Lard.	Lemons.
	Iron.	Lead.	Limes.
4 per cent.	Soap.	Leather	Oranges.
	Sugar.	Linen.	Pineapples.
Chalk.	Wool.	Marble.	Prunes.
Cloves.		Needles.	Raisins.
Cochineal.	19 per cent.	Opium.	Sponges.
Cocoa.		Rice.	Tallow.
Indigo.	Buttons.	Rhubarb.	
Mace.	Cotton lace	Starch.	12 per cent.
Nutmegs.	Hair cloth.	Wheat.	
	Slates.		Steel.
			Tin foil.

QUESTION.—197. How do we find the ad valorem duty on a given quantity of merchandise?

From the foregoing specifications of duties, solve the following

EXAMPLES FOR PRACTICE.

1. What is the duty on 10 sacks of almonds, invoiced at $50 per sack? Ans. $150.

2. What is the duty on 15 casks Port wine, invoiced as containing 45 gallons each, but actually gauging on the average 40 gallons, invoiced price being $2.75 per gallon? Ans. $495.

3. What is the duty on 50 rolls Brussels carpeting, invoiced at an aggregate of $11500, the collector allowing $3500 for damages on shipboard? Ans. $1920.

4. Allen & France imported 1000 drums figs averaging 8½lb. per drum, tare 10%, @ 12cts. per lb.; 50 boxes lemons, invoiced @ $4 per box; 25 boxes oranges @ $3.50 per box; 30 frails dates @ $18 per frail; 10 bbl. currants @ $30 per bbl.; 5 seroons indigo @ $100 per seroon; 5 cases nutmegs @ $87 per case; 20 kegs prunes, invoiced at $12.50 per keg, but 50 per cent. allowed by the collector for damage; 300 boxes Malaga raisins @ $2.50 per box; and 10 sacks cocoa, each weighing 162lb., tare 3%, @ 18 cents per lb. How much duty did Allen & France pay? Ans. $282.35,4+.

5. Clifford & Dewey imported 10 tons bar steel @ $330 per ton; 100 tons bar iron @ $60 per ton; and 20 boxes cutlery, invoiced in the aggregate at $8000. What duty did they pay? Ans. $3756.

6. Bryan & Ogden imported 75 boxes linen, all invoiced at $22500; 5 packages needles, all invoiced at $500; and 100 gross buttons, all invoiced at $2500. What duty did they pay, the collector allowing 20 per cent. on the linen, for damage by wetting? Ans. $3250.

INSURANCE.

Art. 198. INSURANCE is security against pecuniary loss, pledged by one party to another.

The *premium* of insurance is that per cent. of the amount insured, which is paid by the party who receives insurance to the party who gives it.

The *policy* of insurance is the written contract between the insuring and the insured parties.

Insurance has different names, such as *Fire Insurance, Marine Insurance, Health Insurance, Life Insurance,* &c.

QUESTIONS.—198. What is insurance? What is the premium? What is the policy?

Fire Insurance is security against pecuniary loss by fire or lightning.

Marine Insurance is security against pecuniary loss by the perils of navigation.

Health Insurance secures an allowance of money during the sickness of the party insured.

Life Insurance secures a specified sum of money to certain specified survivors, on the death of the party insured.

CASE I.

Art. 199. To compute the premium for a specified time.

Ex. 1. Oliver Bush got his house insured for $1800 for one year, at 1½ per cent. premium. How much premium did he pay? Ans. $27.

WRITTEN PROCESS.

$1800
.015
$27.000

RULE.—*Find the specified per cent. of the amount insured.*

EXAMPLES FOR PRACTICE.

2. J. Miner took out a policy of $1500 for his shop for 9 months at the rate of 3 per cent. per annum. What premium did he pay? Ans. $33.75.

3. Charles Snow had on the ocean a shipment invoiced at $918.75, and took out, at 5 per cent. premium, a policy of insurance for its safe delivery. How much did he pay? and on the ship's being lost, what was the loss of the Insurance Company? Ans. $45.93¾; $872.81¼.

CASE II.

Art. 200. To find the rate per cent. of premium.

Ex. 1. The Hopewell Insurance Co. offered to take a risk of $3500 on A. Catlin's store and goods, for $70, for one year. What was the rate of premium? Ans. 2 per cent.

WRITTEN PROCESS.

3500) 70.00 (.02
7000

MENTAL PROCESS.

Since the premium, $70, is the *product* of $3500 into *rate*, it is plain that rate is obtained by *dividing* the product $70 by *one factor* $3500.

QUESTIONS.—198. What is Fire Insurance? Marine Insurance? Health Insurance? Life Insurance?—199. How do we compute the premium for a specified time?—200. How do we find the rate per cent. of premium?

RULE.—*Divide the premium by the amount insured.*

EXAMPLES FOR PRACTICE.

2. J. Miner's policy of \$1500 for 9 months cost him \$33.75 What was the rate per annum? Ans. 3 per cent.

3. C. Snow's premium for a shipment of \$918.75 was \$45.93¾. What was the rate per cent.? Ans. 5%.

CASE III.

Art. 201. To find what sum must be insured, so as to cover all the expenses of insurance.

Ex. 1. F. Munson's buildings cost him \$7898. If he takes out a policy of insurance, for how much must it be to cover the property, and the policy which cost him \$2, and the premium at 5 per cent.? Ans. \$8315.78,9+.

PROCESS INDICATED.

\$7898 + \$2 = \$7900
\$7900 ÷ .95 = 8315.78,9+

OR BY PROPORTION.

.95 : 1 :: \$7900 : \$8315.78,9+

MENTAL PROCESS.

If Munson pays 5 cents, and in case of loss receives \$1 for it, he really has only the balance, 95 cents. Now, if 95 cents, received in case his buildings are destroyed, require that \$1 be insured, \$7900 received for what the buildings and policy cost him will require the insurance of as many dollars as 95 cents are contained times in \$7900.

RULE.—*Add together the value of the property and the expenses of insuring it, and divide the sum by* 1 *diminished by the rate per cent. of insurance.*

EXAMPLES FOR PRACTICE.

2. Marvin Johnston's factory cost him \$43500. For how much must he insure it to recover its cost and all expenses, the policy costing \$2, and the premium being 4½ per cent.? Ans. \$45551.83,2+.

3. Homer Morris ventured in a ship and cargo \$24800. For how much must he take out a policy, to recover all losses if the vessel is wrecked, the policy costing \$5, and the premium being at 8 per cent.? Ans. \$26961.95,6+.

QUESTION.—201. How do we find the sum to be insured to cover all losses?

COMMISSION AND BROKERAGE.

Art. 202. COMMISSION is an allowance to a person for transacting another's business.

An *agent* or *factor* is a person intrusted with the business of another.

A *broker* is an agent who mediates between the buyer and seller of stocks, notes, bills of exchange, and some other kinds of property. *Brokerage* is the commission of a broker.

Goods sent to an agent are often called a *consignment.*

A *consignor* is a sender or maker of a consignment.

A *consignee* is an agent who receives a consignment.

CASE I.

Art. 203. To find the commission or brokerage on a given sum.

Ex. 1. Daniel Sparks bought for me 82bbl. of pork at $18 per bbl., and charged me 2% commission. How much had I to pay? Ans. $29.52.

PROCESS INDICATED.

$82 \times \$18 = \$1476$

$\$1476 \times .02 = \$29.52$

RULE.—*Find the specified per cent. of the sum on which commission is charged.*

EXAMPLES FOR PRACTICE.

2. F. Farley sold for R. Miller his consignment of 15 bales satinet, realizing $4500. What was Farley's commission at $3\frac{1}{2}$ per cent.? Ans. $157.50.

3. J. Sergeant sold for S. Gano 25doz. hats at an average of $40 per doz. What was Sergeant's commission at 5 per cent.? Ans. $50.

4. Adams & Van Horn consigned 20bbl. coal-oil to B. Pomeroy, who retailed it at $45 per bbl. What was Pomeroy's commission at 4 per cent.? Ans. $36.

5. Z. Holmes & Co. sold for N. Vattell 72 shares of the X. R. R. Company's stock for $35 per share. What was their brokerage at $\frac{1}{8}$ per cent.? Ans. $3.15.

QUESTIONS.—202. What is commission? What is an agent or factor? What is a broker? What is brokerage? What is a consignment? A consignor? A consignee?—203. How do we find the commission or brokerage on given sum?

Case II.

Art. 204. To find the commission or brokerage, when it is included in a given sum

Ex. 1. An agent received $2500 to buy wheat, after deducting his commission at 5%. What sum did he lay out, and how much was his commission?

Ans. Laid out $2380.95,2+; Com. $119.04,7+.

Process Indicated.

1.05 : 1 : : $2500 : $2380.95,2+
$2500 — $2380.95,2+ = $119.04,7+
Proof. $2380.95,2 × .05 = 119.047+
and $2380.95,2 + $119.04,7+ = $2500.

Mental Process.

The agent should charge commission only on the amount he expends. Since his commission is *included* in the $2500, he should not get 5% of *that* sum. Every dollar of the $2500 includes $1 laid out + 5 cents commission. Therefore there will be as many dollars laid out as $1.05 is contained times in $2500; that is, $2380.95,23+. This subtracted from $2500 gives $119.04,7, his commission.

Rule.—*Divide the given sum by* 1 *increased by the per cent. of commission: the quotient is the part invested.*

From the given sum subtract the part invested: the remainder is the commission.

Examples for Practice.

2. Wm. Wolff sent to Trevor & Wise, commission-merchants, $5000 to be expended for cotton, allowing them a commission of 2%. What were their commission, and the sum expended?

Ans. Exp. $4901.96+; Com. $98.04.

3. C. Clarke sends Field & Dunn $659.20 to be expended, if possible, for oats at 20cts. per bushel, after deducting 3% for commission. If they succeed, how many bushels do they purchase? Ans. 3200 bushels.

4. If I send $10500 to Hall & Murray to expend for bacon, after deducting their commission of 5%, how much will be their commission, and how much will remain to be expended as desired? Ans. Com. $500; Exp. $10000.

Question.—204. How do we find the commission when it is included in a given sum?

PROFIT AND LOSS.

Art. 205. In mercantile transactions, gain and loss are usually estimated as a percentage of the sum invested.

Case I.

Art. 206. To find the actual gain or loss, when the cost and rate are known.

Ex. 1. Harlow Childs bought a piece of land for $750, and was at once offered 5% for his bargain. How much did he make by accepting the offer? Ans. $37.50.

Written Process.

$750
.05
$37.50

Rule.—*Find the specified per cent. of the cost.*

Examples for Practice.

2. T. Robertson invested $425 in wheat, and sold out in a short time at 100 per cent. advance. What was his profit?
Ans. $425.

3. Philip Wendell began the year's trade with $984, and, by good fortune, closed it at a profit of 275%. How much did he make? Ans. $2706.

4. H. Fowler gave $40 per share for 15 shares of railroad-stock, and consigned them to Page & Randolph for sale at 10% less than he gave. They charged him ½ per cent. brokerage. What was his whole loss? Ans. $62.70.

5. Harlan Foster bought 475bbl. flour at $5.62½ per bbl., and sold them at a loss of 20%. What was his actual loss?
Ans. $534.37½.

6. A gave $5000 for a city lot, and sold to B at 10% advance; and B sold it to C at 5% advance. What did C give?
Ans. $5775.

7. A gave £1 5s. per bbl. for flour, and sold to B at 2% advance; then B sold to C at 2% loss. What did C give?
Ans. £1 4s. 11d. 3+qr.

Questions.—205. How are gain and loss estimated in trade?—206. How do we find the actual gain or loss when the cost and rate are known?

Case II.

Art. 207. To find the rate per cent. of gain or loss, when the cost and actual gain or loss are known.

Ex. 1. E. Marlin gave \$150 for a horse, and sold him immediately for \$175. What rate of profit did he realize?

Ans. $16\frac{2}{3}$ per cent.

Process Indicated.

175
150
$150)25.00(.16\frac{2}{3}$

By Proportion.

\$150 : \$1 :: \$25 : $.16\frac{2}{3}$

Mental Process.

The gain is \$25. Therefore \$150 gains $\frac{25}{150}$, that is, $\frac{1}{6}$ of itself. But $\frac{25}{150}$ or $\frac{1}{6}$, reduced to hundredths, = $.16\frac{2}{3}$. Or, as \$150 is to \$1, so is \$25 (gained by the \$150) to what is gained by \$1.

Ex. 2. I bought a lot of sugar for \$1600; but it got damaged, and I sold it for \$1200. What per cent. of my money did I lose?

Ans. 25%.

Process Indicated.

\$1600
\$1200
1600)400.00(.25

By Proportion.

\$1600 : \$1 :: 400 : .25

Mental Process.

The loss is \$400. Therefore \$1600 loses $\frac{400}{1600}$, that is, $\frac{1}{4}$ of itself. But $\frac{400}{1600}$ or $\frac{1}{4}$, reduced to hundredths, = .25. Or, as \$1600 is to \$1, so is \$400 (lost by the \$1600) to what is lost by \$1.

Rule.—*Annex ciphers to the actual gain or loss, and divide by the cost.*

Examples for Practice.

3. L. Kilburn bought 25650 feet of lumber, at \$19.20 per thousand, and sold it at \$25 per thousand. What was his rate of profit? Ans. $30\frac{5}{24}$ per cent.

4. If I buy muslin at 8 cents per yd., and sell it at 7 cents per yd., what do I lose per cent.? Ans. $12\frac{1}{2}$ per cent.

5. If I buy coffee at $12\frac{1}{2}$cts. per lb., and sell it at 13cts. per lb., what do I gain per cent.? Ans. 4%.

Question.—207. How do we find the rate per cent. of gain or loss when the cost and actual gain or loss are known?

Case III.

Art. 208. To find that selling price which will realize a certain per cent. gain or loss, when the cost is known.

Ex. 1. A merchant bought a lot of shawls at $16 apiece How must he sell them so as to gain 25 per cent.?

Ans. $20 apiece.

Written Process.

$16 × .25 = $4 gain.
$16 + $4 = $20 price.
Or, $16 × 1.25 = $20

Or, by Proportion.

1 : 1.25 :: $16 : $20

Mental Process.

25% of the cost is $4; which, added to the cost, gives $20 the selling price. Or, if $1 becomes $1.25, $16 will become 16 times $1.25, that is, $20.

Ex. 2. A grocer bought a lot of apples for $15, but had to sell them at 20% loss. What did he get for them?

Ans. $12.

Written Process.

$15 × .20 = $3 loss.
$15 — $3 = $12 price.
Or, 1 — .20 = .80
$15 × .80 = $12

Or, by Proportion.

1 : 1 — .20 :: $15 : $12

Mental Process.

20% of the cost is $3, which, taken from the cost, gives $12 the selling price.

Or, if that which cost $1 sells for $1 — 20 cents, that is, 80 cents, then that which cost $15 will sell for 15 times 80 cents, that is, $12.

Rule.—*Find the specified per cent. of the cost; add it to the cost in case of gain, and subtract it from the cost in case of loss.*

Examples for Practice.

3. If I buy flour at $4.75 per bbl., how must I sell it per bbl. to gain 12%? Ans. $5.32.

4. If I buy calico at 10cts per yd., how must I mark it for sale to realize 25%? Ans. 12½cts.

5. Selling off to close business, how much must I mark linen that cost me 56cts. so as to lose 12½ per cent.? Ans. 49cts.

6. I am willing to lose 10 per cent. on ginghams that cost me 20cts. How must I mark them? Ans. 18cts.

Question.—208. How do we find that price which will realize a certain per cent. gain or loss?

7. R. Elmore buys certain books at 90cts. apiece. How shall he mark them to be able to throw off 10% to his customers, and still gain 10% on his purchase-money? Ans. $1.10.

NOTE.—10% of 90cts. is 9cts.; and 90 + 9 = 99cts. he must get for his book. Now, 99cts. are 90% of what he is to ask, since he is to throw off 10% of *that price*. Hence, .90 : 1 :: 99cts. : $1.10.

8. How shall I mark silks that cost me $1 to be able to allow my customer 5% off, and still make 33%? Ans. $1.40.

9. What must goods which cost $4.50 be marked that the seller may give 10% off, and still make 20%? Ans. $6.

CASE IV.

Art. 209. To find the cost, when the selling price and rate of gain or loss are known.

Ex. 1. If I sell muslin at 12cts. a yd., and thereby gain 20%, what did it cost me? Ans. 10cts.

PROCESS INDICATED.

12cts. ÷ 1.20 = 10cts.

OR, BY PROPORTION.

1.20 : 1 :: 12cts. : 10cts.

MENTAL PROCESS.

If $1\frac{20}{100}$cts. are returned by 1 cent outlay, then 12cts. are returned by as many cents as 1.20 is contained times in 12, that is, by 10cts.

Ex. 2. Sold a lot of wheat for $1872.45, thereby losing 10%: what did I give for the wheat? Ans. $2080.50.

PROCESS INDICATED.

$1872.45 ÷ .90 = $2080.50

OR, BY PROPORTION.

.90 : 1 :: $1872.45 : $2080.50

MENTAL PROCESS.

At 10% loss, $1 returns me 100cts. — 10cts., that is, 90cts. And if 90cts. are returned by $1, then $1872.45 are returned by as many dollars as 90cts. are contained times in $1872.45.

RULE.—*Divide the selling price by* 1 *increased by the gain per cent., or by* 1 *diminished by the loss per cent.*

EXAMPLES FOR PRACTICE.

3. Sold paper at 15cts. per quire, thereby gaining 50%. What did the paper cost me? Ans. 10cts. per quire.

QUESTION.—209. How do we find the cost when the selling price and rate of gain or loss are known?

4. Sold nails at 4½cts. per lb., thereby losing 25%. What did I give? Ans. 6cts. per lb.

5. Sold a lot for $640, thereby realizing 33⅓ per cent. What did I give? Ans. $480.

6. M. Powell sold his cottage for £756 14s., thereby losing 12½ per cent. What did it cost him? Ans. £864 16s.

Case V.

Art. 210. To find the rate of gain or loss at another price, when the selling price and rate of gain or loss are known.

Ex. 1. If I sell wheat at 80cts. a bushel, and thereby gain 25%, what per cent. should I make by selling it at 96cts.?

Ans. 50%.

Process Indicated.	Explanation.
80cts. ÷ 1.25 = 64cts., cost. 96cts. − 64cts. = 32cts., supposed gain. 32cts. ÷ 64cts. = .50 = per cent. that supposed gain is of cost.	We find the cost by Case IV. Then, at 96 cts., the gain is 32cts.; and 32cts. is 50% of the cost, 64cts. by Case II.

Rule.—*Find the cost; then the actual gain at the proposed selling price; then the per cent. which this is of the cost.*

Examples for Practice.

2. If a lot which is sold at $990 realizes 10% gain, what per cent. would it gain if sold at $1050? Ans. 16⅔ per cent.

3. If goods which are sold at $2.60 gain 4%, what would be the per cent. if sold at $2? Ans. 20% loss.

4. If property which is sold at $420 realizes 250%, what would be the per cent. if sold at $660? Ans. 450% gain.

5. If goods which are sold at 21cts. gain 133⅓ per cent., what would be the per cent. if sold at 6cts.?

Ans. 33⅓ per cent. loss.

6. If a share in a bank is sold for $49.75, at ½% loss, what would be the per cent. if sold at $49.62½? Ans. ¾% loss.

7. If a share in a bank is sold for $49.62½, at ¾% loss, what would be the per cent. if sold at $50.12½? Ans. ¼% gain.

Question.—210. How do we find the rate at a supposed selling price, when that at another is known?

INTEREST.

Art. 211. INTEREST is a compensation paid by the borrower to the lender, for the use of money. It is reckoned as a percentage of the sum lent, and pays for its use a certain period of time, as a *month* or a *year*.

The *principal* is the sum lent.

The *amount* is the sum of the principal and interest.

The *rate per cent.* is the sum paid for the use of 100 units of money for a period of time. Thus, 6cts. paid for the use of 100cts., or $6 for the use of $100, *for a year*, is called *six per cent. per annum;* and $1 paid for the use of $100 *for a month* is called *one per cent. a month.*

NOTE.—*Per annum* is Latin, signifying *by the year.*

Legal interest is the rate prescribed by law.

Usury is any rate of interest greater than the legal rate.

TABLE

OF LEGAL RATES OF INTEREST PER ANNUM FOR THE YEAR 1859.

	When parties fix no rate.	Parties allowed by agreement.		When parties fix no rate.	Parties allowed by agreement
England	5 per cent.		Georgia	7 per cent.	
France	5 " "		South Carolina	7 " "	
Louisiana	5 " "	8 per cent.	Texas	8 " "	12 per ct.
Florida	6 " "	8 " "	Alabama	8 " "	
Illinois	6 " "	10 " "			
Arkansas	6 " "	10 " "	Mississippi	8 for money, 6 other contracts.	
Iowa	6 " "	10 " "			
Michigan	7 " "	10 " "	California	10 per cent.	Any per ct.
Wisconsin	7 " "	12 " "	Oregon	10 " "	" "
Washington Ter.	No limit.		Minnesota	12 " "	" "
Utah Territory	" "		New Mexico	12 " "	
Kansas Territory	" "		Other States U.S.	6 " "	
New York	7 per cent.				

Interest, in reference to the mode of computing it, is *simple* or *compound.*

QUESTIONS.—211. What is interest? How is it reckoned? What is prinpal? Amount? Rate per cent.? Per annum? Legal Interest? Usury? How is interest classified?

SIMPLE INTEREST.

Art. 212. SIMPLE INTEREST is interest reckoned on the principal only.

In Simple Interest, that interest which is not paid, when due, is not reckoned as a new loan or principal.

CASE I.

Art. 213. To compute interest for whole periods of time.

Ex. 1. What is the interest of \$137.50 for 4 months, at 1½ per cent. a month? Ans. \$8.25.

WRITTEN PROCESS.

\$137.50 = Principal.
.015 = Rate.
\$2.06250 = Int. for 1mo.
4
\$8.2500 = Int. for 4mo.
137.50
\$145.75 = Am't.

MENTAL PROCESS.

If \$1 pays \$0.015 for 1 month, \$137.50 will pay 137.50 times \$0.015, that is, \$2.0625 for 1 month; and for 4 months 4 times \$2.0625, that is, \$8.25.

Ex. 2. What is the interest of \$650 for 3 years, at 7% a year? Ans. \$136.50.

WRITTEN PROCESS.

\$650 = Principal.
.07 = Rate.
\$45.50 = Int. for 1yr.
3 = No. of yrs.
\$136.50 = Int. for 3yrs.

RULE.—For one period. *Multiply the principal by the rate per cent., expressed decimally.*

For more than one period. *Multiply the interest of one period by the number of periods.*

QUESTIONS.—212. What is simple interest?—213. How do we compute interest for whole periods of time?

Examples for Practice.

3. What are the interest and amount of \$210 for 3 months, at 2% a month? Ans. Int. \$12.60; Am't \$222.60.

4. What are the interest and amount of \$525 for 6 months, at $1\frac{1}{4}$ per cent. a month? Ans. Int. \$39.37½; Am't \$564.37½.

5. What are the interest and amount of \$362.50 for 5yr., at 6% per annum? Ans. Int. \$108.75; Am't \$471.25.

6. What are the interest and amount of \$6550 for 12yrs., at $8\frac{1}{3}$ per cent. per annum? Ans. Int. \$6550; Am't \$13100.

7. What are the interest and amount of \$4725 for 20yrs., at 5% per annum? Ans. Int. \$4725; Am't \$9450.

8. What are the interest and amount of \$1482.25 for 16yrs., at $6\frac{1}{4}$ per cent. per annum?

Ans. Int. \$1482.25; Am't \$2964.50.

9. What are the interest and amount of \$287.50 for 10yrs., at 10% per annum? Ans. Int. \$287.50; Am't \$575.

10. What are the interest and amount of one dime for 100yrs., at 5 per cent. per annum? Ans. Int. \$0.50; Am't \$0.60.

Case II.

Art. 214. To compute interest for any part of a period of time.

Ex. 1. What is the interest of \$125 for 18 days, at $1\frac{1}{4}$ per cent. a month? Ans. \$0.93¾.

Written Process.

\$125 = Principal.
.01¼ = Rate.

125
31.25

5)\$1.5625 = Int. for 1mo.

\$0.3125 = Int. for ⅕mo. or 6 days.
3

\$0.9375 = Int. for ⅗mo. or 18 days.

Mental Process.

Calling 30 days 1 month, 18 days are $\frac{18}{30}$ or ⅗ of 1 month. If 1 month brings \$1.56¼ interest, ⅗ of a month will bring ⅗ of \$1.56¼.

Ex. 2. What is the interest of \$75 for 123 days, at 6% per annum? Ans. \$1.51,6+.

Process Indicated.	Mental Process
$\$75 \times .06 = \$4.50$	The *true* interest for 123 days is $\frac{123}{360}$ of that for a year.
$\$4.50 \div 365 = \$0.012328+ =$ Int. for 1 day.	
$\$0.012328 \times 123 = \$1.51,6+ =$ Int. for 123 da.	

Rule.—*Find the interest for one period, and take that part of it which the given time is of one period.*

Note.—Reckon 30 days a month, if one month is the period; 12 months a year, if 1 year is the period, and the part is expressed in months; or, more accurately, 365 days a year, if 1 year is the period, and the part is expressed in days.

Examples for Practice.

3. What are the interest and amount of \$130 for 21 days, at $1\frac{1}{2}$ per cent. a month? Ans. Int. \$1.36,5; Am't \$131.36,5.

4. What are the interest and amount of \$352.50 for 12 days, at $2\frac{1}{2}$ per cent. a month?

Ans. Int. \$3.52,5; Am't \$356.02,5.

5. What are the interest and amount of \$7650 for 191 days, at 8% per annum? Ans. Int. \$320.25,2; Am't \$7970.25,2.

6. What are the interest and amount of \$6845.75 for 301 days, at 10% per annum?

Ans. Int. \$564.53,99+; Am't \$7410.28,99+.

7. What are the interest and amount of \$2320.80 from March 1 to Nov. 1, at 7% per annum? Note.—See table, Art. 84, note 4.

Ans. Int. \$109.04,5+; Am't \$2429.84,5+.

8. What are the interest and amount of \$118.25 from Apr. 8 to Dec. 15, at 5 per cent. per annum?

Ans. Int. \$4.06,5+; Am't \$122.31,5+.

9. What are the interest and amount of \$92.37½ from Jan. 5, 1860, to Oct. 19, 1860, at 8% per annum?

Ans. Int. \$5.83,1+; Am't \$98.20,6+.

10. What are the interest and amount of \$288.12½ from Feb. 11, 1856, to July 23, 1856, at 9% per annum?

Ans. Int. \$11.58,0+; Am't \$299.70,5.

11. What are the interest and amount of \$427.75 for 7 months, at $4\frac{1}{2}$ per cent. per annum?

Ans. Int. \$11.22,8+; Am't \$438.97,8+.

Question.—214. How do we compute interest for any part of a period of time?

12. What are the interest and amount of $858.20 for 5 months, at 15% per annum?

Ans. Int. $53.63¾; Am't $911.83¾.

Case III.

Art. 215. To compute interest for any time, at any rate.

First Method. By aliquot parts, when the time is expressed in years and parts of a year.

Ex. 1. What is the interest of $425 for 2yr. 5mo. 17da., at 8% per annum? Ans. $83.77,2+.

First Written Process.

	$425	= Prin.
	.08	= Rate.
	$34.00	= Int. 1yr.
	2	= No. of yrs.
	$68	= Int. 2yr.
⅓ of int. 1yr. =	11.3333+	= Int. 4mo.
¼ int. 4mo. =	2.8333+	= Int. 1mo.
½ int. 1mo. =	1.4166+	= Int. 15da.
⅕ int. 1mo. =	.1888+	= Int. 2da.
	$83.77,2+	= Int. 2yr. 5mo. 17da.

Second Written Process.

$425
.08

12	$34.00 × 2	= $68.00	= Int. 2yr.
30	2.8333 × 5	= 14.1666	= Int. 5mo.
	.09444 × 17	= 1.6054	= Int. 17da.

Total interest, $83.77,2+.

Rule.—For one year. *Multiply the principal by the rate per cent., expressed decimally.*

For two or more years. *Multiply the interest of one year by the number of years.*

For months. *Take such a part of the interest of one year as the given number of months is of twelve months.*

Questions.—215. What is the first method of computing interest for any time, at any rate? Why is it inaccurate?

For days. *Take such a part of the interest of one month as the given number of days is of thirty days.*

For the whole time. *Add together the results thus obtained.*

NOTE 1.—Since there are 365 days in a year, 31 days in some months, 30 days in others, and February has sometimes 28 days, at other times 29 days, the method of reckoning 30 days a month, and 12 times 30 days, or 360 days, a year, is inaccurate. It is therefore best not to express parts of a year in months, but to reckon the *days* in that part of the year ending at payment, and then take so many 365ths of one year's interest: as in Case II.

NOTE 2.—Some prefer to find first the interest of $1 or £1 by the above method, and then multiply it by the principal.

EXAMPLES FOR PRACTICE.

2. What are the interest and amount of $768.20, for 1yr. 6mo. 15da. at 7% per annum?

Ans. Int. $82.90,1+; Am't $851.10,1+.

3. What are the interest and amount of $190.50 for 3yr. 4mo. 10da. at 5% per annum?

Ans. Int. $32.01,4+; Am't $222.51,4+.

4. What are the interest and amount of $264.10 for 4yr. 9mo. 20da. at 9% per annum?

Ans. Int. $114.22,3+; Am't $378.32,3+.

5. What are the interest and amount of $0.75 for 5yr. 8mo. 25da. at 10% per annum?

Ans. Int. $0.43,02+; Am't $1.18,02+.

6. What are the interest and amount of $125 from March 1, 1856, to Nov. 3, 1859, at 6% per annum?

Ans. Int. $27.54,1+; Am't $152.54,1+.

7. What are the interest and amount of $1242.80 from July 5, 1859, to Jan. 31, 1860, at 4% per annum?

Ans. Int. $28.44,6+; Am't $1271.24,6+.

8. What are the interest and amount of $15000, from Aug. 11, 1857, to Oct. 13, 1859, at 3% per annum?

Ans. Int. $977.50; Am't $15977.50.

9 What are the interest and amount of $250000 for 6 months at 6% per annum? Ans. Int. $7500; Am't $257500.

10. What are the interest and amount of $1, for 1860yr. at 6% per annum? Ans. Int. $111.60; Am't $112.60.

SECOND METHOD.—By months, when the time is expressed in years and parts of a year.

Ex. 1. What is the interest of $425 for 2yr. 5mo. 17da. at 8%? Ans. $83.77,2+

WRITTEN PROCESS.

$425 = Prin.
.08

12) $34.00 = Int. 1yr.
2.8333+ = Int. 1mo.
$29\frac{17}{30}$ = Whole time in months.
16055 = prod. by $\frac{17}{30}$.
254999
566666
$83.77,2+ = Ans.

EXPLANATION.

Here 2yr. 5mo. 17da. are reduced to months and used as a multiplier to 1 month's interest.

RULE.—*Find the interest for one year, by multiplying by the rate.*

Divide the interest of one year by 12*: the quotient is the interest for one month.*

Multiply the interest of one month by the whole time expressed in months and parts of a month.

EXAMPLES FOR PRACTICE.

2. What are the interest and amount of $1130 for 1yr. 7mo. 21da. at 7% per annum?

Ans. Int. $129.85,5+; Am't $1259.85,5+.

3. What are the interest and amount of 37.37\frac{1}{2}$ for 4yr. 11mo. 15da. at 6% per annum?

Ans. Int. $11.11,9+; Am't $48.49,4+

4. What are the interest and amount of $91.25 from Jan. 1, 1855, to July 1, 1859, at 5%?

Ans. Int. $20.53,1$\frac{1}{4}$; Am't $111.78,1$\frac{1}{4}$.

5. What are the interest and amount of 253.62\frac{1}{2}$ from Oct. 3, 1845, to Aug. 5, 1860, at 4%?

Ans. Int. $150.54+; Am't $404.16,5+.

6. What are the interest and amount of $234.56 from May 18, 1852, to Nov. 25, 1859, at 9%?

Ans. Int. $158.73+; Am't $393.29+.

SPECIAL METHOD for six per cent. in U.S. money.

Ex. 1. What is the interest of $144 for 4yr. 6mo. 19da. at 6%? Ans. $39.336.

QUESTION.—What is the second method of computing interest for any time at any rate?

PROCESS INDICATED.—The interest of $1 for 12mo. is 6cts.; that is, ½ as many cents as months. 4yr. 6mo. = 54mo. One-half of 54 = 27, no. of cts. int. of $1 for 54mo. The interest of $1 for 30da. is 5 mills; that is, ⅙ as many mills as days. 19 ÷ 6 = 3⅙, no. of mills int. of $1 for 19da. 27cts.+3⅙ mills = $0.273⅙, int. of $1 for whole time. 144 × $0.273⅙ = $39.336, int. of $144 for whole time.

RULE.—*Call half of the number of months cents, and one-sixth of the number of days mills, and multiply the sum of these results by the principal.*

NOTE.—Rules like this may be made for 4 per cent. and 3 per cent. Thus, for 4 per cent., *call ⅓ of the number of months cents, and ⅓ of the number of days mills, and multiply their sum by the principal.* Again, for 3 per cent., *call ¼ of the number of months cents, and* $\frac{1}{12}$ *of the number of days mills, and multiply their sum by the principal.*

EXAMPLES FOR PRACTICE.

2. What are the interest and amount of $500 for 2yr. 4mo. 12da. at 6%? Ans. Int. $71; Am't $571.

3. What are the interest and amount of $300 for 3yr. 6mo. 18da. at 6%? Ans. Int. $63.90; Am't $363.90.

4. What are the interest and amount of $225 for 5yr. 8mo. 24da. at 6%? Ans. Int. $77.40; Am't $302.40.

5. What are the interest and amount of $845.25 for 1yr. 10mo. 6da. at 6%?
Ans. Int. $93.82,2+; Am't $939.07,2+.

6. What are the interest and amount of $468.75 from March 15, 1855, to Sept. 4, 1859, at 6%?
Ans. Int. $125.70,3+; Am't $594.45,3+.

7. What are the interest and amount of $197.42 from June 11, 1848, to Dec. 2, 1856, at 6%?
Ans. Int. $100.38,8+; Am't $297.80,8+.

8. What are the interest and amount of $16485 from Feb. 29, 1856, to Jan. 2, 1860, at 6%?
Ans. Int. $3799.79,25 ; Am't $20284.79,25 .

9. What are the interest and amount of $25000 from Jan. 1, to July 1, at 6%? Ans. Int. $750; Am't $25750.

THIRD METHOD.

EX. 1. What is the interest of $308 for 1yr. 5mo. at 11%?
Ans. $47.99,6+.

QUESTIONS.—What is the special method for six per cent.? What other rules might be made?

WRITTEN PROCESS.

$308 = Prin.
.085 = ½ no. of months as cts.

1540
2464

6)26.180 = int. at 6%.

$4.36333 = int. at 1%.
11

$47.99663 = int. at 11%.

MENTAL PROCESS.

Since 11% is ⅙ of 6%, we take ⅙ of the interest at 6% to find that at 11%. If the rate had been 5%, we should have taken ⅚ of the int. at 6%; if 3%, ½, &c.

RULE.—*Find the interest at six per cent, and take as many sixths of this interest as the given rate is of six per cent.*

NOTE 1.—Other rules might be made on the notes of the last rule. Thus, find the interest at 4 per cent., and take as many fourths of this interest as the given rate is of 4 per cent. Again, find the interest at 3 per cent., and take as many thirds of this interest as the given rate is of 3 per cent.

NOTE 2.—Simple inspection of a question will often decide which of the preceding methods of computing interest is preferable for the conditions of that question. The learner should acquaint himself with *all* the methods, and, in any one case, exercise his judgment to select the shortest and most accurate.

EXAMPLES FOR PRACTICE.

2. What are the interest and amount of $155 for 1yr. 6mo. at 7%? Ans. Int. $16.27,5; Am't $171.27,5.

3. What are the interest and amount of $87.87½ for 2yr. 9mo. 6da. at 5%? Ans. Int. $12.15,6+; Am't $100.03,1+.

4. What are the interest and amount of $265.25 for 4yr. 4mo. 4da. at 8%? Ans. Int. $92.18,9+; Am't $357.43,9+.

5. What are the interest and amount of $712.12 for 6yr. 1mo. 3da. at 4%? Ans. Int. $173.51,9+; Am't $885.63,9+.

6. What are the interest and amount of $1428.30 for 9mo. 2da. at 9%? Ans. Int. $97.12,4+; Am't $1525.42,4+.

7. What are the interest and amount of $1800 from April 25, 1856, to April 17, 1859, at 3%?
Ans. Int. $160.80; Am't $1960.80.

8. What are the interest and amount of $4325 from Sept. 5, 1857, to Dec. 25, 1860, at 12%?
Ans. Int. $1715.58+; Am't $6040.58+.

QUESTIONS.—What is the third method? What other methods might be used

9. What are the interest and amount of $6000 from July 1 to Jan 1, at 10%? Ans. Int. $300; Am't $6300.

10. What are the interest and amount of $16666.66,6⅔ from Jan. 1 to April 1, at 7%?

Ans. Int. $291.66⅔; Am't $16958.33⅓.

CASE IV.

Art. 216. To compute interest in English Money.

Ex. 1. What is the interest of £10 12s. 6d. for 1yr. 6mo. at 5%? Ans. 15s. 11d. 1qr.

FIRST METHOD.

£	s.	d.	
10	12	6	= Prin.
		5	= Rate.
100) 53	2	6	
2) 0	10s.	7d.	2qr. = int. 1yr.
0	5	3	3 = int. 6mo.
15	11	1	= Ans.

SECOND METHOD.

12 | 6

20 | 12.5

£10.625 = Prin.

.05 = Rate.

2) .53125 = int. 1yr.

.265625 = int. 6mo.

£0.796875 = whole int

20

s.15.937500

12

d.11.2500

4

qr.1.00

RULE.—*Multiply the principal by the rate as a whole number, and divide the product by* 100. *The quotient is the interest for one year. For other portions of time, proceed by aliquot parts, as in U.S. money.*

Or, *Reduce the shillings, pence, and farthings to the decimal of a pound, and annex this decimal to the pounds. With this result proceed as in U.S. money, and reduce the decimal part of the result to the denominations of English Money.*

' QUESTION.—216. How do we compute interest in English Money?

Examples for Practice.

2. What are the interest and amount of £125 15s. for 3yr. 9mo. at 5%?

Ans. Int. £23 11s. 6d. 3qr.; Am't £149 6s. 6d. 3qr.

3. What are the interest and amount of £62 12s. 6d. for 5yr. 8mo. 15da. at 4%?

Ans. Int. £14 5s. 11d. 3.4qr.; Am't £76 18s. 5d. 3.4qr.

4. What are the interest and amount of £865 5s. 4d. for 2yr. 3mo. 5da. at 3%?

Ans. Int. £58 15s. 3d. 3qr.+; Am't £924 0s. 7d. 3qr.+.

5. What are the interest and amount of £644 18s. for 7 months at 5%?

Ans. Int. £18 16s. 2d. 0.8qr.; Am't £663 14s. 2d. 0.8qr.

Miscellaneous Exercises in Simple Interest.

1. What is the difference of interest of $3500 from Jan. 3 to Dec. 15, as computed by *days* and by *months*, at 6%?

Ans. Days favor Debtor $0.43,1+.

2. From March 3 to Dec. 15?

Ans. Months favor Dr. $0.62,3+.

3. From March 3 to June 15?

Ans. Months favor Dr. $0.33,5+.

4. From Jan. 3 to April 15?

Ans. Days favor Dr. $0.81,5+.

5. From July 3 to Nov. 15?

Ans. Months favor Dr.$0.67,1+.

6. What is the difference of interest of $800 from Jan. 6, 1858, to Dec. 12, 1861, as computed by *months*, and by years, at 7%? Ans. 0. Both $220.26⅔.

7. Which method of computing will, with the least labor, show that $55.72,9+ is the interest of $250 for 4yr. 5mo. 15da. at 5%?

8. Which, that $107.45 is the interest of $300 for 5yr. 11mo 19da. at 6%?

9. Which written process will show, with the fewest figures, that $7.87½ is the interest of $75 for 1yr. 9mo. at 6%?

10. Which, that $325 is the interest of $1000 for 3yr. 3mo. at 10 per cent.?

11. What are the interest and amount of $33⅓ from Jan. 15 to July 21, at 9%? Ans. Int. $1.55; Am't $34.88⅓.

12. What are the interest and amount of $98¼ from Mar. 17 to Oct. 24, at 7%? Ans. Int. $4.14,1+; Am't $102.28,3+

PROMISSORY NOTES.

Art. 217. A PROMISSORY NOTE, or note of hand, is a written promise of payment, given by the borrower to the lender.

The promise of the note is usually either to pay *on demand*, or at a specified time after the date of the note.

When the time of payment is specified in the note, it is customary, in some states, to allow three additional days, called *days of grace*.

A note is said to *mature* on the day on which it is due.

The promise may be to pay the lender; or another person on the lender's order; or to the order of any other person named in the note; or the bearer.

A *negotiable note* is a note that can be sold or transferred.

A *joint note* is a note signed by two or more persons, who are unitedly held for its payment.

A *joint and several* note is a note signed by two or more persons, who are unitedly and individually held for its payment.

The *face* of a note is the sum promised in the note. This sum should always be written *in words* in the text of the note.

The *maker* or *drawer* of a note is the person who signs it.

The *holder* or *payee* is the person to whom the note is to be paid.

An *indorser* is a person who writes his name upon a note or obligation, usually upon its back, thereby becoming responsible for its payment.

A *partial payment* is a payment of part of a note or other legal obligation. If stated on the back of the instrument in writing by the holder, it is called an *indorsement*.

CASE V.

PARTIAL PAYMENTS.

Art. 218. To compute interest when partial payments have been made.

MERCHANTS' RULE.

If partial payments are made upon an obligation, and final settlement is made within a year after the obligation is due, settlement is usually made by the following

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QUESTIONS.—217. What is a promissory note? What promise is made? What are days of grace? When does a note mature? To whom is the promise directed? What is a negotiable note? What is a joint note? A joint and several note? The face of a note? The maker of a note? The holder of a note? An indorser? What is a partial payment? When is it called an indorsement?

RULE.—*Find the amount of the principal at the time of settlement. Then find the amount of each payment from the time it was made until settlement, and subtract the sum of the amounts of the payments from the amount of the principal.*

NOTE.—By this rule we proceed as though a partial payment were a loan from the debtor to the creditor, since it is money paid before settlement. The creditor is thereby prevented from having interest on the interest paid him.

(1.) EXAMPLES FOR PRACTICE.

$2400\frac{00}{100}$. PITTSBURGH, April 1, 1858.

Thirty days after date, I promise to pay, to the order of Fletcher & Phillips, twenty-four hundred dollars, without defalcation, value received. C. W. HENRY.

PAYMENTS.—July 1, 1858, $500. Sept. 1, 1858, $800. Oct. 1, 1858, $500.

How much was due at settlement, Jan. 1, 1859, int. at 6%?

Ans. (By 365ths) $657.13,9.

COMPUTATION.

Principal - - - - - - - - -			$2400
Int. (beginning 33da. after date) from May 3 to Jan. 1 (243da.) - - - - - - - - - -			95.86,8
Am't. of Prin. at settlement - - - - - -			$2495.86,8
First payment - - - - - -	$500		
Int. fr. July 1 to Jan. 1 (184da.) - -	15.123		
Second payment - - - - - -	$800		
Int. fr. Sept. 1 to Jan. 1 (122da.) -	16.044		
Third payment - - - - - -	$500		
Int. fr. Oct. 1 to Jan. 1 (92da.) - -	7.56,2		1838.72,9
Balance due Jan. 1, 1859 - - -			$ 657.13,9

(2.)

$475\frac{50}{100}$. BUFFALO, June 15, 1857.

On demand, for value received, we jointly promise to pay Horace Freeman, or order, four hundred and seventy-five dollars and fifty cents, with interest from date. J. DILWYN.
M. MATSON.

PAYMENTS.—Sept. 15, 1857, $75; Dec. 15, 1857, $75; Mar. 15, 1858, $75.

What was due at settlement, June 15, 1858, int. at 7%?

Ans. (*By calendar months*) $275.91.

QUESTION.—218. What is the merchants' rule for partial payments?

(3.)

$1525\frac{25}{100}$. NEW ORLEANS, Nov. 9, 1857.

For value received, we jointly and severally promise to pay John S. Jones, or order, one thousand five hundred and twenty-five dollars and twenty-five cents, on demand, with interest at five per cent.

B. T. FORTUNE.
J. LEFEVRE.

INDORSEMENTS.—Feb. 23, 1858, received three hundred seventy-five dollars on the within note. Apr. 6, 1858, received two hundred fifty dollars. June 17, 1858, received four hundred twenty-five dollars. Aug. 26, 1858, received two hundred fifty dollars.

How much was due at settlement, Oct. 20, 1858?

Ans. $269.15.

RULE OF THE UNITED STATES COURTS.

Settlements are made by the courts of the United States, and of several individual States, by the following

RULE.—"*When partial payments have been made, apply the payment, in the first place, to the discharge of the interest then due.*

"*If the payment exceeds the interest, the surplus goes toward discharging the principal; and the subsequent interest is to be computed on the balance of the principal remaining due.*

"*If the payment be less than the interest, the surplus of interest must not be taken to augment the principal; but interest continues on the former principal, until the period when the payments, taken together, exceed the interest due, and then the surplus is to be applied toward discharging the principal; and interest is to be computed on the balance, as aforesaid.*"—KENT, Chief Justice.

NOTE.—By this rule the creditor can obtain interest on that interest which is paid him before the end of a year, since he can lend it as soon as received.

EXAMPLES FOR PRACTICE.

(1.)

$3500\frac{00}{100}$. COLUMBUS, O., Nov. 20, 1856.

For value received, I promise to pay W. V. Palmer, or order, thirty-five hundred dollars, on demand, with interest from date at six per cent.

TITUS H. LEE.

QUESTION.—What is the rule of the United States Courts?

On this note the indorsements were, in date and amount, as follows:—May 20, 1857, $100; Nov. 20, 1857, $500; May 20, 1858, $50; Nov. 20, 1858, $950.

How much is due at settlement, May 20, 1859?

Ans. $2365.49,8.

COMPUTATION.

Principal		$3500
Int. fr. Nov. 20, 1856, to May 20, 1857 (6mo.) is $105, or more than the payment; hence,		
Int. of $3500 from Nov. 20, 1856, to Nov. 20, 1857		$ 210
Amount		$3710
First payment	$100	
Second payment	$500	$ 600
Balance, new principal		$3110
Int. of $3110 from Nov. 20, 1857, to May 20, 1858, is $93.30, or more than the payment; hence,		
Int. of $3110 fr. Nov. 20, 1857, to Nov. 20, 1858		$ 186.60
Amount, Nov. 20, 1858		$3296.60
Third payment	$ 50	
Fourth payment	$950	$1000
Balance, new principal		$2296.60
Int. from Nov. 20, 1858 (6mo.)		$ 68.898
Bal. due at settlement, May 20, 1859		$2365.49,8

(2.) What is due on Ex. 1 of the merchants' rule, when computed by the rule of the U.S. courts?

Ans. (By 365ths) $658.32+.

(3.) What is due at settlement June 10, 1859, on a note of $500, dated July 9, 1857, bearing interest at 6% from date, payments having been made as follows:—Oct. 9, 1857, $25; March 2, 1858, $10; July 7, 1858, $30; Jan. 7, 1859, $250?

Ans. (*By calendar months*) $233.78,6.

(4.) What are the dues on Ex. (2) and (3) of the merchants' rule, when computed by the rule of the U.S. courts?

Ans. (2) $276.70+. (3) $270.24,9+ (*By days.*)

CONNECTICUT RULE.

The Supreme Court of the State of Connecticut has adopted the following

RULE.—"*Compute the interest to the time of the first payment; if that be one year or more from the time the interest commenced, add it to the principal, and deduct the payment from the sum total. If there be after-payments made, compute the interest on the balance due to the next payment, and then deduct the payment as above; and in like manner from one payment to another, till all the payments are absorbed: provided the time between one payment and another be one year or more.*

"*But if any payment be made before one year's interest hath accrued, then compute the interest on the principal sum due on the obligation for one year,* (see note,) *add it to the principal, and compute the interest on the sum paid from the time it was paid up to the end of the year; add it to the sum paid, and deduct that sum from the principal and interest added as above.*

"*If any payment be made of a less sum than the interest arisen at the time of such payment, no interest is to be computed, but only on the principal sum for any period.*"

NOTE.—"*If a year does not extend beyond the time of payment; but if it does, then find the amount of the principal remaining unpaid up to the time of settlement, likewise the amount of the payment or payments from the time they were paid to the time of settlement, and deduct the sum of these several amounts from the amount of the principal.*"—Kirby's Reports.

REMARK.—This resembles the merchants' rule, in allowing interest on payments made before the expiration of a year. But the Connecticut rule does not allow interest on payments less than the interest then due. Under this rule, therefore, the lender can, by re-loaning, obtain interest on those payments of interest which are less than the quantity due, if paid before the end of a year.

Ex. 1.

$562\frac{50}{100}$. MIDDLETOWN, August 1, 1854.

For value received, I promise to pay N. Woodard, or order, on demand, five hundred sixty-two dollars and fifty cents, with interest. B. M. RUSSELL.

INDORSEMENTS.—Sept. 1, 1855, received $40; Nov. 1, 1855, received $25; Jan. 1, 1856, received $5; Dec. 1, 1856, received $100; March 16, 1857, received $15; Oct. 25, 1857, received $150; Dec. 25, 1857, received $100.

What is due March 1, 1858? Ans. $235.55,3

COMPUTATION.

Principal			$562.50
Int. from Aug. 1, 1854, to Sept. 1, 1855 (13 months)			36.5625
Am't due Sept. 1, 1855			599.0625
First payment, made Sept. 1, 1855			40
Balance due Sept. 1, 1855			559.0625
Int. from Sept. 1, 1855, to Sept. 1, 1856 (1 year)			33.5438
Amount of principal Sept. 1, 1856			592.606
Second payment		$25.00	
Int. on $25 from Nov. 1, 1855, to Sept. 1, 1856 (10 mo.)		1.25	
Third payment (less than the int. due at Jan. 1, 1856)		5.00	31.25
Balance due Sept. 1, 1856			561.356
Int. from Sept. 1, 1856, to Sept. 1, 1857			33.681
Amount of principal sum Sept. 1, 1857			595.037
Fourth payment		$100	
Int. from Dec. 1, 1856, to Sept. 1, 1857 (9mo.)		4.50	
Fifth payment (less than int. due at March 16, 1857)		15	119.50
Balance due Sept. 1, 1857			475.537
Int. to settlement March 1, 1858 (6mos.)			14.266
Amount of last principal at settlement			489.803
Sixth payment		$150.	
Int. from Oct. 25, 1857 to March 1, 1858 (4⅓mos.)		3.15	
Seventh and last payment		100.	
Int. from Dec. 25, 1857, to March 1, 1858 (2⅓mos.)		1.10	254.25
Balance due at settlement			$235.553

CASE VI.

Art. 219. To find the rate per cent., when the principal, interest, and time are known.

Ex. 1. At what rate per cent. will $800 pay $140 interest in 2½ years? Ans. 7%.

WRITTEN PROCESS.

800 Prin.
025 Int. of $1 at 1% for 2½yr.

$20.000 Int. of $800 at 1% for time.
$140 ÷ $20 = 7 Ans.

$800 as $20 are contained times in $140; that is, 7%.

MENTAL PROCESS.

If $1 at 1% for 2½yr pay 2½cts., $800 will pay 800 times as much; that is, $20. If $20 are 1% of $800 for the time, then $140 are as many per cent. of

RULE.—*Divide the given interest by the interest of the principal at one per cent. for the given time.*

EXAMPLES FOR PRACTICE.

2. At what rate will \$700 pay \$63 interest in $1\frac{1}{2}$ yr.?
Ans. 6% per annum.

3. At what rate will \$922 pay \$53.78$\frac{1}{2}$ interest in 14 months?
Ans. 5% per annum.

4. At what rate per annum will any principal double itself by simple interest in 5, 6, 7, 8, 9, 10, 12, 16, and 20 years?
Ans. 20, $16\frac{2}{3}$, $14\frac{2}{7}$, $12\frac{1}{2}$, $11\frac{1}{9}$, 10, $8\frac{1}{3}$, $6\frac{1}{4}$, and 5%.

CASE VII.

Art. 220. To find the time, when the principal, interest, and rate per cent. are known.

Ex. 1. How long will it take \$150 to gain \$6, at $1\frac{1}{4}$ per cent. a month? Ans. 3mo. 6da.

WRITTEN PROCESS.

\$150 Prin.
.0125 Rate.

\$1.8750 Int. 1 mo.

\$1.875) \$6.0000 (3.2
5625 30
3750 60
3750

MENTAL PROCESS.

If \$1.875 is the interest of the principal for 1 month, \$6 is the interest for as many months as \$1.875 is contained times in \$6; that is, $3\frac{2}{10}$ months, or 3 months 6 days.

RULE.—*Divide the given interest by the interest of the principal for one period of time at the given rate.*

EXAMPLES FOR PRACTICE.

2. How long must \$80 be on interest at 6% per annum to gain \$12? Ans. 2yr. 6mo.

3. In what time will any principal double itself by simple interest at 5, 6, 7, 8, 9, 10, and 12 per cent. per annum?
Ans. 20, $16\frac{2}{3}$, $14\frac{2}{7}$, $12\frac{1}{2}$, $11\frac{1}{9}$, 10, and $8\frac{1}{3}$ yr.

QUESTIONS.—219. How do we find the rate, when principal, interest, and time are known?—220. How do we find the time, when principal, interest, and rate are known?

4. How long will it take \$1374.50 to amount to \$1480.78 at 10% per annum? Ans. 9mo. 8da.

Case VIII.

Art. 221. To find the principal, when the time, rate, and amount or interest are known.

Ex. 1. What principal at 7% will gain \$26.25 in 2½ yr?
Ans. \$150.

Written Process.

\$0.07 Int. of \$1 for 1 yr.
2.5 No. of yrs.

\$0.175) \$26.250 (150
175
875
875
0

Mental Process.

If 17½ cents is the interest of \$1 for the time, \$26.25 must be the interest of as many dollars as \$0.175 is contained times in \$26.25; that is, of \$150.

Ex. 2. What principal at 5% will amount to \$28.75 in 3 years? Ans. \$25.

Process Indicated.

\$28.75 ÷ \$1.15 = 25

Mental Process.

If \$1.15 is the amount of \$1 for 3 years at 5%, \$28.75 is the amount of as many dollars, at the same rate for the same time, as \$1.15 is contained times in \$28.75; that is, of \$25.

Rule.—*Divide the given interest or amount by the interest or amount of one dollar for the rate and time.*

Examples for Practice.

3. What principal at 6% will yield \$1500 a year?
Ans. \$25000.

4. What principal at 5% will produce \$1000 a year?
Ans. \$20000.

5. What principal at 8% will amount to \$17492.27 in 5yr. 3mo? Ans. \$12318.50.

6. What principal at 10% will produce \$172.30 in 1yr. 6mo 22da. Ans. \$1103.70.

Question. — 221. How do we find the principal, when the time, rate, and amount or interest are known?

COMPOUND INTEREST.

Art. 222. COMPOUND INTEREST is interest reckoned on the sum of the principal and its previous unpaid interest. In Compound Interest that interest which is not paid when due is reckoned as a new loan or principal, and bears interest like the first principal.

Art. 223. To compute compound interest for any time at any rate.

Ex. 1. At compound interest at 6% per annum, what is the interest of $450 for 3yr. 6mo. 18da.?

Ans. $103.64,3+.

COMPUTATION.

Principal	$450
Rate	.06
Interest first year	$27.00
Principal added	$450
Amount first year	$477
	.06
Interest second year	$28.62
Principal added	$477.
Amount second year	$505.62
	.06
Interest third year	$30.3372
Principal added	$505.62
Amount third year	$535.9572
Interest of $1 for 6mo. 18da.	.033
	16078716
	16078716
Interest for 6mo. 18da.	$17.6865876
Principal added	$535.9572
Whole amount	$553.6437876
First principal subtracted	$450.
Compound Interest	$103.64,37876

QUESTION.—222. What is compound interest?

NOTE.—If, in the foregoing example, the interest had been made payable semi-annually or quarterly, or at the end of any regularly recurring period, each amount would have been calculated for that period, instead of a year.

RULE.—*Find the amount of the principal for the first period of time, and make it the principal for the second period; then find the amount of this new principal for the second period, and make it the principal for the third period. So continue through all the periods, and through the last fractional period, if there is any. Subtract the first principal from the last amount; the remainder is the compound interest.*

NOTE.—The laws grant simple interest to the creditor in the settlement of his interest-claims. This implies that the price put upon the use of money is not due till that use is finished. Strict simple interest is not consistent with the payment of interest periodically before the principal, because the creditor can reloan the interest-money, and thus get compound interest. But periodical payments of interest are most frequently adopted in business. It is generally thought that compound interest, at the common legal rates for simple interest, would be too heavy a tax upon the productive power of capital.

EXAMPLES FOR PRACTICE.

2. What are the interest and amount of $600 for 2yr. 4mo. 12da., at 5% compound interest, payable annually?
Ans. Int. $73.62,75; Am't $673.62,75.

3. What are the interest and amount of $1000 for 4yr. 8mo. 20da., at 7% compound interest, payable semi-annually?
Ans. Int. $384.09,7+; Am't $1384.09,7+.

4. What are the interest and amount of $225 for 1yr. 11mo. 15da., at 4% compound interest, payable annually?
Ans. Int. $17.97; Am't $242.97.

5. What are the interest and amount of $362.50 for 2yr. 9mo., at 6% compound interest, payable semi-annually?
Ans. Int. $64.04+; Am't $426.54+.

6. What is the difference between the simple and compound interest of $500 for 5 years, at 7%? Ans. 26.27,5+.

7. What is the difference between the simple and compound interest of $1200 for 12 years, at 8%? Ans. $669.80+.

QUESTIONS.—223. How do we compute compound interest? Do the laws grant compound interest to the creditor in settling his interest-claims?

Art. 224. To save labor, compound interest is often computed by the aid of tables.

TABLE,

Showing the amount of $1 or £1 at 3, 4, 5, 6, 7, and 8 per cent. per annum, compound interest, for any number of years, from 1 to 25.

Yr.	3 per cent.	4 per cent.	5 per cent.	6 per cent.	7 per cent.	8 per cent.
1	1.03	1.04	1.05	1.06	1.07	1.08
2	1.060	1.0816	1.1025	1.1236	1.1449	1.1664
3	1.092727	1.124864	1.157625	1.191016	1.225043	1.259712
4	1.125509	1.169859	1.215506	1.262477	1.310796	1.360488
5	1.159274	1.216653	1.276282	1.338226	1.402551	1.469328
6	1.194052	1.265319	1.340096	1.418519	1.500730	1.586874
7	1.229874	1.315932	1.407100	1.503630	1.605781	1.713824
8	1.266770	1.368569	1.477455	1.593848	1.718186	1.850930
9	1.304773	1.423312	1.551328	1.689479	1.838459	1.999004
10	1.343916	1.480244	1.628895	1.790848	1.967151	2.158924
11	1.384234	1.539454	1.710339	1.898299	2.104851	2.331638
12	1.425761	1.601032	1.795856	2.012196	2.252191	2.518170
13	1.468534	1.665074	1.885649	2.132928	2.409845	2.719623
14	1.512590	1.731676	1.979932	2.260904	2.578534	2.937193
15	1.557967	1.800944	2.078928	2.396558	2.759031	3.172169
16	1.604706	1.872981	2.182875	2.540352	2.952163	3.425942
17	1.652848	1.947900	2.292018	2.692773	3.158815	3.700018
18	1.702433	2.025817	2.406619	2.854339	3.379932	3.996019
19	1.753506	2.106849	2.526950	3.025600	3.616527	4.315701
20	1.806111	2.191123	2.653298	3.207135	3.869684	4.660957
21	1.860295	2.278768	2.785963	3.399564	4.140562	5.033833
22	1.916103	2.369919	2.925261	3.603537	4.430401	5.436540
23	1.973587	2.464716	3.071524	3.819750	4.740529	5.871463
24	2.032794	2.563304	3.225100	4.048935	5.072366	6.341180
25	2.093778	2.665836	3.386355	4.291871	5.427432	6.848475

To compute compound interest by this table.

RULE.—For a whole number of years. *Multiply the amount, given in the table, of* $1 *or* £1 *at the given rate for the given number of years, by the principal.*

For a final fraction of a year. *Compute interest on the amount of the previous whole number of years.*

EXAMPLES FOR PRACTICE.

1. What is the interest of $250 for 10 years, at 6% compound interest, payable annually? Ans. $197.71,2.

QUESTIONS.—224. Why is Compound Interest computed by the aid of tables? What is the rule?

WRITTEN PROCESS.

$1.790848	Am't of $1.
250	Prin.
447.712000	Am't of $250.
250	Prin. subtracted.
$197.712	Int. of $250.

MENTAL PROCESS.

The amount of $250 for 10 years is 250 times the amount of $1 for the same time and at the same rate.

Ex. 2. What is the amount, at compound interest, of $1200 for 5yr. 6mo., at 8%, payable annually? Ans. $1833.72,1+.

WRITTEN PROCESS.

$1.469328	Am't of $1 for 5yr.
1200	Principal.
1763.193600	Am't of $1200 for 5yr.
1.04	Am't of $1 for 6mo.
70527744	
17631936	
$1833.721344	Am't of $1200 Ans.

EXPLANATION.

After getting the amount of $1200 for 5yr., it becomes a new principal for the last 6 months, and the interest is calculated on this in the usual way.

Ex. 3. What is the amount, at compound interest, of $75 for 72 years, at 5%, payable annually? Ans. $2515.88,5.

PROCESS INDICATED.

$3.386355 × 75 = $253.976625 Am't for first 25yr.
$3.2251 × 253.976625 = $819.10 Am't for next 24yr.
$3.071524 × 819.10 = $2515.885 Am't for next 23yr.
(72 in all.)

EXPLANATION.

We here divide 72yr. into parts that come within the range of the table; then compute for the first part, use that amount as a principal for the second part, and that amount as a principal for the last part.

4. What is the compound interest of $650 for 7yr. 3mo., at 3%, payable annually? Ans. $155.41,3+.

5. What is the amount of one cent for 100 years, at 6% compound interest, payable annually? Ans. $3.39,3+.

DISCOUNT.

Art. 225. DISCOUNT is a deduction from a price or debt, made by the seller or creditor, for a specific reason. It is of three kinds, viz., *price discount*, *true or equitable discount*, and *bank discount*.

Price discount is a deduction from the price of goods, or the par value of money, stocks, bonds, and drafts. It is reckoned as a percentage of the price or par value.

NOTE.—*Par value* is *nominal* value, and does not always correspond to real value. Thus, the *par value* of a five-dollar bank-bill is $5; but its real value may sometimes be less.

True or equitable discount of a note or account is the interest of the sum remaining after the discount is made, for the time between the payment and maturity of the debt. It proceeds on the supposition that money due at a *future* time is *now* worth only that principal which, with its interest for the time, will amount to the future debt; because the debtor, if he withholds it, or the creditor, if he receives it, may get interest on it for the time.

The *true present worth* of a sum of money due in the future, is that *principal* of which the given sum is the *amount*, for the time intervening.

Art. 226. To compute the true present worth and discount of a given sum, due at a future time.

Ex. 1. I buy goods, on a credit of 4 months, for $5250.50; if I conclude to pay *now*, what discount should I receive, at 6%?

Ans. $102.95,1.

PROCESS INDICATED.

$5250.50 ÷ 1.02 = $5147.54,9+

$5250.50 − $5147.54,9 = $102.95,1

MENTAL PROCESS.

The amount of $1, for the time and rate, is $1.02. Therefore, if that which, four months hence, will be worth $1.02, is *now* worth $1, then that which, four months hence, will be worth $5250.50, is now worth as many dollars as $1.02 is contained times in $5250.50, that is, $5147.54,9. The discount, therefore, must be $5250.50 − $5147.54,9; that is, $102.95,1.

QUESTIONS.—225. What is discount? Of what kinds is it? What is price discount? What is par value? What is true discount? On what supposition does it proceed? Why? What is true present worth?

RULE.—*Divide the given sum by the amount of* $1 *or* £1 *for the given time and rate: the quotient is the present worth.*

Subtract the present worth from the given sum: the remainder is the discount.

NOTE.—This process is similar to Case VIII., Art. 221.

EXAMPLES FOR PRACTICE.

2. What are the present worth and discount of $896.87,5 due 6mo. hence, at 5%? Ans. P.W. $875; D. $21.87,5.

3. What are the present worth and discount of $1315.62½ due 9mo. hence, at 7%? Ans. P.W. $1250; D. $65.62,5.

4. What are the present worth and discount of $1543.56 due 3mo. 3da. hence, at 6%? Ans. P.W. $1520; D. $23.56.

5. Wm. Richards gave a note for his debt of $558 due in 4mo., but the same day changed his mind, searched for and bought his note. How much ought he to have given for it if money then ruled at 10% per annum? Ans. $540.

6. How much, if money was then worth 1½% a month? Ans. $526.41,5+.

7. F. Neal, of London, sold me goods, valued at £100 15s. 6d., on 6 months' credit. How much should I have remitted to him if I concluded to pay him immediately, money being worth 5%? Ans. £98 6s. 4d. nearly.

BANK DISCOUNT, AND BANKING.

Art. 227. BANK DISCOUNT is the interest of the face of a note, deducted from the face of the note at the time of the loan.

A *bank* is an association of persons legally established for the purpose of dealing in money.

A *bank of deposit* is a bank legally empowered to receive and take charge of the money of others.

A *deposit* is money or its equivalent intrusted to the keeping of a bank.

A *depositor* is a person who makes a deposit at a bank.

A *check* is a written order on a bank for money.

QUESTIONS.—226. How do we compute true present worth and discount?—227. What is bank discount? What is a bank? A bank of deposit? What is a deposit? A depositor? A check?

A *bank of discount* is a bank legally empowered to loan money.

A *bank of issue* is a bank legally empowered to pay out its own promissory-notes as money. These notes are frequently called *bank-bills*.

The affairs of a bank are usually managed by a *board of directors*. Their principal officers are a *president*, a *cashier*, who superintends the accounts of the bank, and one or more *tellers* to receive and pay out money.

A bank is said to *discount a note* when it loans money on a note drawn for a definite time, deducting the interest of its face from its face at the time of the loan.

The *avails* or *proceeds* of a note discounted are the face of the note less the discount; that is, the sum received for the note.

In calculating the day of maturity of a note, three days, called *days of grace*, are usually added to the time specified in the note.

Case I.

Art. 228. To find the bank discount and proceeds of a note.

Ex. 1. What are the date of maturity, discount, and proceeds of the following note, discounted Sept. 15, at 6%?

$40\frac{00}{100}$. Allegheny, Sept. 15, 1858.

Sixty days after date, I promise to pay to the order of J. M. Reed forty dollars, without defalcation, value received.

M. Munroe.

Ans. Due Nov. 14/17; Discount, $0.42; Proceeds, $39.58.

Written Process.

$0.0105 Int. of $1 for 63 days.
40 Face of note.

$0.4200 Int. of $40.

$40 — $0.42 = $39.58 Proceeds.

Explanation.

Here 60 days are called 2 months, and interest is calculated for $1 for 2mo. 3da., which is multiplied by the principal. The product, 42cts., is taken as discount from the principal.

Questions.—227. What is a bank of discount? A bank of issue? By whom are banks managed? When does a bank discount a note? What are the proceeds of a note? What are days of grace?

RULE.—*Compute the interest on the face of the note for the time between the day of discounting it and three days after the time mentioned in it: this will be the discount.*

Subtract the discount from the face of the note: the remainder is the proceeds.

NOTE.—Bank discount exceeds true discount by the interest of the true discount for the time. Thus, the bank discount of $106 for a year, (excluding days of grace,) at 6 per cent., is $6.36; while the true discount is $6. The difference, 36cts., is the interest of $6 for a year.

EXAMPLES FOR PRACTICE.

2. Find the maturity, discount, and proceeds of a note drawn for $500, sixty days, dated and discounted Jan. 5, 1859, at 6%.
Ans. Due March 6/9; Discount, $5.25; Proceeds, $494.75.

3. Of a note drawn for $300, ninety days, dated March 10, 1858, and discounted April 13, 1858, at 10%, by days as 365ths.
Ans. Due June 8/11; Dis. $4.84,9+; Proceeds, $295.15+.

4. Of a note drawn for $250, four months, dated July 1, 1858, and discounted Aug. 15, 1858, at 12%, by days as 360ths
Ans. Due Nov. 1/4; Dis. $6.75; Proceeds, $243.25.

5. Of a note drawn for $110, three months, dated and discounted May 2, 1859, at 1½ per cent. a month.
Ans. Due Aug. 2/5; Dis. $5.11,5; Pro. $104.88,5.

6. Of a note drawn for $1750, six months, dated Sept. 7, 1858, and discounted Nov. 22, 1858, at 7% per annum.
Ans. Due March 7/10, 1859; Dis. $36.74,9; Proceeds, $1713.25,1.

7. If, on June 15, you buy a note dated January 15, drawn for $80, eight months, what should you pay for it, money being 2% a month? Ans. $75.04.

CASE II.

Art. 229. To make a note, of which the proceeds shall be a given sum.

Ex. 1. For what sum must I give a note, payable in 90 days, to draw from bank $1969 on a discount of 6%? Ans. $2000.

QUESTIONS.—228. How do we find the bank discount and proceeds of a note? How much does bank discount differ from true discount?

WRITTEN PROCESS.

$1.
.0155 Int. of $1 for 93da.

$0.9845 Proceeds of $1.
.9845)1969.0000(2000
19690

000

PROOF. Int. of $2000 for 93da. = $31.
$2000—$31 = $1969.

MENTAL PROCESS.

If $0.9845 proceeds require $1 of *face*, then $1969 proceeds will require as many dollars of face as $0.9845 is contained times in $1969, that is, $2000.

RULE.—*Divide the given proceeds by the proceeds of one dollar on the given conditions.*

EXAMPLES FOR PRACTICE.

2. For what must a note be given, payable in 60 days, to draw from bank $300, on a discount of 6% per annum?
Ans. By 360ths, $303.18,3+; by 365ths, $303.13,9+.

3. What must be the face of a 4 months note, to draw $800, on a discount of 1% a month? Ans. $834.20,2+.

4. Of a 60 day note, to draw $1536.50, on a discount of 2% a month? Ans. $1603.86,2+.

5. Of a 6 months note, to draw $9700, on a discount of 10% per annum? Ans. $10219.49,07+.

STOCKS.

Art. 230. STOCK is a name applied—

1st. To the capital and privileges of chartered companies, called corporations;

2d. To the bonds issued by governments; and

3d. To the property of persons in business.

The stock of incorporated companies is usually represented by *shares*, each one of which entitles the owner to a definite portion of the profits, losses, or expenses of the concern. The owners of the shares are called *stockholders*.

Stocks are said to be at *par* when shares sell for their cost, or bonds sell for their face; *above par*, or *at a premium*, when shares sell for more than their cost, and bonds sell for more than

QUESTIONS.—229. How do we make a note, of which the proceeds shall be a given sum?—230. What is stock? How is it represented? When are stocks at par? Above par?

their face; *below par*, or *at a discount*, when shares sell for less than their cost, and bonds sell for less than their face.

Premium and *discount* are reckoned as a percentage of the *par value*.

CASE I.

Art. 231. To find the worth of a given number of shares, when the stock is at a premium or discount.

Ex. 1. If I bid off, at auction, 15 shares of bridge stock, par value \$25, at 20% premium, what must I pay for them?

Ans. \$450.

PROCESS INDICATED.

$\$25 \times 15 \times 1.20 = \$450$

or

$\$25 \times 15 = \$375$

$\$375 \times .20 = \$75$

$\$375 + \$75 = \$450$

MENTAL PROCESS.

If \$1 par is worth \$1.20 at 20 per cent. premium, then \$25 par are worth 25 times \$1.20; that is, \$30; and 15 shares are worth 15 times \$30; that is, \$450.

RULE.—*Find the per cent. premium or discount, and add it to the par value when the stock is above par, and subtract it from the par value when the stock is below par.*

EXAMPLES FOR PRACTICE.

2. What cost 24 shares bank stock, par value \$50, at 25% above par? Ans. \$1500.

3. What cost 32 shares railroad stock, par value \$50, at 40% below par? Ans. \$960.

4. What cost 18 shares plank-road stock, par value \$25, at 60% discount? Ans. \$180.

CASE II.

Art. 232. To find the number of shares that can be bought for a given sum.

Ex. 1. How many shares of bank stock, \$50 par, can be bought for \$500, at 25% premium? Ans. 8 shares.

PROCESS INDICATED.

$\$50 \times 1.25 = \$62.50$

$\$500 \div \$62.50 = 8$

MENTAL PROCESS.

If \$1.25 will purchase \$1 par, it will take 50 times \$1.25 to purchase \$50 par; that is, \$62.50; and \$500 will purchase as many shares at that price as \$62.50 is contained times in \$500.

QUESTIONS.—230. When are stocks below par? How are premium and discount reckoned? —231. How do we find the worth of a given number of shares?

RULE.—*Find the price of one share, and divide the given sum by it.*

EXAMPLES FOR PRACTICE.

2. How many shares of gas stock, par value \$25, can be bought for \$440 at 10% above par? Ans. 16 shares.

3. How many shares of museum stock, par value \$100, can be bought for \$1500, at 25% discount? Ans. 20.

4. How many shares of bank stock, par value \$50, can be bought for \$2500, at 25% premium? Ans. 40.

CASE III.

Art. 233. To find the rate of interest obtained by purchasing a given stock at a given price.

Ex. 1. G. Tappan bought copper-mining stock for \$250 per share, and received annual dividends of \$30 per share. What rate of interest did he get? Ans. 12%.

PROCESS INDICATED.	REMARK.
\$30 ÷ \$250 = .12 = 12 per cent.	This is like Case VI. Art. 219, in Interest, which see.

RULE.—*Divide the dividend by the cost of the stock.*

EXAMPLES FOR PRACTICE.

2. Gave \$55 per share for bank stock, par value \$50, and receive \$4 dividend per share semi-annually. What rate of interest do I receive?

Ans. $7\frac{3}{11}$ per cent. per 6mo.; $14\frac{6}{11}$ per annum.

3. Gave \$20 per share for ferry stock, and receive \$1 per share dividend annually. What rate of interest do I receive?

Ans. 5%.

4. Gave \$600 for a 6% railroad bond of \$1000, and drew its interest regularly. What rate of interest did I receive?

Ans. 10%.

CASE IV.

Art. 234. To find the annual income from a given sum, invested in a given stock, at a given price.

Ex. 1. What income is derived from \$17600, invested in government 6% stocks at \$88 for \$100 par? Ans. \$1200.

QUESTIONS.—232. How do we find the number of shares that can be bought for a given sum?—233. How do we find the rate of interest paid by purchasing stock?

PROCESS BY PROPORTION. MENTAL PROCESS.

$88 : $17600 :: $6 : $1200 If $88 yield $6 income, $17600 will yield as many times $6 income as $88 are contained times in $17600; that is, 200 times $6 or $1200.

RULE.—*Divide the given sum by the price of the stock, and multiply the quotient by the rate paid by the stock.*

EXAMPLES FOR PRACTICE.

2. What annual income may be derived from investing $14000 in government 5 per cents. at 70? Ans. $1000.

3. From investing $25000 in stock paying $1 per share, at $20 per share? Ans. $1250.

4. From investing $21200 in government 6 per cents., at $1.06? Ans. $1200.

CASE V.

Art. 235. To find what price must be paid for a given stock that its dividends may be a given rate of interest.

EX. 1. What must I bid for a 6% bond of $1000 that I may receive 10% interest on my money? Ans. $600.

EXPLANATION.—The bond yields 6% of $1000; that is, $60 annually. But $60 is 10 per cent. of what number? ANALYSIS.—If $60 is 10 per cent. of a certain number, $\frac{1}{10}$ of $60, that is, $6, must be 1 per cent. of that number; and 100 times $6, that is $600, must be that number.

Or, 6 per cent. of a number is 10 per cent. of $\frac{6}{10}$ of that number.

RULE.—*Multiply the par value of the stock by that fraction which is made of the fixed interest for a numerator, and the required interest for a denominator.*

EXAMPLES FOR PRACTICE.

2. How much may be paid for bank stock, par value $50, dividend 10%, that the purchaser may realize 8%? Ans. $62.50.

3. For gas stock, par value $25, dividend 8%, to realize 12%? Ans. $16.66⅔.

4. For telegraph stock, par value $100, dividend 12%, to realize 15%? Ans. $80.

QUESTIONS.—234. How do we find the annual income from investing a given sum in stock?—235. How do we find what to pay for a stock to realize a given interest?

CHAPTER XVII.

EQUATION OF PAYMENTS.

Art. 236. EQUATION OF PAYMENTS is the process of finding the proper time for paying two or more debts due at different times.

An *account* is a formal statement, made by one party, of his commercial transactions with another. It has a debtor side, marked *Dr.*, and a creditor side, marked *Cr.* The *Dr.* side shows the items paid or sold to the other party. The *Cr.* side shows the items received from the other party. The date of each transaction is stated in the account.

The *balance* of an account is the excess of the indebtedness of one party in the account over that of the other, and is so called because it would equalize or balance the sides if added to the less side.

An *account current* is an account in which the items are usually allowed interest,* and, if allowed a term of credit, are dated at the time they become *due*, that is, when the term of credit expires. Accounts current are usually settled or balanced at the end of six months or a year.

Equation of Payments is of two kinds,—viz., *Simple* and *Compound*.

SIMPLE EQUATION OF PAYMENTS.

Art. 237. Equation of Payments is called *simple* when the items to be equated are all on one side of the account.

* For the sake of uniformity, and because of its easiness, the interest computed in examples in this chapter should be at 6%, by taking *one-sixth of the number of days as mills*

QUESTIONS.—236. What is Equation of Payments? What is an account? The balance? An account current?—237. What is simple equation of payments?

Art. 238. To find the equitable time for paying debts due at different times.

Ex. 1. Bought $500 worth of goods on a credit of 2 months, and, at the same time, $1000 worth on a credit of 8 months. I afterward concluded to pay both debts at one time. When should it be? Ans. In 6mo. from purchase.

By Interest, (6%.)

$500 6mo. = $15.00
$1000 0mo. = 00
$1500 1mo. = $7.50) $15.00
Backward 2mo.
8 — 2 = 6mo. fr. purchase.

By Products.

$500 \times 6 = 3000$
$1000 \times 0 = 0000$
1500) 3000
Backward 2

Another Method.

$500 \times 2 = 1000$
$1000 \times 8 = 8000$
1500) 9000
Forward 6

Proof.

Int. $500 4mo. = $10. Am't. $510
Int. $1000 2mo. = $10. P.W. $990
$1500 = $1500

Explanation.

My creditor should have the use of the first debt from its maturity to that of the second: worth $15 at 6%. If deprived of this, by putting both payments in one, he should have both long enough to yield $15; that is, as many months as $7.50, the int. of both 1 month, is contained times in $15.

In the first process by products, we represent the use of money *by the creditor* by the product of *money and time*, omitting the common factor *rate*. In the second process by products, we represent the credits enjoyed *by the debtor* in the same way, all his privileges amounting to the use of both debts 6 months. In proving, we use *bank discount* for what is paid before its time, because the allowance of true discount would require a tedious process. In business, bank discount is always given in such cases.

Ex. 2. Sold N. Adair goods on credit as follows:—Jan. 15, $150, four mo.; Feb. 4, $300, four mo.; March 5, $500, five mo.; April 2, $1000, six mo. He afterward wished to pay all at once. When should it be? Ans. Aug. 19.

Due.	First Method. da.	$		Second Method. $	da.	
May 15.	120 ×	150 =	18000	150 × 0	=	0
June 4.	140 ×	300 =	42000	300 × 20	=	6000
Aug. 5.	202 ×	500 =	101000	500 × 82	=	41000
Oct. 2.	260 ×	1000 =	260000	1000 × 140	=	140000
		1950)	421000	1950	)	187000
	From Jan. 15.	215 $\frac{35}{39}$ da.		From May 15.		95 $\frac{35}{39}$ da.

Proof.

Int.	$150	from May 15	to Aug. 19,	(95 $\frac{35}{39}$ da.)	= $2.397+
"	$300	" June 4	"	(75 $\frac{35}{39}$ da.)	= $3.794+
"	$500	" Aug. 5	"	(13 $\frac{35}{39}$ da.)	= $1.158+
Dis.	$1000	" Oct. 2	"	(44 $\frac{4}{39}$ da.)	= $7.350+

Explanation.—We seek such a date that the interest of those debts which are paid after their maturity, for their time over-due, shall equal that of those debts which are paid before their maturity, for their time prepaid. We find this date by *averaging* from an assumed date. In the first method we average from January 15, reasoning as follows:—$150 running 120 days after January 15, equals $1 running 150 × 120, or 18000 days after January 15, &c. All the maturities equal $1 maturing 421000 days after January 15, or $1950 maturing $\frac{1}{1950}$ of 421000, or 215 $\frac{35}{39}$ days after January 15. In the second method, we average in the same way from May 15, and find the same date. For *extreme accuracy*, the proof is here calculated *to the fraction of a day*, (reckoning to and from *the end* of the days of date,) but in practice such exactness is not necessary.

Rule.—*Compute the interest on the items to the last date, and divide it by the interest of the sum of the debts for one period of time. Reckon the quotient back from the last date;* or,

Assume some convenient date from which to reckon, such as the last or the first, or some date before the first or after the last. Multiply each debt by the units of time between the assumed date and the day of that debt's becoming due, and divide the sum of the products by the sum of the debts. When the first or an earlier date is assumed, reckon the quotient forward from it; when the last or a later date is assumed, reckon the quotient backward from it.

Question.—238. How do we find the equitable time for paying debts due at different times?

NOTE 1.—When the quotient has a fraction of a day, neglect it.

NOTE 2.—Reckon fractions of $1 as $1 if more than 50cts., or as 0 if less than 50cts.

NOTE 3.—Some think it best to assume the last day of the year, and determine each product *at the time of the transaction*, thus having the account ready to equate at any time.

Let the pupil, by several assumed dates, equate each of the following

EXAMPLES FOR PRACTICE.

3. Holt & Ward's account against me is as follows:—

1859. Jan. 4.	M'dse	(4mo. cr.)	$240
Feb. 16.	"	(3mo. cr.)	$150
March 23.	"	(4mo. cr.)	$500
April 14.	"	(6mo. cr.)	$750
May 12.	"	(4mo. cr.)	$400
June 1.	"	(4mo. cr.)	$250

When should I pay if it is agreed to equate? Ans. Aug. 23.

4. My account against Collins & Park is as follows:—

1859. Jan. 13.	M'dse	(4mo. cr.)	$125
Feb. 10.	"	(6mo. cr.)	$800
March 8.	"	(4mo. cr.)	$440
April 28.	"	(3mo. cr.)	$230
May 20.	"	(4mo. cr.)	$200
June 10.	"	(2mo. cr.)	$100

When should they pay if it is agreed to equate?

Ans. July 31.

COMPOUND EQUATION OF PAYMENTS.

Art. 239. Equation of Payments is called *compound* when the items to be equated are on both sides of an account; that is, when there are both debits and credits.

Art. 240. To find the equitable time for paying the balance of an account current.

Ex. 1. On May 12, 1858, A buys $2000 worth of goods of B, on a credit of 6 months. Two months afterward, A pays $500, and, 1 month after that, $500 more. How long should A keep the balance beyond the term of credit?

Ans. 3½ months; or till Feb. 27, 1859.

QUESTION.—239. When is equation of payments compound?

FORM OF THE ACCOUNT.

Dr. *A in Acc't Current with B.* *Cr.*

1858.		$	ct.	1858.		$	ct.
Nov. 12.	To M'dse.	2000		July 12.	By Cash	500	
				Aug. 12.	" "	500	
				1859.			
				Feb. 27.	" " *bal.*	1000	

PROCESS BY INTEREST.

\$500 4mo. = \$10
\$500 3mo. = 7.50
\$1000 *bal.* 1mo. = \$5)17.50
3½mo.

PROCESS BY PRODUCTS.

500 × 4 = 2000
500 × 3 = 1500
Bal. 1000)3500
3½mo.

MENTAL PROCESS.

A loses and B enjoys the interest of \$500 4mo. before due, and \$500 3mo. before due; in all, \$17.50. A should keep the balance, \$1000, till its interest is \$17.50.

Here notice that interest is owed by *one party*, and the balance by *another*, and *the quotient* 3½ *months is reckoned forward.*

Ex. 2. When did the *balance* of the following account become due, considering each item due at date?

Dr. *R. Sample in Acc't Cur. with B. Mason & Co.* *Cr.*

1858.		da.	Int.	$	ct.	1858.		da.	Int.	$	ct.
Jan. 6.	To M'dse.	135	1.92	85	25	Jan. 13.	By M'dse.	128	1.60	75	
" 8.	" "	133	1.77	79	75	Apr. 7.	" "	44	.32	43	87
Feb. 4.	" "	106	.35	20		May 12.	" Cash	9	.21	142	13
" 7.	" "	103	3.69	215							
Apr. 3.	" "	48	1.52	190	05						
May 21.	" "			29	95						

Ans. 119da. back from May 21; that is, Jan. 22, 1858;
Or, 16da. forward from Jan. 6; that is, Jan. 22, 1858.

PROCESS BY INTEREST.

Sum of int. of Drs. to May 21 (see acc't) \$9.25
" " Crs. " " \$2.13
Int. of bal. \$359 for 1 day \$0.05983)\$7.12
119da. nearly

FIRST METHOD BY PRODUCTS.

$	da.		$	da.	
85	× 135	= 11475	75	× 128	= 9600
80	× 133	= 10640	44	× 44	= 1936
20	× 106	= 2120	142	× 9	= 1278
215	× 103	= 22145	261		12814
190	× 48	= 9120			
30	× 0	= 0			
620		55500			
261		12814			
359		) 42686 (119da.			

Reckoned back from May 21.

SECOND METHOD BY PRODUCTS.

$	da.		$	da.	
85	× 0	= 0	75	× 7	= 525
80	× 2	= 160	44	× 91	= 4004
20	× 29	= 580	142	× 126	= 17892
215	× 32	= 6880	261		22421
190	× 87	= 16530			
30	× 135	= 4050			
620		28200			
261		22421			
359		) 5779 (16+da.			

Reckoned forward from Jan. 6.

EXPLANATION.—In the process by interest we compute interest on each item to May 21, the last date. The debits have $7.12 more interest than the credits, and there are $359 more debits than credits. Sample therefore owes $359, balance, + $7.12 interest. His $359 should therefore be considered due far enough back of May 21 to produce $7.12. In the first method by products, we represent the use of money by each party in the product of the debt into its time before May 21. It appears that Sample has had benefit equal to the use of $1 for 42686 days, or $359 for 119 days. It should therefore be considered as having fallen due 119 days previous to May 21, and he should be charged interest for that time. In the second method by products, the items are supposed to be furnished Jan. 6, by which supposition Sample is deprived of the use of $359 for 16 days. He should therefore keep it for 16 days beyond Jan. 6.

Here notice that both interest and balance are owed by one party, and that the quotient, 119 days, is reckoned *backward* from the last date, May 21.

RULE.—*Compute the interest of all the items from their dates to that of the last item. Divide the balance of interest by the interest of the balance of items for one of the periods of time used in computing the interest of the items.*

Reckon the quotient backward from the last date when the balance of items and that of interest are owed by one party, but forward when the balance of items is owed by one party, and that of interest by the other; or,

Find the products, for both sides of the account, from one convenient date, by the rule for Simple Equation, and divide the difference of the amounts of the debit and credit products by the balance of items.

Reckon the quotient backward,

1st. *When the assumed date is the last given, or a later date, and the larger sum of items is on the side of the larger sum of products.*

2d. When the assumed date is the first given, or a prior date, and the smaller sum of items is on the side of the larger sum of products.

Reckon the quotient forward,

1st. When the assumed date is the last given, or a later date, and the smaller sum of items is on the side of the larger sum of products.

2d. When the assumed date is the first given, or a prior date, and the larger sum of items is on the side of the larger sum of products.

NOTE.—If the equitable time for paying a balance comes before an account is actually finished, it is not absurd, since it merely indicates that interest is to be charged on that balance from the equitable time to settlement.

EXAMPLES FOR PRACTICE.

3. What is the balance of the following account, and the equitable time for its payment?

Dr. *W. Post in Acc't Cur. with D. Hall.* *Cr.*

1859.		$	ct.	1859.		$	ct.
Jan. 11.	To M'dse.	400		Mar. 1.	By Cash	500	
" 29.	" "	200		Apr. 15.	By M'dse.	250	
Mar. 7.	" "	500		May 18.	" M'dse.	375	
" 22.	" "	800		June 8.	By Cash	400	
May 13.	" "	300					

Ans. Bal. $675. Due Nov. 26, 1858.

4. What is the balance of the following account, and the equitable time for its payment, each item which is on credit having been dated at the time of the transaction, instead of the time when it fell due?

Dr. *L. Beck in Acc't with G. Hays.* *Cr.*

1859.		$	ct.	1859.		$	ct.
May 10.	To M'dse. (3mo. cr.)	900		Jan. 4.	By M'dse.	1200	
June 4.	To M'dse. (3mo. cr.)	700		" 10.	" "	1400	
July 6.	To Cash	800		" 24.	" "	2400	
" 14.	To M'dse. (3mo. cr.)	400		Mar. 4.	" "	800	
Nov. 6.	To Cash	200		Apr. 1.	" "	200	

Ans. Balance $3000. Due June 29, 1858.

QUESTION.—240. How do we find the equitable time for paying the balance of an account current?

Art. 241. The object of equating an account current is to lead the parties in account to a just settlement of their claims upon each other. If the parties adjust their claims and close account on the day found by equating, (provided that day is subsequent to the transactions of the account,) *the true balance of the account is the balance of items*, because the holder of that balance has then held it till its interest has equaled the interest owed him.

If the parties settle before the equitable time, the holder of the balance should be allowed discount for the time for which he has lost the use of it by paying it before it is due.

If the parties settle after the equitable time, the holder of the balance should be charged interest on it for the time that he holds it beyond the equitable time.

NOTE.—The rules in common use allow the debtor *bank discount* on debts paid before their time. The allowance of *true discount* would require a tedious process.

Art. 242. To find the true balance of an account current at settlement.

RULES.—I. By equation of time.

Find the equitable time for the payment of the balance of items. If settlement is at that time, that balance is the true balance.

If settlement is before the equitable time, deduct from the balance of items its interest for the time from settlement to the equitable time.

If settlement is after the equitable time, add to the balance of items its interest from the equitable time to settlement.

II. By computing the items to the time of settlement.

Find the interest of the items from the time they are due to settlement, reckoning in with the interest of the opposite side that of items paid before due. Add to that side, which produces the greater amount of interest, the excess of its interest over that of the other side, and then find the difference of the two sides; or,

Multiply each item by the units of time from the time of being due to settlement, putting the products of those paid before due

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QUESTIONS.—241. What is the object of equating an account current? When is the balance of items the true balance? Why? What should be done if the parties settle before the equitable time? If after the equitable time?—242. How do we find the true balance of an account current at settlement?

with the products of the other side. Find the difference of the sums of these products, and multiply it by the interest of one dollar for one of the units of time. Add this product to that side which produces the greater sum of products, and then find the difference of the sides.

NOTE.—That method should be pursued which requires the least labor, since both methods bring the same result.

EXAMPLES FOR PRACTICE.

1. What is the true balance of the following account at settlement, July 1, 1859, each item *of merchandise* having a credit of 4 months?

Dr. *H. Caldwell in acc't with R. Munroe.* *Cr.*

1859.		$	cts.	1859.		$	cts.
Jan. 1.	To Md'se.	570		Feb. 1.	By Cash.	300	
March 1.	" "	980		April 1.	By Md'se.	700	
April 1.	" "	800		May 20.	By Cash.	400	

Ans. By rule first, due $0\frac{6}{9}\frac{8}{5}$ days after Aug. 1; that is, Aug. 2. Discount of balance \$950 for $31\frac{6}{9}\frac{8}{5}$ days, to July 1, \$4.952+. True balance \$945.047+.

PROCESS BY INTEREST (6%).

Due.	Debits.	$	Due.	Credits.	$
May 1.	$570 (Int. 61da.)	5.7155	Feb. 1.	$300 (Int. 150da.)	7.3972
July 1.	$980 (" 0 ")	0	May 20.	$400 (Int. 42da.)	2.7616
Aug. 1.	$800 { disc't 31da. $4.0766 carry to credits. }	3.5671	Aug. 1.	$700 { disc't 31da. $3.5671 carry to debits. }	4.0766
	$2350	$9.2826		$1400	$14.2354
	1404.9528			4.9528	9.2826
	$945.047 = Bal.			$1404.9528	$4.9528

EXPLANATION.—Caldwell owes \$5.7155 interest + \$3.5671 discount to Munroe on \$700 paid 31da. before due. Therefore Munroe's discount is added among the interest owed by Caldwell, making \$9.2826. Also Munroe owes (\$7.3972 + \$2.7616) interest + \$4.0766 discount to Caldwell on \$800 paid 31da. before due, making \$14.2354. Excess owed by Munroe, \$4.9528, which, added to his items, makes his whole debt \$1404.9528. This, taken from \$2350, Caldwell's debt, leaves Caldwell debtor \$945.047, giving the true balance at settlement, July 1, 1859. The above interest is computed in days as 365ths for the sake of accuracy. When the interest is computed at 6%, it is generally best to take one-sixth of the number of days as mills. It is plain that parties may allow other rates of interest than 6%.

PROCESS BY PRODUCTS.

570 × 61	=	34770	300 × 150	=	45000
980 × 0	=	0	400 × 42	=	16800
800 × 31 (carry over)		21700	700 × 31 (carry over)		24800
2350		56470	1400		86600
1404.952+			4.952		56470
945.047 True bal.			1404.952		30130

Int. of $1 for 1da. = .00016438

Int. bal. $4.952+

EXPLANATION.—The products represent the use of $1 for as many days as there are units in the products. The credits, therefore, use $1 for 30130 days more than the debits do. Now, $\frac{1}{365}$ of 6cts. (the interest of $1 for 1yr.) is $0.00016438+, which is the interest of $1 for 1 day; and for 30130 days, it would be 30130 times as much: that is, $4.952+.

NOTE.—In computing the true balance of the following examples, reckon one-sixth the number of days as mills, since that gives less trouble than other methods.

2. What is the true balance of the following account at settlement, July 1, 1859? Ans. $1410.71+.

Dr. *J. S. Smith in acc't cur. with Wm. English.* *Cr.*

1859.		$	cts.	1859.		$	cts.
Jan. 14.	To Md'se. (@ 2mo.)	600		May 6.	By Cash.	800	
Feb. 5.	" " (@ 4mo.)	800		" 8	By Md'se. (@ 2mo.)	600	
March 2.	" " (@ 3mo.)	900		June 4.	" " (@ 2mo.)	300	
April 12.	" " (@ 3mo.)	1200		" 20.	By Cash.	400	

3. What is the true balance of the following account at settlement, July 1, 1859? Ans. $2033.23+.

Dr. *F. Ames in acc't cur. with Hall & Lee.* *Cr.*

1859.		$	cts.	1859.		$	cts.
Jan. 8.	To Md'se. (@ 4mo.)	800		Jan. 12.	By Cash.	300	
Feb. 9.	" " (@ 6mo.)	900		" 14.	By Md'se. (@ 3mo.)	400	
Mar. 12.	" " (@ 8mo.)	1200		Feb. 16.	By Cash.	500	
April 4.	" " (@ 4mo.)	1000		Mar. 16.	" "	600	

CHAPTER XVIII.

CURRENCIES. EXCHANGE.

CURRENCIES.

Art. 243. The currency of a country is its money Trade often makes it necessary to reduce the money of one country to its value in that of another. Coin may have different values,—viz.:

An *intrinsic* value, or that of the metal in the coin;

A *commercial* value, or the worth of the coin as a medium of trade; and

A *legal* value, or that established by law.

Table of U.S. Custom-House Value of some Foreign Coins.

Pound Ster. or Sovereign, (Eng.)	$4.84	Rix dollar, or thaler of Prussia, and Northern States of Germany	$0.69
Pound Ster. Brit. Prov. Canada, Nova Scotia, N. Bruns., Newfdld	$4.00	Ruble, silver, Russia	$0.75
Dollar. Mexico, Peru, Chili, Central America	$1.00	Guilder, florin, Netherlands	$0.40
Specie dollar, Denmark	$1.05	Florin, South Germany	$0.40
Specie dollar, Sweden and Norway	$1.06	Milree, Madeira	$1.00
Milree, Portugal	$1.12½	Franc, France	$0.186
Pagoda. India	$1.84	Ducat, Naples	$0.80
Tael, China	$1.48	Real of Plate, Spain	$0.10
Rupee, Bengal	$0.50	Lira, Tuscany	$0.16
Rupee, British India	$0.41¼	Ounce, Sicily	$2.40
Rix dollar, Bremen	$0.78¾	Doubloon, Mexico	$15.60

Art. 244. Before the adoption of Federal Money by the United States in 1786, the currency of the colonies was in pounds, shillings, and pence, English Money. But, as the colonies issued bills of credit, called *paper money*, beyond their power to redeem them, the currency depreciated in value,—more in some colonies than in others. Some people, in small transactions, still reckon in these currencies at their depreciated values, which are stated in the following

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Questions.—243. What is currency? Intrinsic value? Commercial value? Legal value?—244. What was the colonial currency?

TABLE OF OLD CURRENCIES OF THE UNITED STATES.

English Money.	£1 = $4\frac{2}{3}\frac{1}{3}$ = $4.84	New England..	£1 = 3\frac{1}{3}$ = 3.33\frac{1}{3}$
	1s. = 0.24\frac{1}{5}$	Virginia.....	
	$1 = £$\frac{2}{1}\frac{5}{2}\frac{5}{1}$ = 4$\frac{1}{1}\frac{6}{2}\frac{0}{1}$s.	Kentucky.....	1s. = 0.16\frac{2}{3}$
		Tennessee.....	$1 = £$\frac{3}{1}\frac{}{0}$ = 6s.
Georgia and So. Carolina.....	£1 = 4\frac{2}{7}$ = 4.28\frac{4}{7}$	New Jersey.....	£1 = 2\frac{1}{3}$ = 2.66\frac{2}{3}$
	1s. = 0.21\frac{3}{7}$	Pennsylvania...	
	$1 = £$\frac{7}{3}\frac{}{0}$ = 4s. 8d.	Delaware.....	1s. = 0.13\frac{1}{3}$
		Maryland.....	$1 = £$\frac{3}{4}$ = 7s. 6d.
Canada, Nov. Sc.. N. Bruns., and N'foundland.	£1 = $4.00	New York	£1 = 2\frac{1}{2}$ = $2.50
	1s. = $0.20	N. Carolina.....	
	$1 = £$\frac{1}{4}$ = 5s.	Michigan	1s. = 0.12\frac{1}{2}$
		Ohio	$1 = £$\frac{2}{5}$ = 8s.

EXERCISES.

1. How much is 6d. English? Ans. $0.12,1.
2. How much is 6d. New England currency? 9d.?
3. How much is $4\frac{1}{2}$d. New England cur.? Ans. $6\frac{1}{4}$cts.
4. How much is $3\frac{1}{2}$d. Georgia currency? Ans. $6\frac{1}{4}$cts.
5. How much is 6d. Canada currency? Ans. 10cts.
6. How much is 6d. New York cur.? Ans. $6\frac{1}{4}$cts.
7. How much is 4s. New Eng. cur.? New York cur.?
8. How much is *two and sixpence* N.Y.? Ans. $31\frac{1}{4}$cts.
9. How much is *four and sixpence* N.E.? Ans. 75cts.
10. How much is *two and three pence* N.E.? Ans. $37\frac{1}{2}$cts.

REDUCTION OF CURRENCIES.

Art. 245. Reduction of Currencies is finding the value of one currency in the denominations of another.

Art. 246. To reduce a given sum in one currency to terms of another.

Ex. 1. What is the value, in U.S. money, of £37 7s. 6d. sterling? Ans. $180.89,5.

WRITTEN PROCESS.

£37 7s. 6d. = £37.375

£37.375 ÷ $\frac{2\,5}{1\,2\,1}$ = $180.89,5

Or, £37 = 37 × $4.84 = $179.08

7s. = 7 × 0.24\frac{1}{5}$ = 1.69,4

6d. = $\frac{1}{2}$ of 0.24\frac{1}{5}$ = 0.12,1

Total, $180.89,5

MENTAL PROCESS.

If £$\frac{2\,5}{1\,2\,1}$ = $1, then £37.375 equal as many dollars as $\frac{2\,5}{1\,2\,1}$ is contained times in 37.375, that is, $180.89,5.

Or, since £1 = 1\frac{2\,5}{1\,2\,1}$, £37.375 = 37.375 times 1\frac{2\,5}{1\,2\,1}$, that is, $180.89,5.

QUESTION.—245. What is reduction of currencies?

RULE.—*Multiply the given sum by the value of a unit of itself in the currency to which it is to be reduced;* or,

Divide the given sum by that number of its own denomination which makes one of the denomination to which it is to be reduced.

NOTE 1.—Either compound numbers must first be reduced to one denomination, or the reduction must be by parts.

NOTE 2.—In 1789 Congress fixed the pound sterling at $\$4.44\frac{4}{9}$. It is still customary to reckon on that value as a base, and to add a per cent. to bring the pound to its commercial value. Thus, when £1 = $4.86, custom adds to $\$4.44\frac{4}{9}$ a sufficient per cent. of itself, viz., $9\frac{35}{100}$ per cent., to bring it to $4.86. In such cases $\$4.44\frac{4}{9}$ is called the *par* value of £1, and the per cent. added is the *premium*.

EXAMPLES FOR PRACTICE.

2. What is the custom-house value of £345 15s. 2d.?
Ans. $1673.47+.

3. When the commercial value of £1 is $4.86, what is the value of £120 8s. 6d.? Ans. $585.26,55.

4. What is the custom-house value of $500 in English money?
Ans. £103 6s. 1d. $1\frac{1}{2}$+qr.

5. What is the commercial value of $862.50 in English money, £1 being worth $4.85? Ans. £177 16s. 8d. 1+qr.

6. What is the custom-house value of 10000 francs?
Ans. $1860.

7. What is the commercial value of 750 francs, that of 1 franc being $18\frac{3}{4}$ cents? Ans. $140.62\frac{1}{2}$.

8. What is the value of 1575 silver rubles? Ans. $1181.25.

9. What is the value of $9540 in silver rubles?
Ans. 12720 rubles.

10. What is the custom-house value of 1820 Prussian thalers?
Ans. $1255.80.

11. Convert 4000 ducats to U.S. money. Ans. $3200.

12. Convert $2500 to florins. Ans. 6250 florins.

13. Convert 1936 Bengal rupees to £. Ans. £200.

14. Convert £700 to Bengal rupees. Ans. 6776.

15. Convert 1000 India pagodas to florins. Ans. 4600.

16. Convert 4784 florins to India pagodas. Ans. 1040.

17. Convert £3700 to taels. Ans. 12100.

18. Convert 2420 taels to £. Ans. £740.

QUESTION.—246. How do we reduce one currency to another?

EXCHANGE.

Art. 247. EXCHANGE is the method of effecting a transfer of money to a person in a distant place by means of *drafts*.

A *draft* is a written order, directed by one person to another, to pay money to a third person. It is also called a *check*, and, when it is directed to a person in a distant place, it is called a *bill of exchange*.

The *drawer* is the person who makes the order.

The *drawee* is the person to whom the order is directed.

An *inland draft* or bill of exchange is one of which the drawer and drawee reside in the same country.

This is sometimes called *home* or *domestic exchange*.

A *foreign* bill of exchange is one of which the drawer and drawee reside in different countries.

Exchange is *at par* when the price of a draft is the sum ordered in it, or the *face* of it. It is *at a premium* or *above par* when the price of it is more, and *at a discount* or *below par* when the price of it is less, than the face of it.

The *rate of exchange* is a per cent. of the face of the draft, reckoned as premium or discount.

NOTE.—Of two cities or countries compared, the one that imports from the other more than it exports to it will pay a premium for its exchange upon the other, while the one that exports to the other more than it imports from it will get exchange upon the other at a discount.

INLAND EXCHANGE.

Art. 248. To find the value of an inland draft, or bill of exchange.

RULE.—*Multiply the face of the draft by the rate of exchange, and add the product to the face in the case of premium, or subtract the product from the face in the case of discount.*

EXAMPLES FOR PRACTICE.

1. At $\frac{1}{8}$ per cent. premium, what was paid for the following draft? Ans. $725.70,6.

QUESTIONS.—247. What is exchange? A draft? The drawer? The drawee? An inland draft? A foreign bill of exchange? When is exchange at par? At a premium? At a discount? What is the rate of exchange?—248. How do we find the value of an inland draft?

$724\frac{80}{100}$. PITTSBURGH, May 19, 1859.

Please to pay, at sight, to the order of Horace Simpson, seven hundred and twenty-four dollars eighty cents, and charge the same to the account of

S. NEWMAN & SONS.

To Messrs. GRAY, M'NEAL & Co., Bankers, New York.

2. What would be paid for the foregoing, at $\frac{1}{8}\%$ discount?
Ans. $723.89,4.

3. If the foregoing draft had been "thirty days after sight," the premium 1%, and Gray & Co. had, on sight, discounted it at 12% per annum for 33 days, how much less would the receiver get than the sender paid? Ans. $15.22+.

4. What costs a sight draft of $8350.25, at $\frac{3}{8}\%$ premium?
Ans. $8381.56+.

5. What costs a 30 day bill of $5625.75, at $\frac{3}{4}\%$ discount, interest off at 6%? Ans. $5552.61+.

NOTE.—Compute interest and discount on the face of the note.

6. What costs a 60 day check of $732.87, at $1\frac{1}{4}\%$ premium, interest off at 6%? Ans. $734.33+.

FOREIGN EXCHANGE.

Art. 249. To secure against loss or delay in sending a foreign bill of exchange, two or more similar ones are drawn, and sent by different conveyances. These are called *First of Exchange, Second of Exchange,* &c., and each states that it is payable only in case the others are unpaid. When one of this set has been paid, or accepted, by the drawee, the others are worthless. The sum mentioned in a bill of exchange is in the currency of the country to which it is directed.

The *par of exchange* is the *intrinsic* value of the money of one country in the denominations of that of another.

The *course of exchange* at any time is the number of units of one money paid at that time, in its country, for a certain number of units of another, paid in its country.

Thus, $1 paid in the United States may, at one time, purchase the payment in Paris of 5 francs 20 centimes, at another 5 francs 12 centimes, &c.

QUESTIONS.—249. How are foreign bills of exchange made and sent? What is the par of exchange? The course of exchange?

TABLE OF SOME FOREIGN CURRENCIES.

FRANCE.—1 franc = 10 decimes = 100 centimes = 20 sous = ... $0.18.6.
HAVANA and SPAIN.—1 dollar = 8 reals plate = 20 reals vellon = ... $1.
LISBON and BRAZIL.—1 milree = 1000 rees = ... $1.12.
BELGIUM.—1 florin = 40 grotes = 100 centimes = 20 stivers = ... $0.40.
GENOA.—1 lira = 100 centesimi = ... $0.18,6.
LEGHORN.—1 lire = 20 soldi = 240 denari = ... $0.87.
NAPLES.—1 ducato = 10 carlini = 100 grani = ... $0.80.
RUSSIA.—1 silver ruble = 100 copecs = ... $0.75.
CONSTANTINOPLE.—1 piastre = 40 paras = 120 aspers = ... $0.05.
TRIESTE.—1 florin = 60 creutzers = 240 pfennings = ... $0.48½ nearly.
VENICE.—1 lira = 100 centesimi = 1000 millesimi = ... $0.16
BERLIN.—1 thaler = 30 groschen = 360 pfennings = ... $0.69.
BRITISH INDIA.—1 rupee = 16 annas = 192 pice = ... $0.44½.
CANTON.—1 tael = 10 mace = 100 candarines = 1000 cash = ... $1.48.

Art. 250. To compute foreign exchange.

RULE.—*To the value of a unit of the foreign money add its premium, or from it subtract its discount, and then proceed as in reduction of currencies.*

EXAMPLES FOR PRACTICE.

1. What is the value of the following bill of exchange at 9½ per cent. premium? Ans. $4866.66⅔.

Exchange £1000. NEW YORK, June 16, 1859.

At sight of this first of exchange (second and third of the same date and tenor unpaid) pay to the order of Wm. Hill one thousand pounds, and place the same to our account.

To Messrs. BARING & BLISS, Bankers, London. } BALDWIN & SHERMAN.

2. What is the value of a bill of exchange on Paris for 1700 francs at 5⅓ francs for a dollar, exchange on Paris being 1% discount? Ans. $315.56¼.

3. Of a bill from Paris on New York for $500 at 5⅓ francs for a dollar, exchange on New York being 3% premium?
Ans. 2746 francs 66⅔ centimes.

4. Of a bill on Amsterdam for 2000 florins, 5 florins being equal to $2, exchange being 2½% premium?
Ans. $820.

5. Of a bill from Amsterdam on New York for $168.10,5, exchange being 5% premium? Ans. 441fl. 5st.+.

6. Of a bill from Lisbon on New Orleans for $720, exchange being 5% discount? Ans. 610⅔ milrees.

QUESTION.—250. How do we compute foreign exchange?

ARBITRATION OF EXCHANGE.

Art. 251. ARBITRATION OF EXCHANGE is the method of effecting exchange with a country through the currencies of other countries, that is, by a *chain*, or succession, of exchanges.

Art. 252. To arbitrate exchange.

Ex. 1. A merchant in New York wishes to pay a debt in Canton, China, of 1000 taels, through London and Calcutta, exchange on London being 10% premium, from London to Calcutta £1 for 11 rupees, and from Calcutta to Canton 7 rupees for 2 taels. How many dollars will this cost him in New York?

Ans. $\$1555.55\frac{5}{9}$.

PROCESS INDICATED.

$$1000 \times \tfrac{7}{2} \times \tfrac{1}{11} \times \$4.44\tfrac{4}{9} \times 1.10 = \$1555.55\tfrac{5}{9}.$$

MENTAL PROCESS.—1000 taels in China will cost him $\frac{7}{2}$ as many rupees : $\frac{7}{2}$ of 1000 rupees = $\frac{1}{11}$ as many pounds : $\frac{1}{11}$ of $\frac{7}{2}$ of 1000 pounds = $\frac{1}{11}$ of $\frac{7}{2}$ of $1000 \times \$4.44\frac{4}{9} \times 1.10 = \$1555.55\frac{5}{9}$.

RULE.—*Multiply the given sum by the ratios of the currencies intervening between it and the money sought.*

EXAMPLES FOR PRACTICE.

2. If I wish to pay in Lisbon 1680.5 milrees, and the direct ratio is $112 for 100 milrees; but to Liverpool it is $4.84 to £1, from Liverpool to Paris £1 to 25 francs, from Paris to Lisbon 6 francs to 1 milree, what should I lose by remitting successively through Liverpool and Paris (excluding brokerage) instead of directly? Ans. $69.90,8.

3. If A in London draws 1500 dollars Spanish on E in Madrid by ordering him to remit to D in Amsterdam, and ordering B in Paris to draw on D in Amsterdam, and remit to A in London; how much does he gain by this circuit, 1 Spanish dollar being worth 2 florins 15 stivers, 11 florins worth 26 francs, and 25 francs worth £1; but *directly* 40 Spanish dollars being worth £9? Ans. £52 10s.

4. A of New York collects 39200 thalers of C of Berlin, through B of London. What draft does B send to A, allowing 7 thalers 5 groschen for £1, and $4.86 for £1, B charging $\frac{1}{4}$% for brokerage? Ans. $26516.61+.

QUESTIONS.—251. What is arbitration of exchange?—252. How do we arbitrate exchange?

CHAPTER XIX.

ALLIGATION.

Art. 253. ALLIGATION is that part of Arithmetic which treats of compounding different ingredients with reference to their values and that of the compound.

CASE I.

Art. 254. To find the value of a unit of the mixture, when the ingredients and their prices are known.

RULE.—*Find the value of each ingredient, and divide the sum of their values by the sum of the ingredients.*

NOTE.—This is merely a case of *average*, and is generally called *Alligation Medial.*

EXAMPLES FOR PRACTICE.

1. If I mix 10lb. tea at 50cts. per lb. with 30lb. at 75cts. per lb., and 50lb. at $1 per lb., what is 1lb. of the mixture worth?

Ans. 0.86\frac{1}{3}$.

2. If I mix 18gal. alcohol 75% strong with 30gal. 95% strong, 12gal. 85% strong, and 6gal. 98% strong, what per cent. of alcohol in the mixture? Ans. 88%.

CASE II.

Art. 255. To find the quantities of ingredients, of known value, required to make a mixture of a given value.

Ex. 1. I have alcohol which cost 80cts. per gallon; how much water must be added to make it cost 60cts. per gallon?

Ans. $\frac{1}{3}$ as much water as alcohol.

QUESTIONS.—253. What is Alligation?—254. How do we find the value of unit of the mixture when the ingredients are known?

WRITTEN PROCESS.

$$60\begin{cases}80) & 60 \text{ of alcohol.}\\ 00) & 20 \text{ of water.}\end{cases}$$

PROOF. 60gal. × .80 = $48.00
20 " × 0 = 0
80) 48.00
.60

EXPLANATION.

The alcohol at 60cts. loses 20cts. per gal.; the water at 60cts. gains 60cts. It therefore takes 3gal. alcohol to lose as much as 1gal. water gains. The written process obtains *one set* of quantities in this ratio, by assuming 20 for water, since water must gain 20 for the alcohol; and by assuming 60 for alcohol, since it must lose 60 for the water.

Ex. 2. Four simples being 4cts., 6cts., 8cts., and 10cts. per unit, how much of each will make a unit of the mixture worth 7cts.?

RATIO ANSWERS.

1st. An equal quantity of each.
2d. The 4 and 10ct. each $\frac{3}{8}$, the 6 and 8ct. each $\frac{1}{8}$ of the whole.
3d. The 4 and 10ct. each $\frac{1}{8}$, the 6 and 8ct. each $\frac{3}{8}$ of the whole.
4th. The 4 and 10ct. each $\frac{4}{14}$, the 6 and 8ct. each $\frac{3}{14}$ of the whole.
5th. The 4 and 10ct. each $\frac{1}{10}$, the 6 and 8ct. each $\frac{4}{10}$ of the whole.
6th. The 6 and 10ct. each $\frac{4}{12}$, the 4ct. $\frac{3}{12}$, the 8ct. $\frac{1}{12}$ of the whole.
7th. The 4 and 8ct. each $\frac{4}{12}$, the 6ct. $\frac{1}{12}$, the 10ct. $\frac{3}{12}$ of the whole.

ILLUSTRATIVE PROCESSES.

$$7\begin{cases}4\\6\\8\\10\end{cases}\quad\begin{aligned}1+3&=4\\1+3&=4\\3+1&=4\\3+1&=4\\ &\overline{16}\end{aligned}\qquad 7\begin{cases}4\\6\\8\\10\end{cases}\quad\begin{aligned}&3\\&1\\&1\\&3\\ &\overline{8}\end{aligned}\qquad 7\begin{cases}4\\6\\8\\10\end{cases}\quad\begin{aligned}&1\\&3\\&3\\&1\\ &\overline{8}\end{aligned}$$

$$7\begin{cases}4\\6\\8\\10\end{cases}\quad\begin{aligned}3+1&=4\\&\quad 3\\&\quad 3\\3+1&=4\\ &\overline{14}\end{aligned}\qquad 7\begin{cases}4\\6\\8\\10\end{cases}\quad\begin{aligned}&1\\3+1&=4\\3+1&=4\\&1\\ &\overline{10}\end{aligned}$$

$$7\begin{cases}4\\6\\8\\10\end{cases}\quad\begin{aligned}&3\\3+1&=4\\&1\\3+1&=4\\ &\overline{12}\end{aligned}\qquad 7\begin{cases}4\\6\\8\\10\end{cases}\quad\begin{aligned}3+1&=4\\&1\\3+1&=4\\&3\\ &\overline{12}\end{aligned}$$

EXPLANATION.—In the first process, if we connect the 4 with the 8 and consider these exclusively, 4 gains 3cts., and 8 loses 1ct. Therefore 3 of 8 and 1 of 4 will give a mixture at 7cts. Proof, $3 \times 8 + 4 \times 1 = 28$; $28 \div (3 + 1) = 7$. Again, connect 4 with 10, and consider them exclusively; 4 gains 3cts., and 10 loses 3cts. Therefore 3 of each will give a mixture at 7cts. Proof, $3 \times 10 + 3 \times 4 = 42$; $42 \div (3 + 3) = 7$. Now, connecting 6 with 10 and 8, we get in the same way 1 of 10 and 3 of 6, and 1 of 8 and 1 of 6. Of each simple, therefore, we get $3 + 1 = 4$: that is, *any* equal quantity of each will answer.

The other six processes illustrate other methods of connecting the simples.

RULE.—*Write the prices of the ingredients in a column on the right of a vertical line, and the price of the mixture on the left of it.*

Draw a line connecting each price less with one or more of those greater than that of the mixture.

Write the difference between the price of the mixture and that of each ingredient opposite every price with which it is connected.

The sum of such differences opposite any price may be taken as the quantity of that ingredient; or the ratio of the sum of such differences opposite any price to the sum of the differences opposite all the prices is the ratio of that ingredient to the whole mixture.

PROOF.—Multiply the obtained quantities by the prices, and divide the sum of the products by the sum of the quantities. The quotient should be the price of the mixture.

NOTE 1.—Such questions have an unlimited number of *particular* answers.

NOTE 2.—This and the following cases are often called *Alligation Alternate.*

EXAMPLES FOR PRACTICE.

3. What relative quantities of sugar costing 8, 9, and 10cts. per lb. will make a mixture costing 9¼cts. per lb.?

Ans. 1 of 8ct.; 1 of 9ct.; 2 of 10ct.

4. In what proportion must water be mingled with spirits to reduce the cost from 85cts. to 55cts. per gallon?

Ans. 6 of water; 11 of spirits.

5. In what proportion should copper costing 24cts. per lb. be melted with zinc costing 6cts. per lb., to make brass at 16cts. per lb., exclusive of the cost of manufacture?

Ans. 4 zinc; 5 copper.

QUESTION.—255. How do we find the quantities of ingredients when the rice and that of the mixture are known?

Case III.

Art. 256. To find the quantities of ingredients, of known values, required to make a certain quantity of mixture of given value.

Ex. 1. How can simples at 4, 6, 8, and 10cts. be mixed so as to make 48 units at 7cts. per unit?

Explanation.—This is Ex. 2 of Case II., with the *quantity* of the mixture known. For one solution, take the 2d set of ratios. The 4ct. simple is $\frac{3}{8}$ of the whole. Now, if 48 is the whole, $\frac{3}{8}$ of 48, that is, 18, is the quantity for the 4 and 10ct. simples; and $\frac{1}{8}$ of 48, that is, 6, is the quantity for the 6 and 8ct. simples. The other ratios give other quantities.

Rule.—*Find the ratios by Case II., and take those fractions of the given whole mixture.*

Examples for Practice.

2. How can pure gold (24 carats fine) be melted with alloys at 18 carats, 16 carats, and 14 carats, to make 3lb. of 15 carats fine?

Ans. Of 24c., 18c., and 16c., each $2\frac{1}{4}$oz.; of 14c. 2lb. $5\frac{1}{4}$oz.

3. It is related of Archimedes that he detected for Hiero, King of Syracuse, the dishonesty of his jeweler in making a crown. If the king gave the jeweler $\frac{1}{2}$lb. of silver $10\frac{1}{2}$ times heavier than water, and 5lb. of gold $19\frac{1}{4}$ times heavier than water, and the crown, when returned, was $14\frac{5}{8}$ times heavier than water, though weighing as much as the ingredients furnished, how much of the king's gold had the jeweler replaced by silver? Ans. 2lb. 4oz. 17pwt. $17\frac{1}{7}$gr.

Case IV

Art. 257. Knowing the quantity of one ingredient, the prices of all, and the price of the mixture, to find the quantities of the other ingredients and of the mixture.

Remark.—This is a case of particular supposition, one of which is discussed under Case II

Rule.—*Find the ratios by Case II., and from the given value of one ratio determine the values of the other ratios and of the whole.*

Questions.—256. What is Case III? What is the rule?—257. What is Case IV.? What is the rule?

EXAMPLES FOR PRACTICE.

1. If you buy 80bbl. flour at $7.20 per bbl., and flour falls to $5 per bbl., how many more should you then buy, so as to sell without loss when it rises to $6 per bbl.? Ans. 96bbl.

2. How much water will reduce 29gal. of alcohol 96% strong to 87% strong? Ans. 3gal.

CASE V.

Art. 258. Knowing the price of each ingredient and of the mixture, and the quantities of two or more ingredients, to find the quantities of the other ingredients and of the mixture.

Ex. 1. I bought of a certain stock at one time 4 shares at $80, at another 8 shares at $89. The stock falling to $60, how many shares must I buy at that price to sell the whole without loss when the stock rises to $73? Ans. 12 shares.

WRITTEN PROCESS.

$4 \times 80 = 320$
$8 \times 89 = 712$

12)1032

86

$73 \begin{cases} 60 \\ 86 \end{cases}$ 13 13

26

$13 : 13 :: 12 : 12$

PROOF.—12 shares @ $86 = $1032
12 " " 60 = 720

24 ")1752

73

EXPLANATION.

By Case I. each of my 12 shares cost $86. I may consider these 12 shares as one ingredient at the price $86. Then, by Case II., 12 shares are $\frac{13}{26}$ of the whole. Then $\frac{13}{26} : \frac{13}{26} :: 12$ shares : 12 shares.

RULE.—*Find the quantity and average price of the known ingredients by Case I., and treat the result as the quantity and price of a single ingredient by Case IV.*

EXAMPLES FOR PRACTICE.

2. How much alcohol 96% strong must be put with 42gal. 75% strong and 14gal. 68% strong, to raise the mixture to 82% strong? Ans. 35gal.

3. Having 2pwt. gold 22 carats fine, and 4pwt. 14 carats fine, how much of each of two other kinds, 18 and 15 carats fine, must be taken to make the alloy 17 carats fine?

Ans. 14pwt. of 18c.; 6 pwt. of 15c.

QUESTIONS.—258. What is Case V.? What is the rule?

CHAPTER XX.

INVOLUTION. EVOLUTION.

INVOLUTION.

Art. 259. INVOLUTION is the process of finding a power of a number.

A *power* of a number is a product whose factors are that number. Multiplying these factors is called *involving* the number. The number itself is called the *root* of the power.

Art. 260. Powers are named from the number of times the root is used as a factor in producing them; thus,

2, used once, gives 2, the 1st power.
2, used twice, gives $2 \times 2 =$ 4, the 2d power, or *square*.
2, used three times, gives $2 \times 2 \times 2 =$ 8, the 3d power, or *cube*.
&c. &c.

The *index* or *exponent* of a power is that number which indicates its degree or name. It is a small number placed at the right of the root; thus, 7^2 indicates the square of 7; $(\frac{3}{4})^5$ indicates the 5th power of $\frac{3}{4}$, &c.

NOTE.—The second power of a number is called its *square*, because the process of finding the area of a square surface is like that of finding the second power of a number expressing the side of a square. The third power of a number is called its *cube*, because in the process of finding the volume of a cube we use the length of its edge three times as a factor, as in finding the third power.

A *perfect* power is a number which can be produced by the multiplication of the proper number of equal factors. Thus, 25 is a *perfect square*, being equal to 5×5.

An *imperfect* power is a number which cannot be produced by the proper number of equal factors. Thus, 25 is an *imperfect cube*.

Art. 261. *The product of two or more powers of a number is a power of that number indicated by the sum of the indices of the powers used as factors;* thus, $4^2 \times 4^3 = (4 \times 4) \times (4 \times 4 \times 4) = 4 \times 4 \times 4 \times 4 \times 4 = 4^5 = 4^{2+3}$.

QUESTIONS.—259. What is Involution? A power? Involving? What is the involved number called?—260. How are powers named? What is an index? What is a perfect power? An imperfect power?—261. What is the product of two or more powers? Show it.

Art. 262. To find a required power of a number.

RULE.—*Use the number as a factor as many times as is indicated by the name of the power.*

NOTE.—The labor of finding high powers may sometimes be lessened by using the principle in Art. 261. Thus, to find the 4th power, *multiply the square by itself;* to find the fifth power, *multiply the square by the cube,* &c.

EXAMPLES FOR PRACTICE.

	Find				Find	
1.	16^2	Ans.	256.	9.	$(\frac{2}{3})^4$	Ans. $\frac{16}{81}$.
2.	9^3	Ans.	729.	10.	1.2^3	Ans. 1.728.
3.	5^4	Ans.	625.	11.	$(3\frac{2}{3})^5$	Ans. $662\frac{185}{243}$.
4.	4^5	Ans.	1024.	12.	$(\frac{1}{6})^7$	Ans. $\frac{1}{4782969}$.
5.	3^6	Ans.	729.	13.	$(.003)^3$	Ans. .000000027.
6.	2^7	Ans.	128.	14.	8^8	Ans. 16777216.
7.	1^8	Ans.	1.	15.	$(3^4)^2$	Ans. 3^8 or 6561.
8.	25^1	Ans.	25.	16.	101^5	Ans. 10510100501.

Art. 263. *The square of a number has twice as many figures as the number, or one less than twice as many.*

ILLUSTRATION.—The least integer of one figure is 1, and its square is 1; the greatest is 9, and its square is 81. The least integer of *two* figures is 10, and its square is 100, a number of *three* figures; the greatest is 99, and its square is 9801, a number of *four* figures, &c. Therefore the squares of all numbers between 1 and 9 must be either of one or two figures; and those of numbers between 10 and 99 must be either of three or four figures, &c.

Art. 264. *The cube of a number cannot have more than three times as many figures as the number, nor fewer than three times as many less two.*

ILLUSTRATION.—Least integer 1, its cube 1; greatest 9, its cube 729, a number of three figures. Therefore the cubes of numbers between 1 and 9 must be of one, two, or three figures. Least integer of two figures 10, its cube 1000; greatest 99, its cube 970299. Therefore the cubes of numbers between 10 and 99 must be of four, five, or six figures.

Art. 265. *The square of any number is equal to the square of the local value of its first left-hand figure; plus twice the product of its value into that of the second; plus the square of that of the second; plus twice the product of that of the first and second into the local value of the third; plus the square of*

QUESTIONS.—262. How do we find a power of a number?—263. How many figures has the square of a number?—264. How many figures may a cube have?

that of the third; plus twice the product of those of the first three into that of the fourth; plus the square of that of the fourth; and so on.

ILLUSTRATION.

$$\begin{array}{rl} 123= & 100+20+3 \\ 123= & 100+20+3 \\ \hline 369= & (100\times3)+(20\times3)+3^2 \\ 2460= & (100\times20)+20^2+(20\times3) \\ 12300= & 100^2+(100\times20)+(100\times3) \\ \hline 15129= & 100^2+2(100\times20)+20^2+2(100\times3)+2(20\times3)+3^2 \end{array}$$

Art. 266. *The cube of any number is equal to the cube of the local value of its first left-hand figure; plus three times the square of that of the first into that of the second; plus three times that of the first into the square of that of the second; plus the cube of that of the second; plus three times the square of that of the sum of the first two into that of the third; plus three times that of the sum of the first two into the square of that of the third; plus the cube of that of the third; and so on.*

ILLUSTRATION.

$$\begin{array}{rl} 529=23^2= & 20^2+2(20\times3)+3^2 \\ 23= & 20+3 \\ \hline 1587= & (20^2\times3)+2(20\times3^2)+3^3 \\ 10580= & 20^3+2(20^2\times3)+(20\times3^2) \\ \hline 12167=23^3= & 20^3+3(20^2\times3)+3(20\times3^2)+3^3 \end{array}$$

Let the pupil illustrate the principle by the cube of a number of more than two figures.

EVOLUTION.

Art. 267. EVOLUTION is the process of finding a root of a number.

A *root* of a number is that number which, used as a factor a certain number of times, will produce it.

Art. 268. Roots are named from the number of times they must be used as factors to produce a number.

Thus, the *square root* must be used as a factor *twice*, the cube root *three times*, the fourth or biquadrate *four* times, &c.

QUESTIONS.—265. What does the square of any number equal?—266. What does the cube of any number equal?—267. What is Evolution? A root?—268. How are roots named?

Rational roots are roots which can be exactly obtained.

Surd roots, or *surds*, are roots which cannot be exactly obtained.

The symbol $\sqrt{\ }$ is called the *radical sign*, and signifies the *square root* of the number before which it is placed. Thus, $\sqrt{25}$ is read *the square root of* 25.

The radical sign, with a small figure over it, signifies other roots. Thus $\sqrt[3]{27}$ is read *the cube root of* 27; $\sqrt[4]{81}$ signifies the *fourth root of* 81, &c.

The *index* or *exponent* of a root is that number which indicates its degree or name. It is often written as a denominator to the index of a power. Thus $9^{\frac{1}{2}}$ signifies the square root of the first power of 9, that is, $\sqrt{9}$; and $8^{\frac{2}{3}}$ signifies the cube root of the square of 8, that is, $\sqrt[3]{64}$, &c.

EXTRACTION OF THE SQUARE ROOT.

Art. 269. To extract the square root of a number is to resolve that number into two equal factors.

The multiplication table furnishes the following

ROOTS.	1	2	3	4	5	6	7	8	9	10
SQUARES.	1	4	9	16	25	36	49	64	81	100

CASE I.

Art. 270. To extract the square root of an integral, decimal, or mixed number.

Ex. 1. What is the square root of 529? Ans. 23.

WRITTEN PROCESS.

$$\begin{array}{r} \overbrace{529}(23 \\ 4 \\ \hline 43)129 \\ \quad 129 \\ \hline \end{array}$$

EXPLANATION.

By Art. 263, the root of 529 must contain two figures, because the units of the root could not, when squared, make the *three* figures in 529. Hence, also, we must get the units of the root from 29, and its tens from 5. Now, the greatest perfect square in 5 is 4, whose root is 2, that is, 2 *tens*. Put 2 in the root, and take its square 4 from 5, leaving 1; then bring down 29, making 129. By Art. 265, this 129 must contain *twice the product of* 20 *into the units, plus the square of the units.* Therefore, to find the units, we must divide 120 by $2 \times 20 = 40$. The quotient is 3. Now, $3 \times 40 + 3^2 = 129$, which leaves no remainder. In the *written process*, we do both multiplications at once, by annex-

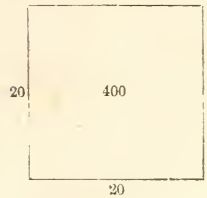
QUESTIONS.—268. What are rational roots? Surds? What is an index or exponent of a root? How is it often written?—269. What is extracting the square root?

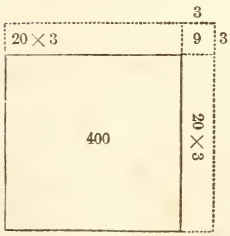
ing 3 to 40, making 43, before multiplying by 3. The operations may be represented thus:—

$$\begin{array}{rl|l} & \widehat{5}\widehat{29} & 20 = 2 \text{ tens.} \\ & 400 & 3 = 3 \text{ units.} \\ 40 & 129 & 23 \text{ Ans.} \\ 2 \times 20 \times 3 = & 120 & \\ 3^2 = & 9 & \\ & 129 & \end{array}$$

ILLUSTRATION BY AREAS.

529 can be represented by the area of a square surface, whose side is required. (See Art. 75, Note 1.) The first part of the root, that is, 20, is an *approximation*, exhausting 400 units of area. This leaves 129 units of area to be added to two sides of the square, 400, to make it equal 529, *still keeping it a square*. This implies *two side-pieces*, each 20 long, and a *square corner-piece* between them with sides equal to their breadth. Therefore the length of the two side-pieces is $2 \times 20 = 40$. Dividing 120, their *approximate* area, by 40, their length, gives 3, their breadth. Now, the three added pieces are





$20 \times 3 = 60$ one side-piece.
$20 \times 3 = 60$ other side-piece.
$3 \times 3 = 9$ corner-piece.
129 total.

This makes the whole completed side $20 + 3 = 23$, and exhausts the area. If this first effort had not exhausted the area, we should have had to apply the remainder to the sides of the area already found, in the same manner, and so on with every successive remainder.

Ex. 2. What is the $\sqrt{1448.5636}$? Ans. 38.06.

WRITTEN PROCESS.

$$\begin{array}{rl|l} & \widehat{14}\widehat{48}.\widehat{56}\widehat{36} & (38.06 \\ & 9 & \\ 2 \times 3 = 6: & 68 \mid 548 & \\ & \quad 544 & \\ 2 \times 38 = 76: & 760 \mid 45636 & \\ 2 \times 380 = 760: & 7606 \mid 45636 & \\ & & \end{array}$$

EXPLANATION. Since this number has a decimal of four places, its square root must have two decimal places. Because, by Art. 148, a *product* has as many decimal places as are in *both factors*, and the factors of a square are the root

taken twice. Since the *tenths* of the root, squared, can furnish no further decimal than *hundredths*, that is, two decimal places, and the *hundredths* of the root, squared. can furnish no more than *four* decimal places, &c., we must get the tenths of the root from the first two decimal places, the hundredths of the root from the next two, &c. Hence, we mark off the decimal by twos from the point to the right. Again, although 6 goes in 54 *nine* times, we write 8 in the quotient, because, *on trial*, 9 gives a product greater than 548. Again, when 76 goes in 45 no times, we write 0 in the root, and also annex it to 76, making 760, then find the next root-figure. This saves a useless multiplying by 0, &c.

RULE.—*Beginning at the right of the units, mark off the number into periods of two figures each; when the number of figures is odd, the last-formed period will have but one figure.*

Find the greatest square number in the left-hand period, write its root as the first figure of the answer, subtract the square from the period, and to the remainder annex the second period for a dividend.

At the left of this dividend write as a trial divisor double the root found; find how many times the trial divisor is contained in all but the right-hand figure of the dividend, and annex the quotient both to the root found, as the next figure of the answer, and to the trial divisor.

Multiply the divisor thus formed by the last figure annexed to the root; subtract the product, if possible, from the dividend, and to the remainder annex the next period for a new dividend.

Double the whole root found, for a new trial divisor, and proceed as before, till all the periods have been used.

When the product is greater than the dividend, erase the figure that produced it, and put a figure of less value in both the root and trial divisor, till the product is small enough for subtraction.

When the trial divisor is not contained in the part of the dividend before mentioned, annex a cipher both to the root and trial divisor, bring down the next period, and proceed as usual.

Point off from the right of the root as many figures for decimals as there were decimal periods used in the operation.

NOTE 1.—If, in marking off, the last decimal place has but one figure, annex a cipher to fill the period.

NOTE 2.—In surds, the root can be closely approximated by using decimal periods of ciphers.

NOTE 3.—In extracting the square root of a large number, we can, after finding more than the first half of the root, lessen labor by constantly dividing the last remainder by the last divisor, except its right-hand figure, as in Art. 152.

NOTE 4.—*The square root of a perfect square may be found by resolving*

it into its prime factors, and finding the product of one of each two equal factors. Thus $64 = (2 \times 2) \times (2 \times 2) \times (2 \times 2)$; and its square root $= 2 \times 2 \times 2 = 8$.

Examples for Practice.

What is the square root

Of	Ans.	Of	Ans.
3. 6561?	81.	10. 1.234?	1.11085+.
4. 10201?	101.	11. 2?	1.4142135+.
5. 152.2756?	12.34.	12. 629?	25.07987+.
6. 3455881?	1859.	13. .000729?	.027.
7. 1521.468036?	39.006.	14. .000001?	.001.
8. 4326400?	2080.	15. Find $(36)^{\frac{3}{2}}$.	216.
9. 23.222761?	4.819.	16. Find $(\sqrt{65536})^{\frac{1}{2}}$	16.

Case II.

Art. 271. To extract the square root of common fractions and mixed numbers.

Ex. 1. What is $\sqrt{1\frac{21}{144}}$? Ans. $1\frac{1}{12}$.

Process Indicated.

$$\sqrt{\frac{121}{144}} = \frac{\sqrt{121}}{\sqrt{144}} = 1\frac{1}{12}$$

Mental Process.

Since $1\frac{21}{144}$ is assumed as a square, it must be the product of two equal factors. Now, by Art. 120, the numerator 121 must be the square of the numerator, and the denominator 144 must be the square of the denominator of the root. Therefore the terms of the root are found by extracting the root of each term of $1\frac{21}{144}$ separately.

Ex. 2. What is $\sqrt{6\frac{1}{4}}$? Ans. $2\frac{1}{2}$.

First Method.

$$6\frac{1}{4} = \frac{25}{4}. \quad \sqrt{\frac{25}{4}} = \frac{5}{2} = 2\frac{1}{2}$$

Second Method.

$$6\frac{1}{4} = 6.25. \quad \sqrt{6.25} = 2.5 = 2\frac{1}{2}$$

Rules.—I. For a fraction.

Make the square root of the numerator a new numerator, and that of the denominator a new denominator; or,

First reduce the fraction to a decimal, and then find its square root.

II. For a mixed number.

Reduce it to an improper fraction or to a mixed decimal number, and find its square root in this form.

Questions.—270. How do we extract the square root of a number, either integral, decimal, or mixed? What may be done with surds? How can we lessen labor in large numbers? Give note 4.—271. How do we extract the square root of common fractions and mixed numbers?

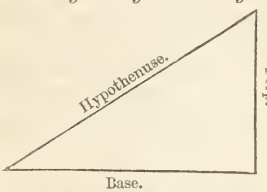
EXAMPLES FOR PRACTICE.

3. Find $\sqrt{\frac{3}{4}\frac{6}{9}}$. Ans. $\frac{6}{7}$.
4. Find $\sqrt{\frac{6}{1}\frac{4}{0}\frac{4}{0}}$. Ans. $\frac{4}{5}$.
5. Find $\sqrt{\frac{8}{2}\frac{1}{2}\frac{1}{5}}$. Ans. $\frac{3}{5}$.
6. Find $\sqrt{169\frac{1}{16}}$. 13.0024+.
7. Find $\sqrt{\frac{2}{5}}$. Ans. .632+.
8. Find $\sqrt{196\frac{9}{100}}$. A.14.003+
9. Find $\sqrt{5\frac{2}{3}}$. Ans. 2.38+
10. Find $\sqrt{596\frac{3}{4}\frac{7}{9}}$. Ans. $24\frac{3}{7}$.
11. Find $\sqrt{\frac{83\frac{2}{17}}{452\frac{9}{17}}}$. Ans. $\frac{3}{7}$.

APPLICATIONS OF SQUARE ROOT.

Art. 272. A PLANE TRIANGLE is a plane figure bounded by three straight lines called its *sides*.

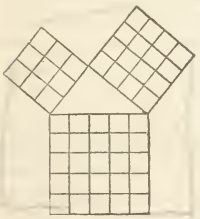
A *right-angled triangle* is a triangle two of whose sides are perpendicular to each other.



The *hypothenuse* is the longest side of a right-angled triangle, and is opposite the right angle. The *base* is the side on which the figure is supposed to rest.

Art. 273. *The square of the hypothenuse is equal to the sum of the squares of the perpendicular sides.*

ILLUSTRATION.



3 linear units squared = 9 sq. units.
4 " " " = 16 " "

Sum = 25 " "
Hyp. (5 lin. un.) sq. = 25 " "

The example here given is only an instance of a general truth, proved in Geometry.

RULES.—I. To find the hypothenuse.

Extract the square root of the sum of the squares of the other two sides.

II. To find one perpendicular side.

From the square of the hypothenuse take the square of the given side, and extract the square root of the remainder.

III. To find the side of a square equal to a given area.

Find the square root of that area.

QUESTIONS.—272. What is a plane triangle? What is a right-angled triangle? The hypothenuse? The base?—273. To what is the square of th hypothenuse equal? How do we find the hypothenuse? How one perpendicu lar side? How the side of a square of given area?

Examples for Practice.

1. How long is the side of a square acre?

Ans. 12rd. 10ft. 8.52+in.

2. How long is the side of a square ten-acre field?

Ans. 40rd.

3. How much longer is the side of a square whose area is 2 sq. miles, than that of a sq. mile? Ans 132rd. 9ft. 0.57+in.

4. If an army of 7076 men, including 20 officers, should arrange the private soldiers in a solid square, how many men would stand in rank and file? Ans. 84.

5. How long must a brace be which is inserted into two timbers, at the distance from their right-angled junction of 15 inches on one and 18 inches on the other? Ans. 23.43+in.

6. What is the length of a rafter, the roof being 10ft. high and the building 30ft. wide? Ans. 18ft. $\frac{3}{4}$in. nearly.

7. How high will a ladder 65ft. long reach upon a building, its foot resting 9ft. from the building?

Ans. 64ft. $4\frac{1}{2}$in. nearly.

8. What is the longest straight line that can be drawn on the floor of a room 16ft. long and 14ft. broad?

Ans. 21ft. $3\frac{1}{10}$in. nearly.

9. What is the longest straight line that can be contained in the same room if it is 12ft. high? Ans. 24ft. 5in. nearly.

10. If a boy can reach 7ft. from the ground, and swings on a rope 48ft. long, attached to the top of a pole 50ft. high, what is the greatest distance that he can touch ground from the bottom of the pole? Ans. 22ft. 11in. nearly.

11. A farmer, having a lot 80rd. long by 20rd. wide, removed the fence to enclose an equal square: how many rails had he left if 15 rails fenced 2rd.? Ans. 300 rails.

12. How many more bricks will it take to build the walls of a house 75ft. long, 25ft. wide, and 25ft. high, than a square house of the same area of floor and the same height, allowing 18 bricks per square foot of wall? Ans. 12058 nearly.

Art. 274. *The areas of similar figures are to each other as the squares of the like dimensions of the figures.* Thus, a square whose side is twice that of another has four times the area; a circle whose diameter is twice that of another has four times the area; three times the diameter, nine times the area, &c. This is proved in Geometry.

Question.—274. How are the areas of similar figures to each other?

EXAMPLES FOR PRACTICE.

1. How much larger is a circle whose diameter is 5in. than a circle whose diameter is 2in.? Ans. $6\frac{1}{4}$ times.

2. What is the hole bored by an auger $1\frac{1}{2}$in. diameter, compared with that bored by a gimlet $\frac{1}{8}$in. in diameter?

Ans. 144 times larger.

3. If a hose 1in. diameter discharge a barrel of water in 5 minutes, how large must the hose be which would discharge 3 barrels in the same time and under the same circumstances?

Ans. 1.73+in.

4. If a half-inch pipe fill a cask in $11\frac{1}{4}$ minutes, how long would it take a $\frac{3}{4}$-inch pipe? Ans. 5 minutes.

5. How much more weight ought a round bar of metal $1\frac{1}{2}$in. in diameter to sustain, than a wire of the same metal $\frac{1}{12}$ of an inch in diameter? Ans. 225 times.

6. If a bar of iron 1in. square sustain 68000lb., how large must be a square bar of the same material, to sustain $128562\frac{1}{2}$lb.?

Ans. $1\frac{3}{8}$in.

7. If, with a certain steam-pressure, the force on a piston-head 5in. in diameter is $1963\frac{1}{2}$lb., how large must be the piston which, with the same steam-pressure, would receive 11309.76lb.?

Ans. 12in. diameter.

8. If a hall 100ft. long and 50ft. broad can accommodate 1500 persons, how large must the floor of a similar hall be to accommodate 1000 persons?

Ans. 81ft. 7.7in. long, 40ft. 9.8in. broad.

9. If a car whose end is 8ft. high and 9ft. broad meets at a certain velocity in still air a resistance, from the air, of 300lb., how large must be the end of a similar car, to receive, in the same circumstances, a resistance of 400lb.?

Ans. 9ft. 2.8in. high, 10ft. 4.7in. broad.

10. If a triangular sail, 16ft. broad at base, move a certain boat in a wind of a certain velocity, 8 miles per hour, how broad must a similar sail be to move the same boat 10 miles per hour under the same circumstances, not reckoning any increased resistance of the water? Ans. 17ft. 10.6+in.

EXTRACTION OF THE CUBE ROOT

Art. 275. To extract the cube root of a number is to resolve that number into three equal factors.

QUESTION.—275. What is extracting the cube root of a number?

The integers from 1 to 10 inclusive are the cube roots of the numbers here set beneath them.

Roots,	1	2	3	4	5	6	7	8	9	10.
Cubes,	1	8	27	64	125	216	343	512	729	1000.

Case I.

Art. 276. To extract the cube root of an integral, decimal, or mixed number.

Ex. 1. What is the cube root of 12167? Ans. 23.

Written Process.

$\overbrace{12}\overbrace{167}(23$

Cube of the tens = 8

$3\times20^2 =$ 1200 trial d'r. | 4167

$3\times20\times$units $3 =$ 180 | 4167

(units $3)^2 =$ 9

1389 true d'r.

Explanation.

By Art. 264, this number must have a root of two figures,—viz., of *units* and *tens;* and the three right-hand figures (167) must contain the cube of the *units* of the root; the other two figures (12) must contain the cube of the *tens*. Therefore, in general, each *group of three figures* successively toward the left, must furnish a figure of the cube root. Hence, mark off the number from units into groups of three. By Art. 266, this number must contain

1st, The cube of the tens.

2d, 3 times the square of the tens $\times$ the units.

3d, 3 times the tens $\times$ the units $\times$ the units.

4th, The cube of the units = square of the units $\times$ the units.

The root of 8, the greatest cube in the second period, (12,) will give 2, the *tens* of the whole root. Taking 8 from 12, we have 4167 from which the units of the root must be obtained. This 4167 must contain the last three of the before-mentioned products, of which the *units* are one factor.

Hence, dividing by one of the factors,—*viz.*, $3\times$ *sq. of the tens*,—we shall *probably* obtain the other factor,—viz., the *units*. On filling out the required products, they do not amount to more than the dividend: hence 3 is the units of the root. But if the products filled out should in any case amount to more than the dividend, it is evident that we must take a less figure for the units. In this written process, we delayed multiplying by the units till we found the sum of the other factors, thus multiplying them all at once; but each product could have been filled out by itself. Thus performed, the whole operation would be represented in full, thus:—

			$\overbrace{12167}$	20
Cube of the tens =			8000	3
3×20^2	= 1200 trial divisor.		4167	23
$3 \times 20^2 \times 3$	=	3600		
$3 \times 20 \times 3^2$	=	540		
3^3	=	27	4167	

Illustration by Solids.

The number 12167 can be represented by a cubical block whose volume is found by cubing the number representing the length of one edge, and it is required to find that edge. The first part of the root—viz., 20—is an *approximation*, exhausting $20 \times 20 \times 20 = 8000$ units of volume. (See fig. 1.) This leaves 4167 units of volume to be added to the sides of the approximating cube 8000, to make it equal to 12167, *still keeping it a cube.*

Fig. 1.

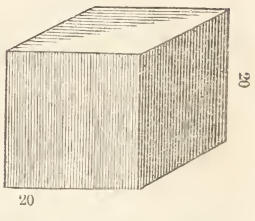
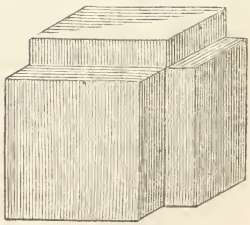


Fig. 2.



This implies three side-slabs, each having one side $20 \times 20 = 400$. (See fig. 2.) Also three equal corner-pieces, each 20 long, and having both breadth and thickness equal to the thickness of the slabs. (See fig. 3.)

Fig. 3.

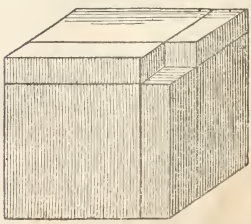
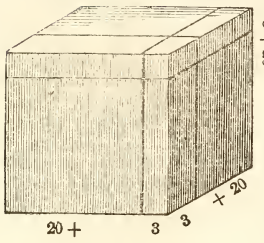


Fig. 4.



Also one corner-cube to fill the vacancy left by the three corner-pieces. (See fig. 4.)

Therefore, dividing the additional volume 4167 by the area of one side of the slab-additions, will give, either nearly or exactly, the thickness of these slabs. In the example it gives it *exactly*,—viz., 3. The added pieces, therefore, are in volume, as follows :—

$20 \times 20 \times 3$ = first side slab,	= 1200
$20 \times 20 \times 3$ = second "	= 1200
$20 \times 20 \times 3$ = third "	= 1200
$20 \times 3 \times 3$ = first corner-piece,	= 180
$20 \times 3 \times 3$ = second "	= 180
$20 \times 3 \times 3$ = third "	= 180
$3 \times 3 \times 3$ = corner-cube	= 27
Total,	4167

Therefore the approximate edge 20, when completed, is $20 + 3 = 23$. It is plain that if the 4167 had not exhausted the given volume, the remainder should have been again applied to the three enlarged sides of the first approximate cube, in the same way as the first remainder was applied; and so with every successive remainder.

Ex. 2. What is the cube root of 12.167? Ans. 2.3.

Explanation. Here the processes must be as before: but, as the period 167 is *decimal*, its root must be *tenths*, because the cube of tenths cannot be lower than thousandths, since $.9 \times .9 \times .9 = .729$. Hence the answer must be 2.3. If there had been three more decimal places in the number, they would have furnished *hundredths* in the root; three more, *thousandths*, &c.; since the cube of hundredths gives *millionths*, or six places, the cube of thousandths gives nine places, &c.

Rule.—*Beginning at the right of the units, mark off the number into periods of three figures each: the last-formed period will have less than three figures, when the number of figures is not divisible by* 3

Find the greatest cube number in the left-hand period, write its cube root as the first figure of the answer, subtract the cube from the period, and to the remainder annex the second period for a dividend.

Square the root found, annex two ciphers, and multiply the result by 3 for a trial divisor.

Find how many times the trial divisor is contained in the dividend, and write the quotient as the next figure of the root.

To the trial divisor add three times the product of the former part of the root (with a cipher annexed) by the last figure of the root, and the square of the last figure.

Multiply this sum by the last figure of the root, subtract the product, if possible, from the dividend, and to the remainder annex the next period for a new dividend. Proceed as before till all the periods have been used.

When the product is greater than the dividend, erase the figure that produced it, and with a figure of less value recalculate the additions to the trial divisor, till the product is small enough for subtraction.

When the trial divisor is not contained in the dividend, annex a cipher to the root, two ciphers to the trial divisor, bring down the next period to the right of the dividend, and proceed as usual.

Point off from the right of the root as many figures for decimals as there are decimal periods in the operation.

NOTE 1.—In surds, the root can be closely approximated by using decimal periods of ciphers.

NOTE 2.—If, in marking off, the last decimal period has less than three figures, fill the period with ciphers.

NOTE 3.—It is plain that we can *cube the whole root* after getting any figure of it by the trial divisor, and for a new dividend subtract that cube from all the periods that have been used.

NOTE 4.—In finding cube root to a certain number of decimal places, the labor can be shortened after getting *one more than* half the places, by annexing one cipher to the last dividend, dividing by the trial divisor, taking the quotient as a new root figure, multiplying trial divisor by it, subtracting product, annexing one cipher to remainder, and so on; or,

After finding one more than half the places, divide the last remainder continually by the last divisor, omitting each time two right-hand figures from the divisor, and one from the remainder.

NOTE 5.—*The cube root of a perfect cube may be found by resolving it into its prime factors, and finding the product of one of each three equal factors.* Thus $64 = (2 \times 2 \times 2) \times (2 \times 2 \times 2,)$ and its cube root $= 2 \times 2 = 4$.

QUESTION.—276. How do we extract the cube root of an integral, decimal or mixed number?

Examples for Practice.

3. Find $\sqrt[3]{.123}$ to five decimal places.

Common Process.

.123 (.49731
64
4800|59000
1080|
81|53649
5961| 5351000
720300|
10290|
49|5114473
730639| 236527000
74102700|
44730|
9|222442317
74147439| 14084683000
7419218700|
14910|
1|7419233611
7419233611|6665449389

Short Process.

.123 (.49732
64
4800|59000
1080|
81|53649
5961| 5351000
720300|
10290|
49|5114473
730639| 236527
21919
1733
146

What is the cube root

	Of	Ans.		Of	Ans.
4.	4913?	17	11.	1234567?	107.276572+
5.	1061208?	102	12.	44.6?	3.546322+
6.	46656?	36	13.	12345678?	231.120418+
7.	102503.232?	46.8	14.	11?	2.22398+
8.	130393258.550829?	507.09	15.	25?	2.92401+
9.	8.144865728?	2.012	16.	2?	1.25992104+
10.	483.921516051?	7.851	17.	6?	1.817120+

Case II.

Art. 277. To extract the cube root of a common fraction, or of a mixed number.

Ex. 1. What is the cube root of $1\frac{2}{1}\frac{5}{6}$? Ans. $\frac{5}{6}$.

PROCESS INDICATED.

$$\sqrt{\frac{125}{216}} = \frac{\sqrt[3]{125}}{\sqrt[3]{216}} = \frac{5}{6}$$

MENTAL PROCESS.

Since $\frac{125}{216}$ is assumed as a cube, it must be the product of three equal fractions, which would be the cube of their numerator over the cube of their denominator. Therefore, the cube root of a fraction is the cube root of its numerator over the cube root of its denominator.

Ex. 2. What is the cube root of $15\frac{5}{8}$? Ans. $2\frac{1}{2}$.

FIRST METHOD.

$$15\tfrac{5}{8} = \tfrac{125}{8}. \quad \sqrt[3]{\tfrac{125}{8}} = \tfrac{5}{2} = 2\tfrac{1}{2}$$

SECOND METHOD.

$$15\tfrac{5}{8} = 15.625. \quad \sqrt[3]{15.625} = 2.5 = 2\tfrac{1}{2}$$

RULES.—I. For a fraction.

Make the cube root of the numerator a new numerator, and the cube root of the denominator a new denominator; or,

First reduce the fraction to a decimal, and then find its cube root.

II. For a mixed number.

Reduce it to an improper fraction, or to a mixed decimal number, and find its cube root in this form.

EXAMPLES FOR PRACTICE.

3. $\sqrt[3]{\frac{594}{1408}}$	=	$\frac{3}{4}$.	10. $\sqrt[3]{31\frac{30}{686}}$	=	$3\frac{1}{7}$.
4. $\sqrt[3]{\frac{5488}{13718}}$	=	$\frac{14}{19}$.	11. $\sqrt[3]{\frac{1331}{4913}}$	=	$\frac{11}{17}$.
5. $\sqrt[3]{\frac{250}{686}}$	=	$\frac{5}{7}$.	12. $\sqrt[3]{\frac{4}{15}}$	=	.64365958+.
6. $\sqrt[3]{\frac{8}{9}}$	=	.9614997+.	13. $\sqrt[3]{\frac{152}{513}}$	=	$\frac{2}{3}$.
7. $\sqrt[3]{\frac{729}{4096}}$	=	$\frac{9}{16}$.	14. $\sqrt[3]{2\frac{43}{343}}$	=	$1\frac{2}{7}$.
8. $\sqrt[3]{5\frac{9}{50}}$	=	1.73025+.	15. $\sqrt[3]{1\frac{1}{20}}$	=	1.01639+.
9. $\sqrt[3]{285\frac{3}{4}}$	=	6.58661+.	16. $\sqrt[3]{7\frac{6}{7}}$	=	1.988+.

Art. 278. *The volumes of similar solids are to each other as the cubes of their corresponding linear dimensions.*

Thus, the volume of one cube is to that of another as the cube of the edge of the first is to the cube of the edge of the second; and the volume of one globe is to that of another as the cube of the diameter of the first is to the cube of the diameter of the second; &c. This is proved in Geometry.

QUESTIONS.—277. How do we find the cube root of a common fraction, or of a mixed number?—278. What ratio have similar solids?

Examples for Practice.

1. If a cannon-ball 6 inches in diameter weighs 32lb., what is the weight of a ball 3 inches in diameter, of the same material? Ans. 4lb.

2. On the same conditions, what would be the diameter of a ball that weighed 256lb.? Ans. 12in.

3. What is the edge of a cubical box that contains 1qt. dry measure? Ans. $4\frac{1}{16}$ in. nearly.

4. What would be the side of a cubical box that would hold 500 bushels of coal, Pa. measure? Ans. 9ft. 2.3in.+.

5. How far down a stack of hay, shaped like a cone or pyramid, must I take, to take $\frac{1}{8}$ of the whole?
Ans. $\frac{1}{2}$ of the height.

6. How many worlds like ours could be contained in the space occupied by the sun, the earth being 7912 miles in diameter, and the sun being nearly 886144 miles in diameter?
Ans. 1404928.

7. How many worlds like our moon would it take to make a mass equal in size to our earth, the moon's diameter being about 2083 miles? Ans. 55 nearly.

8. If a cylinder $18\frac{1}{2}$ inches diameter and 8 inches deep contain a Winchester bushel, what are the dimensions of a cistern of proportional parts, that will contain 1000 Winchester bushels?
Ans. 6ft. 8in. deep; 15ft. 5in. diameter.

9. What is the difference between half a cubic yard and a half-yard cube? Ans. $10\frac{1}{8}$ cubic feet.

10. If the gas which fills a balloon, 26.7 feet in diameter, has an upward force of 700lb., how large must be the balloon that, when filled with the same quality of gas, would have an upward force of a ton? Ans. 37.8+ft. diam.

11. A stove dealer made four sizes of a certain pattern of stove. The smallest was 20in. broad on the top, and weighed 300lb. If the other three were proportionally made, how broad were they, their weights being 400, 500, and 600lb.?
Ans. 22+in.; 23.71+in.; 25.19+in.

12. Comparing the principle involved in Ex. 5 and 6 of Art 274 with that of this article, and assuming a limit to the strength of materials, prove,

1st, That there must be a limit to the proportional growth of the branches of trees and the bodies of animals;

2d, That there must be a limit to the size of ships, spans of suspension bridges, and other structures, if enlarged proportionally;

3d, That a bar, or rope, cannot, by increasing its diameter, be made strong enough to support the weight of an unlimited length of itself.

EXTRACTION OF ANY ROOT.

Art. 279. To extract any root of a number.

A method of extracting *any* root of a number was invented by Mr. Horner, and is called

Horner's Method.

Rule.—*Begin as many columns of numbers as there are units in the index of the root, by writing the given number as the head of the right-hand column, and ciphers as the head of the others.*

Beginning at the right of units, mark off the number into as many periods as possible of as many figures each as there are columns.

The root of the greatest perfect power, of the given degree, in the left-hand period is the first figure of the answer.

Add this figure to the term of the first column, for its next term. Multiply this sum by the root figure; add the product to the term in the second column; multiply this sum by the same root-figure, and add the product to the term in the third column; and so proceed till you place the last product under that period in the right-hand column from which the root figure was found. Subtract the last product from the period above it.

Add the same root-figure to the last term of the first column, for its next term. Multiply the sum by the root-figure; add the product to the last term of the second column; and so proceed, ending in the last column but one. Repeat the process, ending each time one column farther to the left, till you at last merely begin it in the left-hand column.

Annex one cipher to the last term in the first column, two ciphers to that in the second, and so on; and to that in the last column annex the next period for a dividend. Divide this by the last term of the previous column as a trial divisor (making allow-

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Questions.—279. Repeat Horner's Method. What other method is there for roots with composite exponents?

ance for completing it) *for the next figure of the root, with which do as with the first figure, and so on till all the periods have been used.*

NOTE 1.—This method, though applicable to any root, is especially useful in extracting roots whose exponent is a *prime number*, and also when the number whose root is desired is an imperfect power.

NOTE 2.—When the exponent of the required root is a composite number, that root may be found by extracting successively the roots expressed by the factors of its exponent. Thus, to find the fourth root, take the *square root of the square root;* to find the *sixth* root, take the *cube root of the square root,* or the *square root of the cube root,* &c.

NOTE 3.—For the sake of practice, the pupil is recommended to solve, by this method, several of the examples in cube root. For a method of contraction adapted to Horner's method, see Higher Arithmetic.

EXAMPLES FOR PRACTICE.

1. What is the fifth root of 184528125? Ans. 45.

0	0	0	0	$\overline{184528125}$ (45
4	16	64	256	1024
4	16	64	256	82128125
4	32	192	1024	82128125
8	48	256	12800000	
4	48	384	3625625	
12	96	640000	16425625	
4	64	85125		
16	16000	725125		
4	1025			
200	17025			
5				
205				

2. What is the $\sqrt[3]{}$ 123456789? Ans. 497.93+.
3. The 4th root of 390625? Ans. 25.
4. The 6th root of 225199600704? Ans. 78.
5. The 7th root of 987654321? Ans. 19.27+

CHAPTER XXI.

SERIES. ANNUITIES. PERMUTATION. COMBINATION.

SERIES.

Art. 280. A SERIES, or *progression*, is a succession of numbers, each of which has the same relation to the preceding number. Such numbers are said to be *in progression*.

The *terms* of a series are the numbers which compose it.

An *ascending* series is a series composed of terms each of which is larger than the preceding term.

A *descending* series is a series composed of terms each of which is smaller than the preceding term.

The *extremes* of a series are its first and last terms.

The *means* of a series are all the terms except the first and last.

In reference to the ratio of the terms, series are *Arithmetical* or *Geometrical*.

ARITHMETICAL SERIES.

Art. 281. An Arithmetical series is a series, between each of whose terms and the preceding term there is a constant *difference*, called the *common difference*. Thus, 2 is the common difference of the ascending series 2, 4, 6, 8, 10, &c., and of the descending series 10, 8, 6, 4, 2.

Art. 282. To find any required term of a series, when the first term and common difference are known.

Ex. 1. What is the 25th term of 4, 6, 8, &c.? Ans. 52.

PROCESS INDICATED.	EXPLANATION.
$25 - 1 = 24$ $(24 \times 2) + 4 = 52$ &c. Hence the following	The *second* term is the first, plus *once* the common difference; the *third* term is the first, plus *twice* the common difference,

QUESTIONS. 280.—What is a series? Its terms? An ascending series? A descending series? Extremes? Means? How are series classified?—281. What is an arithmetical series?

RULE.—*Multiply the common difference by the number of terms less one, and add the product to the first term in an ascending series; but subtract it from the first term in a descending series.*

EXAMPLES FOR PRACTICE.

2. What is the 100th term of 2, 5, 8, &c.? Ans. 299.
3. What is the 75th term of 501, 497, 493, &c.? Ans. 205
4. If you begin with spending a half-dime, and increase the quantity a half-dime every day, how much would you spend on the 365th day? Ans. $18.25.

Art. 283. To find the sum of an arithmetical series.

Ex. 1. What is the sum of 1, 3, 5, 7, &c. to the 6th term? Ans. 36.

ILLUSTRATION.

1	3	5	7	9	11
11	9	7	5	3	1
12	12	12	12	12	12

Here we make the series descending, and write its terms under the given ascending series. On adding the terms thus arranged, the sums are equal,—viz.: 12 for every double term. Hence there are as many *twelves* in twice the given series as there are terms in it, that is, $6 \times 12 = 72$. Hence the sum of the series is $72 \div 2 = 36$.

RULE.—*Multiply half the sum of the extremes by the number of terms, or the sum of the extremes by half the number of terms.*

EXAMPLES FOR PRACTICE.

2. What is the sum of 100 terms of 2, 5, 8, &c.? Ans. 15050.
3. Of 75 terms of 501, 497, 493, &c. Ans. 26475.
4. How much would you spend in a common year, beginning with spending 5cts. and increasing 5cts. every day? Ans. $3339.75.
5. How many strokes of a common clock in 12 hours? Ans. 78.

Art. 284. To find the common difference, when the extremes and number of terms are given.

Ex. 1. What is the common difference of a series whose first term is 6 and tenth term 33? Ans. 3.

QUESTIONS.—282. How do we find any term of a series?—283. How do we find the sum of a series?

PROCESS INDICATED.—$(33 - 6) \div (10 - 1) = 27 \div 9 = 3$

RULE.—*Divide the difference of the extremes by the number of terms less one.*

EXAMPLES FOR PRACTICE.

2. What is the common difference of a series whose first term is 2 and fiftieth term $26\frac{1}{2}$? Ans. $\frac{1}{2}$.

3. Of a series whose first term is 8 and 100th term $416\frac{3}{8}$? Ans. $4\frac{1}{8}$.

Art. 285. To find the number of terms, when the extremes and common difference are known.

Ex. 1. Beginning with \$2, and paying \$2 more every week, in how many weeks will my payment be \$100?

Ans. 50 weeks.

PROCESS INDICATED.—$(100 - 2) \div 2 = 49$. $49 + 1 = 50$.

RULE.—*Divide the difference of the extremes by the common difference, and add* 1 *to the quotient.*

EXAMPLES FOR PRACTICE.

2. What term of the series 1, $1\frac{1}{2}$, 2, $2\frac{1}{2}$, &c. is 38?

Ans. The 75th.

3. What term of the series 100, 96, 92, &c. is 20?

Ans. The 21st.

GEOMETRICAL SERIES.

Art. 286. A Geometrical series is a series each of whose terms may be formed from the preceding by a constant multiplier, called the *ratio*.

When the ratio is greater than unity, the series is ascending, when less, descending.

Thus,

2, 4, 8, 16, &c. is ascending, with the ratio 2.

16, 8, 4, 2, is descending, with the ratio $\frac{1}{2}$.

Art. 287. To find any required term of a series, when the first term and the ratio are known.

Ex. 1. What is the 10th term of 2, 6, 18, &c.?

Ans. 39366.

QUESTIONS.—284. How do we find the common differences?—285. How do we find the number of terms?—286. What is a geometrical series?—287. How do we find any term of a series?

Process Indicated.	Explanation.
$3^{10-1} = 3^9 = 19683$ $19683 \times 2 = 39366$	The *second* term is the first term into 3 : the *third* term is the first term into $3 \times 3 = 3^2$; the *fourth* term is the first term into $3 \times 3 \times 3 = 3^3$, &c. Hence the tenth term is the first term into 3^9. Hence the following

Rule.—*Multiply the first term by that power of the ratio whose exponent is one less than the number of the required term*

Examples for Practice.

2. What is the 7th term of the series 4, 12, 36, &c.?
Ans. 2916.

3. What is the 7th term of the series 2916, 972, &c.?
Ans. 4.

4. If I buy 10 acres land, giving \$2 for the first, \$4 for the second, \$8 for the third, &c., what will the tenth acre cost?
Ans. \$1024.

5. The successive annual amounts of \$1 at 6% compound interest being \$1, \$1.06, \$1.06,^{2} &c., what is the amount of \$100 for 5yr.? Ans. \$133.82,25+.

6. What is the amount of \$500 at 5% compound interest for 10yr.? Ans. \$814.44,7+.

Art. 288. To find the sum of a geometrical series.

Ex. 1. What is the sum of the series 2, 8, 32, 128?
Ans. 170.

Illustration.

$512 + 128 + 32 + 8$	= 4 times the series.
$128 + 32 + 8 + 2$	= once the series.
$512 - 0 - 0 - 0 - 2 = 510$	= 3 times the series.
$510 \div 3 = 170$ = the sum of the series.	

Rule.—*Multiply the last term by the ratio, and divide the difference between this product and the first term by the difference between the ratio and unity.*

Note.—If a descending series is infinite, the last term is 0.

Examples for Practice.

2. What is the sum of 7 terms of the series 4, 12, 36, &c.?
Ans. 4372.

3. What is the sum of 7 terms of the series 2916, 972, &c.?
Ans. 4372.

4. What is the sum of $\frac{1}{4}$, $\frac{1}{8}$, $\frac{1}{16}$, &c., to infinity? Ans. $\frac{1}{2}$.

Question.—288. How do we find the sum of a geometrical series?

5. What is the sum of $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$, &c.; that is, of the repetend decimal .333, &c. to infinity? Ans. $\frac{3}{9}$ or $\frac{1}{3}$.

6. What is the sum of $\frac{27}{100}$, $\frac{27}{10000}$, &c.; that is, of the repetend decimal .2727, &c. to infinity? Ans. $\frac{3}{11}$.

7. A farmer agreed to pay a thrasher for 18 days' work, as follows:—2 grains of wheat the first day, 6 the second, 18 the third, &c.: what would the wages be if 7000 grains of wheat make a pound, 60 pounds a bushel, and the price is $1 a bushel?

Ans. $922.42,9+.

ANNUITIES.

Art. 289. An *annuity* is a sum of money payable at regular periods of time.

Note.—An annuity at first signified a sum payable annually.

A *perpetuity* is an annuity which is to continue forever.

An *annuity certain* is an annuity continuing a definite number of years.

An *annuity deferred*, or *in reversion*, is one which is to commence after the lapse of a definite time.

An *annuity forborne*, or *in arrears*, is one whose payments have not been made when due.

The *amount* of an annuity forborne is the sum of the payments, and their interest for the time they have been withheld.

The *initial worth* of an annuity is its value at its beginning.

The *present worth* of a perpetuity deferred is such a sum as, when put at interest for the time deferred, would amount to the initial worth of the perpetuity.

The present worth of an annuity certain is that sum which, if put at interest till the end of the annuity or the time it is forborne, would amount to the value of the annuity at that time.

An annuity commences one period of time before the first payment.

ANNUITIES AT SIMPLE INTEREST.

Art. 290. To find the amount of an annuity certain or forborne, at simple interest.

Ex. 1. A tenant withheld an annual rent of $100 for 6 years, and then settled with the landlord at 6% simple interest. What did he pay him? Ans. $826.

Questions.—289. What is an annuity? A perpetuity? An annuity certain? An annuity deferred? An annuity forborne? The amount of an annuity forborne? What is the initial worth of an annuity? The present worth of a perpetuity deferred? Of a deferred annuity certain? When does an annuity commence?

EXPLANATION.—The first \$100 amounted in 6 years to \$136; the second, in 5 years, to \$130; the third, in 4 years, to \$124; the fourth, in 3 years, to \$118; the fifth, in 2 years, to \$112; the sixth, in 1 year, to \$106; and the seventh \$100 was due without interest. These numbers form an arithmetical series with a common difference of \$6. Hence the sum is found by Art. 283 thus:—

$$\frac{100 + 136}{2} = 118. \quad 118 \times 7 = 826 \text{ Ans.}$$

RULE.—*Make the interest of one period the common difference, and the annuity the first term of an arithmetical series; then find its last term and sum.*

EXAMPLES FOR PRACTICE.

2. A guardian was bound to pay \$300 a year to an orphan, but withheld payment for 9 years. What was then due to the orphan; int. 6%? Ans. \$3810.

3. A farmer purchased 200 acres of land at \$100 per acre, and agreed to pay for it in 10 equal annual installments; but, there being some doubt about the title, he withheld the installments 12 years from the time the first was due, till the title was perfect. What was then due; int. 7%? Ans. \$30500.

ANNUITIES AT COMPOUND INTEREST.

CASE I.

Art. 291. To find the amount of an annuity forborne for a definite time, at compound interest.

Ex. 1. A tenant withheld an annual rent of \$100 for 6 years, and then settled with the landlord, paying 6% compound interest. What did he pay him? Ans. \$839.38,3.

EXPLANATION.—The seventh \$100 was due without interest; the sixth, in 1 year, amounted to $\$100 \times 1.06$; the fifth, in 2 years, to $\$100 \times 1.06 \times 1.06 = \$100 \times (1.06)^2$; the fourth, in 3 years, to $\$100 \times (1.06)^3$, &c. This forms a geometrical series with a ratio of 1.06. Hence, by Art. 288, the sum is found thus:—

$$\frac{\$100 \times (1.06)^6 \times 1.06 - 100}{1.06 - 1} = \frac{100 \times 1.50363 - 100}{.06} = \frac{50.363}{.06}$$

= \$839.38,3. This process being tedious, it is customary to use a

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QUESTION.—290. How do we find the amount of an annuity forborne, at simple interest?

Table,

Showing the amount of $1 or £1 annuity per annum, at compound interest, for any number of years not more than fifty.

Years.	5 per cent.	6 per cent.	7 per cent.	Years.	5 per cent.	6 per cent.	7 per cent.
1	1.000 000	1.000 000	1.000 000	26	51.113 454	59.156 383	68.676 470
2	2.050 000	2.060 000	2.070 000	27	54.669 126	63.705 766	74.483 823
3	3.152 500	3.183 600	3.214 900	28	58.402 583	68.528 112	80.697 691
4	4.310 125	4.374 616	4.439 943	29	62.322 712	73.639 798	87.346 529
5	5.525 631	5.637 093	5.750 739	30	66.438 848	79.058 186	94.460 786
6	6.801 913	6.975 319	7.153 291	31	70.760 790	84.801 677	102.073 041
7	8.142 008	8.393 838	8.654 021	32	75.298 829	90.889 778	110.218 154
8	9.549 109	9.897 468	10.259 803	33	80.063 771	97.343 165	118.933 425
9	11.026 564	11.491 316	11.977 989	34	85.066 959	104.183 755	128.258 765
10	12.577 893	13.180 795	13.816 448	35	90.320 307	111.434 780	138.236 878
11	14.206 787	14.971 643	15.783 599	36	95.836 323	119.120 867	148.913 460
12	15.917 127	16.869 941	17.888 451	37	101.628 139	127.268 119	160.337 400
13	17.712 983	18.882 138	20.140 643	38	107.709 546	135.904 206	172.561 020
14	19.598 632	21.015 066	22.550 488	39	114.095 023	145.058 458	185.640 292
15	21.578 564	23.275 970	25.129 022	40	120.799 774	154.761 966	199.635 112
16	23.657 492	25.670 528	27.888 054	41	127.839 763	165.047 684	214.609 570
17	25.840 366	28.212 880	30.840 217	42	135.231 751	175.950 645	230.632 240
18	28.132 385	30.905 653	33.999 033	43	142.993 339	187.507 577	247.776 496
19	30.539 004	33.759 992	37.378 965	44	151.143 006	199.758 032	266.120 851
20	33.065 954	36.785 591	40.995 492	45	159.700 156	212.743 514	285.749 311
21	35.719 252	39.992 727	44.865 177	46	168.685 164	226.508 125	306.751 763
22	38.505 214	43.392 290	49.005 739	47	178.119 422	241.098 612	329.224 386
23	41.430 475	46.995 828	53.436 141	48	188.025 393	256.564 529	353.270 093
24	44.501 999	50.815 577	58.176 671	49	198.426 663	272.958 401	378.999 000
25	47.727 099	54.864 512	63.249 030	50	209.347 976	290.335 905	406.528 929

Illustration.—To perform Ex. 1 by this table, we take out 8.393838 for 7 years at 6%. Then $8.393838 \times \$100 = \$839.3838$ Ans.

Rule.—*Make the amount of one dollar for one period of time the ratio, and the annuity the first term of a geometrical series; then find its last term and sum;* or,

Find in the table the amount of one unit of money for the given time and rate, and multiply it by the annuity.

Examples for Practice.

2. What is the amount of an annuity of $125 forborne 7 years, at 7% compound interest? Ans. $1081.75+.

3. If you should begin at 25 years of age to save from your profits in business $200 per year, and keep it at compound interest at 6%, what would you realize from it at 50 years of age? Ans. $10972.90+.

4. What, if you should begin at 21 years of age, and save $300 per year, at 6%, till 55 years of age? Ans. $31255.12+

Question.—291. How do we find the amount of an annuity forborne for a definite time, at compound interest?

5. What, from $100 a year, at 7%, from 20 to 60 years of age? Ans. $19963 51+.

6. What, from $500 a year, at 5%, from 30 to 50 years of age? Ans. $16532.97+.

Case II.

Art. 292. To find the worth of a perpetuity.

Ex. 1. B could not sell me a certain lot, but gave me a perpetual lease of it. What did I pay him, to make my rent $300, money being worth 6%? Ans. $5000.

Process Indicated.	Mental Process.
$300 ÷ .06 = $5000	Since I lose the interest of the money invested, the question is, "How much principal produces $300 in a year at 6%?" This is like Case VIII. of Interest.

Rule.—*Divide the annuity by the interest of a dollar for the period at the given rate.*

Examples for Practice.

2. What should I pay for a perpetual lease worth $1000 per annum, if money is worth 8%? Ans. $12500.

3. What, if the rent is payable semi-annually? Ans. $12750.

Note.—In this case the *annual* rent is $1020, reckoning 6 months' interest on the first $500.

4. A man reckoned that, by living in his own house, he paid a rent of $600; "For," said he, "I could invest its worth at 10%." What was it worth? Ans. $6000.

Case III.

Art. 293. To find the present worth of a deferred perpetuity.

Ex. 1. A inherited a perpetuity of $600 a year, to commence when he was 25 years of age. He sold it when he was 23 years of age, money being worth 6 per cent. per annum. What did he receive?

Process Indicated.

$600 ÷ .06 = $10000 = initial worth.

$10000 ÷ $1.123606 = $8899.91,6 = present worth at the time of sale.

Question.—292. How do we find the initial worth of a perpetuity?

MENTAL PROCESS.—At 23 years of age, his annuity was two years deferred. The amount of $1 at compound interest, for 2 years, is $1.123606, which is its worth 2 years hence. If $1.123606, due 2 years hence, is worth $1 now, $10000, due 2 years hence, is worth as many dollars now as $1.123606 is contained times in $10000, that is, $8899.91,6.

RULE.—*Divide the worth by the amount of a dollar, at compound interest, for the time that the perpetuity is deferred.*

EXAMPLES FOR PRACTICE.

2. What is the present value of a perpetuity of $1080 a year, at 6%, deferred 10yr.? Ans. $10051.10+.

3. Of a perpetual ground-rent of $320 a year, to commence in 3yr. 4mo., at 7%? Ans. $3646.56.

4. Of a perpetuity of $500 a year, deferred 8yr., at 6%? Ans. $5228.44.

CASE IV.

Art. 294. To find the present worth of an annuity certain.

RULE.—*Divide the amount of the annuity for the time it is to run by the amount of a unit of money running at compound interest from the present time to the termination of the annuity;* or,

If the annuity is immediate, find, in the table of initial values, that of one unit of money for the time and rate, and multiply it by the annuity.

If the annuity is deferred, find, in the table of initial values, that of a unit of money commencing immediately and continuing till the annuity commences, and also till the annuity terminates, and multiply the difference of these initial worths by the given annuity.

EXAMPLES FOR PRACTICE.

1. Having sold a farm for $3500, to be received in 7 equal annual installments, I wish to sell my claims on the debtor for cash. What should I receive, at 6%? Ans. $2791.19+.

2. If you are entitled to $500 a year, to commence in 5 years and continue for life, and your chance of life is reckoned at 45 years, for how much should you sell your title, to lose nothing, interest at 7%? Ans. $4752.66,3+

QUESTIONS.—293. How do we find the present worth of a deferred perpetuity?—294. How do we find the present worth of an annuity certain?

Case V.

Art. 295. To find the initial worth of an annuity certain.

Rule.—*If the annuity is immediate, the initial is the same as the present worth.*

If the annuity is deferred, find, in the table of initial values, that of one unit of money for the time and rate, and multiply it by the annuity.

Table,

Showing the initial worth of an annuity of $1 or £1 per annum, for any number of years not more than fifty.

Years.	5 per cent.	6 per cent.	7 per cent.	Years.	5 per cent.	6 per cent.	7 per cent.
1	0.952 381	0.943 396	0.934 579	26	14.275 185	13.003 166	11.825 779
2	1.859 410	1.833 393	1.808 017	27	14.643 034	13.210 534	11.986 709
3	2.723 248	2.673 012	2.624 314	28	14.898 127	13.406 164	12.137 111
4	3.545 951	3.465 106	3.387 209	29	15.141 074	13.590 721	12.277 674
5	4.329 477	4.212 364	4.100 195	30	15.372 451	13.764 831	12.409 041
6	5.075 692	4.917 324	4.766 537	31	15.592 811	13.929 086	12.531 814
7	5.786 373	5.582 381	5.389 286	32	15.802 677	14.084 043	12.646 555
8	6.463 213	6.209 794	5.971 295	33	16.002 549	14.230 230	12.753 790
9	7.107 822	6.801 692	6.515 228	34	16.192 204	14.368 141	12.854 009
10	7.721 735	7.360 087	7.023 577	35	16.374 194	14.498 246	12.947 672
11	8.306 414	7.886 875	7.498 669	36	16.546 852	14.620 987	13.035 208
12	8.863 252	8.383 844	7.942 671	37	16.711 287	14.736 780	13.117 017
13	9.393 573	8.852 683	8.357 635	38	16.867 893	14.846 019	13.193 473
14	9.898 641	9.294 984	8.745 452	39	17.017 041	14.949 075	13.264 928
15	10.379 658	9.712 249	9.107 898	40	17.159 086	15.046 297	13.331 709
16	10.837 770	10.105 895	9.446 632	41	17.294 368	15.138 016	13.394 120
17	11.274 066	10.477 260	9.763 206	42	17.423 208	15.224 543	13.452 449
18	11.689 587	10.827 603	10.059 070	43	17.545 912	15.306 173	13.506 962
19	12.085 321	11.158 116	10.335 578	44	17.662 773	15.383 182	13.557 908
20	12.462 210	11.469 421	10.593 997	45	17.774 070	15.455 832	13.605 522
21	12.821 153	11.764 077	10.835 527	46	17.880 067	15.524 370	13.650 020
22	13.163 003	12.041 582	11.061 241	47	17.981 016	15.589 028	13.691 608
23	13.488 574	12.303 379	11.272 187	48	18.077 158	15.650 027	13.730 474
24	13.798 642	12.550 358	11.469 334	49	18.168 722	15.707 572	13.766 799
25	14.093 945	12.783 356	11.653 583	50	18.255 925	15.761 861	13.800 746

Examples for Practice.

1. What is the *initial* worth of an annuity of $600, to continue 4yr., at 6% compound interest; and what is its *present worth*, if it is to begin 2yr. hence?

Ans. I. W. $2079.06+. P. W. $1850.35+.

2. What is the initial worth of an annuity of $400, to continue 7yr., at 5% compound interest; and what is its present worth if it is to begin 3yr. hence?

Ans. I. W. $2314.54,9+; P. W. $1999.39+.

Question.—295. How do we find the initial worth of an annuity certain?

PERMUTATIONS.

Art. 296. PERMUTATIONS are changes in the arrangement of a given number of things.

ILLUSTRATION.—Two letters, *a* and *b*, can be arranged in two ways; thus, *ab*, *ba*. That is, in the ordinary way of writing letters, *b* can be put only on the *right* and *left* of *a*. A third letter, *c*, can be put in *three new positions* in both *ab* and *ba*, viz., on the *left*, the *right*, and *between*; thus, *cab*, *acb*, *abc*, *cba*, *bca*, *bac*. A fourth letter, *d*, can take *four new positions* in each of these six arrangements, viz., at the *left*, the *right*, and *between each two*. Hence the changes for

Two things, are $1 \times 2 = 2$: and for
Three things, are $1 \times 2 \times 3 = 6$: and for
Four things, are $1 \times 2 \times 3 \times 4 = 24$: &c. &c.

CASE I.

Art. 297. To find the possible permutations of a given number of things taken all at a time.

RULE.—*Multiply together the numbers of the natural series* 1, 2, 3, *&c. till as many have been used as there are units in the given number of objects.*

EXAMPLES FOR PRACTICE.

1. In how many different ways can the 8 notes of an octave be arranged? Ans. 40320.

2. How many integral numbers can be expressed by different arrangements of the nine digits? Ans. 362880.

3. A lady changed every day the order of 7 beads upon a string. How long before she finished the possible methods?
Ans. 13yr. 292da.

CASE II.

Art. 298. To find the possible permutations of a given number of things taken a given less number at a time.

ILLUSTRATION.—If, for example, we require all the possible arrangements of *six* letters by *twos*, it is plain that *each* letter of the six, by joining with each of the other five, would make 5 arrangements; and 6 letters would make $6 \times 5 = 30$ permutations. Again, were we to arrange 6 letters by *threes*, we could join to each of the 30 permutations by twos, each of the four remaining letters, making 4 arrangements for each of the 30; then the whole number would be $6 \times 5 \times 4 = 120$ permutations.

QUESTIONS.—296. What are permutations? Give the illustration.—297. What is Case I.? Give the rule.—298. What is Case II? Give the illustration

RULE.—*Multiply together the numbers of the decreasing natural series, whose greatest term is the number of things given, and whose number of terms is the number of things to be taken at a time.*

EXAMPLES FOR PRACTICE.

1. How many integral numbers can be expressed by the nine digits, taken four at a time? Ans. 3024.
2. How many, taken six at a time? Ans. 60480.
3. How many, taken seven at a time? Ans. 181440
4. How many, taken eight at a time? Ans. 362880.
5. How many, taken three at a time? Ans. 504.

COMBINATIONS.

Art. 299. COMBINATIONS are groups so formed from the members of a given number of things that all the members of one group are not the same as those in another.

ILLUSTRATION.—Of the three letters *a b c*, *ab*, *ac*, and *bc* are three combinations. They would not become different combinations by changing the *order* of their letters, but would then be merely different *permutations*.

Art. 300. To find the number of possible combinations of a given number of things taken a given number at a time.

ILLUSTRATION.—The *permutations* of five things, taken two at a time, are 20. Now, since the same *combination* of two things may have *two permutations* or modes of arrangement, the possible *combinations* of five things, taken two at a time, are $20 \div 2 = 10$; that is, $(5 \times 4) \div (1 \times 2)$.

RULE.—*Divide the number of possible permutations of the whole number, taken the given number at a time, by the number of possible permutations of as many things as are taken at a time.*

EXAMPLES FOR PRACTICE.

1. How many combinations of the ten Arabic figures, taken seven in a set? Ans. 120.
2. Of the nine digits, four in a set? Ans. 126.
3. Of the nine digits, six in a set? Ans. 84.
4. Of the nine digits, seven in a set? Ans. 36.

QUESTIONS.—298. Give the rule.—299. What are combinations? Give the illustration.—300. How do we find the number of possible combinations of a given number of things, taken a given number at a time?

CHAPTER XXII.

MENSURATION.

Art. 301. MENSURATION treats of the measurement of magnitudes.

Measuring is applying to a magnitude an assumed *unit of measure*, of the same kind as the magnitude, to find the number of times the unit is contained in the magnitude.

In reference to the kind of magnitude, mensuration is of *lines*, *surfaces*, and *solids*. The mensuration of lines requires no discussion in this chapter.

MENSURATION OF SURFACES.

Art. 302. A *surface* is a magnitude having length and breadth, but no thickness.

The unit of measure of a surface is generally a *square*.

Area is quantity of surface.

A *quadrilateral* is a surface bounded by four sides.

A *parallelogram* is a quadrilateral whose opposite sides are parallel.

The *altitude* of a parallelogram is the perpendicular distance from the base to the opposite side.

Art. 303. To find the area of a parallelogram.

RULE.—*Multiply the base by the altitude expressed in the same kind of linear units. The product expresses square units of the same name as the linear.*

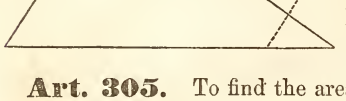
NOTE.—For illustration of *one kind* of parallelogram, see Art. 75, Note 1. The general proposition is proved in Geometry.

1. What is the area of a parallelogram 5½ch. long and 3½rd. broad? Ans. 1R. 37P.

2. How much less land will be enclosed by the fence of a right-angled field 15rd. long and 8rd. broad, if the angles be so changed that the longest sides are 5rd. apart? Ans. 1R. 5P.

QUESTIONS.—301. Of what does mensuration treat? What is measuring? Of what kinds is mensuration?—302. What is a surface? The unit of surface measure? Area? A quadrilateral? A parallelogram? The altitude of a parallelogram?—303. How do we find the area of a parallelogram?

Art. 304. A trapezoid is a quadrilateral having only two parallel sides. Its area is equivalent to that of a parallelogram of the same altitude whose base is the *average* length of the two parallel sides of the trapezoid. (See figure.)



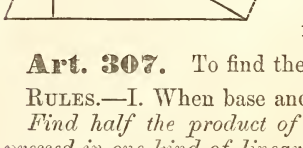
Art. 305. To find the area of a trapezoid.

RULE.—*Multiply half the sum of the parallel sides by their perpendicular distance from each other, expressed in the same linear measure.*

1. How many square feet in a tapering board 12ft. long, 15in. wide at one end and 18in. wide at the other? Ans. $16\frac{1}{2}$.

2. How many square feet in the two largest sides of a four-sloped roof, 40ft. long at the eaves and 20ft. long at the ridge, the rafter being 15ft. long? Ans. 900.

Art. 306. The *altitude* of a triangle is the perpendicular distance from the base to the opposite angle.



The area of a triangle is equivalent to half that of a parallelogram of equal base and altitude. (See figure.)

Art. 307. To find the area of a triangle.

RULES.—I. When base and altitude are known.

Find half the product of base and altitude, both being expressed in one kind of linear unit.

II. When the three sides are known.

From half the sum of the three sides take each side separately; multiply together the three remainders and the half sum, and find the square root of the product.

1. Find the area of a triangle whose base is 50ft. and altitude 40ft. Ans. 1000 sq. ft.

2. Of a triangle whose sides are 10rd., 16rd., and 24rd. Ans. 58.09 sq. rd.

Art. 308. A *trapezium* is a quadrilateral having no parallel sides.

QUESTIONS.—304. What is a trapezoid? To what is its area equivalent?—305. How do we find the area of a trapezoid?—306. What is the altitude of a triangle? The area of a triangle?—307. How do we find the area of a triangle?—308. What is a trapezium?

A *diagonal* is a line joining the opposite angles of a figure.

Art. 309. To find the area of a trapezium.

RULE.—*Draw a diagonal, and find the sum of the areas of the two resulting triangles.*

1. Find the area of a trapezium whose diagonal is 50ft., and whose sides are, on one side of it, each 35ft., and on the other, 36ft. and 40ft. respectively. Ans. 1325.53+ sq. ft.

2. Of a trapezium whose diagonal is 40rd., the altitude from it of the triangle on one side being 42rd., and on the other 38rd. Ans. 10A.

Art. 310. A *polygon* is a surface bounded by more than 4 sides.

A *pentagon* is a polygon of 5 sides; a *hexagon*, of 6 sides; a *heptagon*, of 7 sides; an *octagon*, of 8 sides; a *nonagon*, of 9 sides; a *decagon*, of 10 sides; an *undecagon*, of 11 sides; a *dodecagon*, of 12 sides, &c.

A *regular* polygon is one that has all its sides equal and all its angles equal.

An *irregular* polygon is one that has unequal sides and unequal angles.

The *perimeter* of a figure is the sum of its sides.

Art. 311. To find the area of a polygon.

RULE.—*If the polygon is irregular, separate it into triangles by diagonals, then find the sum of these triangles. If the polygon is regular, separate it into triangles, by drawing lines from the centre to each angle; then find half the product of the perimeter and the perpendicular from the centre to one side.*

1. What is the area of a regular hexagon whose sides are 20ft. long, the perpendicular upon them from the centre being 17.32ft.? Ans. 1039.2 sq. ft.

QUESTIONS.—309. What is a diagonal? How do we find the area of a trapezium?—310. What is a polygon? A pentagon, &c.? An irregular polygon? The perimeter of a figure?—311. How do we find the area of a polygon?

2. An irregular quadrilateral has a 10ft. diagonal, and two sides on one side of it are $10\frac{3}{4}$ft. and $12\frac{1}{2}$ft. respectively; on the other, $9\frac{3}{4}$ft. and $11\frac{1}{4}$ft. respectively. What is its area?

Ans. 97.3073 sq. ft.

Art. 312. The *circumference* of a circle is the line that bounds it.

The *diameter* of a circle is a straight line passing through the centre and terminating at each end in the circumference.

The *radius* of a circle is a straight line terminating at the centre and circumference: its length is half that of the diameter.

Art. 313. To find the circumference of a circle.

RULE.—*Multiply the diameter by* 3.1415926.

NOTE.—This is not *exact*. The ratio of diameter to circumference has been found to hundreds of decimal places, without obtaining a complete result. It is often near enough to take it as 3.1416.

1. The earth's radius being 3956 miles, what is its circumference? Ans. 24856.28+ miles.

2. What is the length of tire on a wheel whose radius is $2\frac{1}{2}$ft.? Ans. 15.7+ ft.

Art. 314. To find the diameter of a circle.

RULES.—I. From the circumference.

Divide the circumference by 3.1415926, *or multiply the circumference by* .31830989.

II. From the area.

Divide the area by .785398, *and find the square root of the quotient.*

NOTE.—It is often sufficient to divide by .7854.

1. What is the diameter of a circle whose circumference is 31ft.? Ans. 9.86+ ft.

2. Of a circular farm of 100 acres? Ans. 35.68+ chains

Art. 315. To find the area of a circle.

RULES.—I. *Multiply the square of the diameter by* .785398.

II. *Multiply the square of the radius by* 3.1415926.

QUESTIONS.—312. What is the circumference of a circle? The diameter? The radius?—313. How do we find the circumference of a circle? Is it exact?—314. How do we find the diameter of a circle?

III. *Multiply the square of the circumference by* .079577.

IV. *Multiply half the diameter by half the circumference.*

V. *Multiply the circumference by one-fourth the diameter, or the diameter by one-fourth of the circumference, or the diameter by the circumference and take one-fourth of the product.*

1. What is the area of a circle whose circumference is 12ft.? Ans. 11.459+ sq. ft.
2. Of a circle whose diameter is 10ft.? Ans. 78.53+ sq. ft.
3. Of a circle whose radius is 1¼ft.? Ans. 4.9087+ sq. ft.

Art. 316. To find the side of a square whose area is equal to that of a given circle.

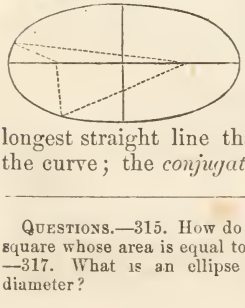
REMARK.—This cannot be *exactly* found, because the ratio of diameter to circumference has not been exactly determined. Since the side of a square is equal to the square root of its area, we must take, in this case, the square root of the area of the given circle; that is, the square root of the product of .785398163397, into the square of the diameter. or an equivalent expression derived from the rules in Art. 315. Hence, the square root of .785398163397 being .886227+, the reason of the first rule following is plain. The learner can find that for the second and other methods.

RULES.—I. *Multiply the diameter by* .886227.

II. *Multiply the circumference by* .282094.

1. What is the side of a square, whose area equals that of a circle 100rd. in circumference? Ans. 28.2+rd.
2. Of a square whose area equals that of a circle of 10ft. radius? Ans. 17.72+ft.
3. Comparing a circle with a square, which contains the greater surface with the same perimeter?

Art. 317. An *ellipse* is an oval figure bounded by a curved line, such that the sum of the distances from any point in the curve to two points within, called *foci*, is the same as for any other point in the curve. The *transverse diameter* is the longest straight line that passes through the centre, limited by the curve; the *conjugate diameter* is the shortest.



QUESTIONS.—315. How do we find the area of a circle?—316. The side of a square whose area is equal to that of a given circle? Why is this not exact? —317. What is an ellipse? The transverse diameter? The conjugate diameter?

Art. 318. To find the area of an ellipse.

RULE.—*Multiply the transverse diameter by the conjugate, and that product by* .785398.

1. What is the area of an ellipse whose diameters are 18ft. and 24ft.? Ans. 339.29+ sq. ft.

2. Of an elliptical table-top, 4ft. long and 2½ft. wide?
Ans. 7.85+ sq. ft.

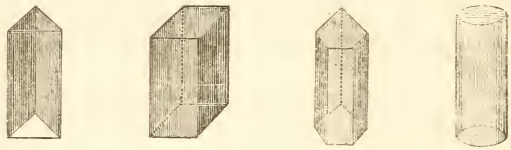
MENSURATION OF SOLIDS.

Art. 319. A *solid* is a magnitude having length, breadth, and thickness.

A *prism* is a solid whose ends are equal and similar plane figures, and whose sides are parallelograms. (See figs. 1–3.)

A *cylinder* is a regular solid whose ends are equal and parallel circles. (See fig. 4.)

Fig. 1. Fig. 2. Fig. 3. Fig. 4.



The *altitude* of a prism or cylinder is the perpendicular distance between the planes of its ends.

Art. 320. To find the surface of a prism or cylinder.

RULE.—*Multiply the perimeter of its end by its altitude, and to the product add the area of both ends.*

1. How much tin will make a cylindrical bucket and cover 10 inches in diameter, and 12 inches high? Ans. 3.7+ sq. ft.

2. What is the surface of a rectangular log 20ft. long, whose sides are 15in. and 18in. broad? Ans. 113 sq. ft.; 108 sq. in.

Art. 321. To find the volume of a prism or cylinder.

RULE.—*Multiply the area of its base by its altitude.*

1. What is the capacity of a triangular penstock 10ft. long, whose sides are 3, 4, and 5ft. broad? Ans. 60 cu. ft.

QUESTIONS.—318. How do we find the area of an ellipse?—319. What is solid? A prism? A cylinder? The altitude of a prism or cylinder?—320 How do we find the surface of a prism or cylinder?—321. How do we find th volume of a prism or cylinder?

2. Of a round bucket 6 inches in diameter and 6 inches high?
Ans. 2.93+quarts.

Art. 322. A *pyramid* is a solid having one end triangular, square, or polygonal, and triangular sides whose vertices unite at the other end in a common point. (See fig. 1.)

A *cone* is a solid having one end circular and its convex surface tapering regularly to a point at the other end. (See fig. 2.)

The *altitude* of a pyramid or cone is the perpendicular distance from the vertex to the plane of the base.

The *slant height* of a pyramid is the distance from the vertex to the middle of one of the sides of the base.

The *slant height* of a cone is the distance from the vertex to the circumference of the base.

Fig. 1.

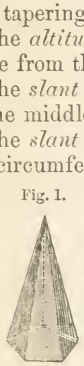


Fig. 2.

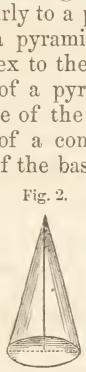


Fig. 3.

Fig. 4.

A *frustum* of a pyramid or cone is the part that remains after the top is cut off parallel to the base. (See figs. 3 and 4.)

Art. 323. To find the convex surface of a pyramid or cone.

RULE.—*Multiply the perimeter of the base by half of the slant height.*

1. How many yards of canvas 24in. wide will make a conical tent 50ft. in diameter, the slant height being 80ft.?
Ans. $1047\frac{1}{3}$ nearly.

Art. 324. To find the volume of a pyramid or cone.

RULE.—*Multiply the area of the base by one-third of the altitude.*

1. What is the volume of a square pyramid each side of whose base is 30ft., and whose altitude is 25ft.? Ans. 7500 cu. ft.

Art. 325. To find the surface of a frustum of a pyramid or cone.

QUESTIONS.—322. What is a pyramid? A cone? Altitude? What is the slant height of a pyramid? Of a cone? What is a frustum?—323. How do we find the surface of a pyramid or cone? 324. The volume of a pyramid or cone?

RULE.—*Multiply the sum of the perimeters of the two ends by half the slant height, and to the product add the areas of the two ends.*

1. What is the surface of a frustum of a square pyramid whose slant height is 10ft., each side of the lower base being 3ft. 4in. and of the upper base 2ft. 2in.? Ans. $125\frac{2}{3}\frac{9}{6}$ sq. ft.

Art. 326. To find the volume of a frustum of a pyramid or cone.

RULE.—*Find the square root of the product of the areas of the two ends; to this root add the two areas, and multiply the sum by one-third of the altitude of the frustum.*

1. What is the volume of a beam 24ft. long, 15in. square at one end and 6in. square at the other? Ans. 19.5 cu. ft.

Art. 327. A *sphere* is a solid every part of whose surface is equally distant from a point within, called the centre.

A *diameter* of a sphere is a straight line passing through the centre and terminating at each end in the surface.

A *radius* of a sphere is a straight line terminating at the centre and surface.

A *segment* of a sphere is a part of a sphere included between two parallel planes, one of which may touch, or both of which may pass through, the sphere.

The *height* of a segment of a sphere is the perpendicular distance between its including planes.

Art. 328. To find the surface of a sphere.

RULE.—*Multiply the diameter by the circumference.*

1. What is the surface of the earth, if its diameter is 7912 miles? Ans. 196662892 sq. miles.
2. What is the surface of the sun, if its diameter is 885000 miles? Ans. 2460573864135 sq. miles.

Art. 329. To find the volume of a sphere.

RULE.—*Multiply the cube of the diameter by* .52359878.

1. What is the volume of the earth?
Ans. 259332807530 cu. miles.

QUESTIONS.—325. What is the surface of a frustum?—326. How do we find the volume of a frustum?—327. What is a sphere? Its diameter? Radius? Segment? What is the height of a segment?—328. How do we find the surface of a sphere?—329. The volume of a sphere?

Art. 330. To find the curved surface of a segment of a sphere.

RULE.—*Multiply the height of the segment by the circumference of the sphere.*

1. What is the surface of the North Temperate Zone, if the height of that segment is 2046 miles?

Ans. 50855950.21+ sq. miles.

Art. 331. To find the volume of a segment of a sphere.

RULE.—*Multiply the sum of the areas of the two bases by half the height, and to this product add the volume of a sphere whose diameter is the height.*

1. What is the volume of the North Temperate Zone, if the radius of its southern base is 3634 miles, that of its northern base 1580 miles, and its height 2046 miles?

By Art. 315, R. II. (See Key.) Ans. 54949520078+cu. miles.

Art. 332. To find the contents, in board measure, of boards and square timber.

RULE.—*Multiply together the length in feet, the average width in inches, and the thickness in inches, and divide the product by* 12.

NOTE.—The unit of board measure is a square foot of a board 1 inch thick. The average width is half the sum of the widths of the ends.

1. How many feet in an inch board 16ft. long, 16in. broad at one end and 12 at the other? Ans $18\frac{2}{3}$.
2. How many feet in a three-inch plank of the same length and breadth? Ans. 56.
3. In a log 20ft. long and 16in. by 18in. broad? Ans. 480.

Art. 333. To gauge a cask made of curved staves.

RULE.—*Multiply together the length in inches, the square of the mean diameter, and .7854. The result is cubic inches.* Or, *Multiply together the length in inches, the square of the mean diameter in inches, and .0034. The result is U.S. wine gallons.*

QUESTIONS.—330. How do we find the curved surface of a segment?—331. The volume of a segment?—332. How do we find the board measure of lumber? What is the unit of board measure? The average width of a board —333. How do we gauge a cask made of curved staves?

NOTE.—The mean diameter is found by adding to the head diameter $\frac{2}{3}$ of the difference between the head and bung diameters if the staves are much curved, but $\frac{3}{5}$ of that difference if the staves are but little curved. It is manifest that this is reducing the cask to a cylinder of average diameter. The .0034 of the rule is the value of .7854 ÷ 231, the cubic inches in a wine gallon U.S.

1. What is the capacity of a cask whose head diameter is 15 inches, bung diameter 21 inches, and length 40 inches?

Ans. 49 wine gal. nearly.

2. Of a cask whose head diameter is 45in., bung diameter 50in., and length 60in.? Ans. 470 wine gal. nearly.

MISCELLANEOUS EXERCISES.

1. If boarding during Jan., Feb., and March, 1856, was \$2.75 per week, during April, May, and June, \$3 per week, during July, Aug., and Sept., \$3.37½ per week, and during Oct., Nov., and Dec., \$3.75 per week, what was the average per week during the year? Ans. $\$3.22\frac{23}{366}$.

2. What is the difference between 109 quadrillions English and French? Ans. 108999999891 French quadrillions.

3. What number is that of which $\frac{7}{8}$ exceeds $\frac{4}{5}$ by 90?

4. $1\frac{1}{2} + 2\frac{1}{3} + 3\frac{7}{8} + 4\frac{5}{12} =$ what? Ans. $12\frac{1}{8}$.

5. $3\frac{7}{8} + 5\frac{11}{12} + \frac{4}{5}$ of $\frac{3}{7}$ of $10\frac{1}{2} - (1\frac{3}{4} + \frac{3}{5}$ of $7\frac{1}{2} + 5) =$ what? Ans. $2\frac{17}{120}$.

6. Having 12 logs, of which 4 are 26ft. long, 5 others $32\frac{1}{2}$ft. long, and the remainder 39ft. long, and knowing that the diameter of each is large enough to furnish 10 fence-posts, how shall I cut the logs so as to get the greatest number of posts of the greatest length? Ans. 6½ft. long; 590 posts.

7. If pewter is composed of 112 parts tin, 15 lead, and 6 brass, how much of each will make 665lb. of pewter?

Ans. Tin, 560lb.; lead, 75lb.; brass, 30lb.

8. A, B, and C agreed to furnish \$300 of their minister's salary, in shares which should be inversely as the distance they must come to hear him. A came $\frac{3}{4}$ of a mile, B 1½ miles, and C 2¼ miles. What was the share of each?

Ans. A $\$163.63\frac{7}{11}$; B $\$81.81\frac{9}{11}$; C $\$54.54\frac{6}{11}$.

9. If A and B can dig a drain in 4 days, B and C in 6 days, and A and C in 5 days, in what time could they do it together, and separately?

Ans. All $3\frac{9}{37}$da.; A $7\frac{1}{17}$da.; B $9\frac{3}{13}$da.; C $17\frac{1}{7}$da.

QUESTION.—333. How do we find the mean diameter?

10. The hour and minute hand of a clock are together at 12 o'clock: when are they again together?

Ans. 1 o'c. 5m. $27\frac{3}{11}$sec.; 2 o'c. 10m. $54\frac{6}{11}$sec.; 3 o'c. 16m. $21\frac{9}{11}$sec., &c. &c.

11. What o'clock is it when $\frac{1}{5}$ of the time past noon equals $\frac{1}{13}$ of the time to midnight? Ans. 3 o'c. 20m.

12. The velocity of sound is variously stated as 1090ft. per second, 1120ft., and 1142ft. In estimating the distance of a gun whose report is heard in 5 seconds, what differences would result from using these numbers? Ans. 150ft.; 110ft.; 260ft.

13. If a clock strikes the hours from 1 to 24, how many strokes does it make? Ans. 300.

14. What is the greatest common divisor of $\frac{15}{16}$, $2\frac{1}{4}$, 4, and $5\frac{1}{3}$? Ans. $\frac{1}{48}$.

15. What is the least common multiple of $\frac{1}{6}$, $1\frac{1}{2}$, and $5\frac{1}{4}$? Ans. $10\frac{1}{2}$.

16. What length of tether should you give your horse, that his grazing may be confined to a circular acre? Ans. 117.75+ ft.

17. A party composed of ladies and gentlemen, the former being twice as many as the latter, spent £3 5s. 4d.; and, on settling, each gentleman paid as many pence as there were gentlemen in the party. How many persons were in the party? Ans. 84.

18. Of a certain pole, 5% stood in earth, 10% in water, and 42ft. 6in. above water: what was the length of the pole? Ans. 50ft.

19. If a landlord's net rents, after deducting 10cts. per dollar for taxes and repairs, are $3654.72, what is the amount of his rents? Ans. $4060.80.

20. If an animalcule is $\frac{1}{24000}$ of an inch in diameter, and the thickness of a membrane in his body is $\frac{1}{150}$ of his diameter, what is the thickness of the membrane? Ans. $\frac{1}{3600000}$in.

21. How shall a house that cost $3000 be rented, that the landlord may annually expend 5% of the rent, and still realize 10% on his investment? Ans. $315.78,9+.

22. E. Everson bought a house and lot for $2500, and let the property at $250 a year, but paid annually for insurance $7.50, repairs $12.50, and taxes 18 mills on a dollar. What net income did he receive? Ans. $7\frac{2}{5}$%.

23. If 168 men, working 10 hours a day, can in 5 days dig a trench of 5 degrees of hardness, 105ft. long, 9ft. wide, and 6ft. deep, in how many days of 12 hours each will 240 men dig a

trench 108ft. long, 15ft. wide, and 9ft. deep, of 6 degrees of hardness? Ans. 9 days.

24. If you buy cloth at $1.12 per yd., how must you sell it to gain 25%? Ans. $1.40.

25. If, by buying at 10% discount a stock whose par value is $25, and selling it at 20% premium, I make $375, what per cent. do I make upon my investment, and how many shares do I buy? Ans. 33⅓%; 50 shares.

26. I sold goods at 10% gain. Had they cost me $150 more, I should have lost 10% What did they cost?
Ans. $675.

27. If an article had cost me 20% less, my gain would have been 30% more. What was the gain per cent.? Ans. 20%.

28. What are the interest and amount of $9162 for 3 months, at 1½% a month? Ans. Int. $412.29; Am't $9574.29.

29. The hands employed in a certain mill are men at 1s. 4d. and women at 11d. per day, and their wages average 1s. 2d. 2qr. The number of women being 21, what is that of the men?
Ans. 49.

30. A, B, and C divided profits as follows:—A's stock had been in 6 months, and his share was $1200; B in 5 months received $1500; C's stock was $4000: it had been in 9 months, and he received $2100. What was the whole stock, and what were the shares of A and B?

Ans. Whole, $12571.42+; A $3428.57+; B $5142.85+.

31. What are the interest and amount of $4650 for 3yr. 4mo. 12da., at 7%? Ans. Int. $1095.85; Am't $5745.85.

32. What are the interest and amount of £429 17s. 4d. from June 29, 1858, to Feb. 12, 1859, at 5%?
Ans. Int. £13 8s. 6¼d.; Am't £443 5s. 10¼d.

33. What are the interest and amount of £43 3s. from June 20 to Nov. 8, at 6%? Ans. Int. £1; Am't £44 3s.

34. What is the true balance of the following account at settlement Dec. 10, 1859, interest being allowed at 4%?

Dr. *S. Bell, London, in Acc't Cur. with T. Shaw, N.Y.* *Cr.*

1859.		£	s.	d.	1859.		£	s.	d.
Feb. 10.	To M'dse.	186	7	6	Mar. 24.	By M'dse.	166	13	4
" 25.	" "	214	10		April 6.	By Cash.	347	18	
June 20.	" "	415	8	4	Sept. 26.	" "	200		
Dec. 10.					Dec. 10.				

35. By the rule of the U. S. Courts, what was due on the following note, Jan. 1, 1854, int. at 6%?

$1250\frac{00}{100}$. BRIGHTON, Jan. 1, 1853.

For value received, I promise to pay Merwin Beachford, or order, twelve hundred and fifty dollars, without defalcation, with interest at 6%. ASA KEMPTON.

Indorsements. April 1, 1853, rec'd $175.75; Aug. 1, 1853, $20; Nov. 1, 1853, $360.

Ans. $758.76,7+.

36. For how much must a note at 30 days be drawn, that the proceeds may be $1500, after discount at 1% a month?

Ans. $1516.68.

37. If I conclude to pay now three installments of $400 each, due in 3 months, 6 months, and 9 months, how much should I pay, discounting at 6%? Ans. $1165.21+.

38. If, by receiving $1.25 for every day he works, and by spending 50cts. every day he is idle, a man makes $19 in 25 days, how many days is he idle? Ans. 7.

39. If an animalcule is $\frac{1}{3500}$ of an inch in diameter, how many such would make a cubic inch? Ans. 42875000000.

40. If $500 be put at compound interest at 6% when a person is 21 years of age, to what will it amount when he is 50 years of age? Ans. $2709.19,4.

41. What is the value of $(2.25)^5$? Ans. $57\frac{681}{1024}$.

42. If light, heat, and gravity vary inversely as the square of the distance from their sources,—

1st. How much heat would you receive from the fire if, being 3ft. from it, you should remove $1\frac{1}{2}$ft. more? Ans. $\frac{4}{9}$ as much.

2d. How much light would fall on the page of a book, 4ft from the candle, if removed 1ft. more? Ans. $\frac{16}{25}$ as much.

3d. How much if, being 7ft. distant, it is put at 5ft. 3in. distance? Ans. $1\frac{7}{9}$ times as much.

4th. How much less are the light, heat, and attraction of the sun at the planet Neptune, 2850000000 miles from it, than at the earth, 95000000 miles from it? Ans. $\frac{1}{900}$ as much

5th. At Uranus, 1820000000 miles from it?

Ans. $\frac{1}{367}$ nearly.

6th. At Saturn, 900000000 miles from it?

7th. At Jupiter, 490000000 miles from it?

8th. At Mars, 144000000 miles from it?

9th. How much more at Venus, 69000000 miles from it?

10th. At Mercury, 37000000 miles from it?

11th. If a body weighing 100lb. on the earth, 4000 miles from its centre, be removed to the distance of the moon, 240000 miles, what would it then weigh? Ans. $7\frac{1}{9}$dr.

43. The square of the time of the revolution of one planet around the sun is to the square of the time of another as the cube of the mean distance of the first is to the cube of the mean distance of the second. The earth being assumed to be 95 millions of miles from the sun, its time of revolution 365.2563744 days, and Jupiter's time being 4332.5848032 days, what is the distance of Jupiter from the sun?

44. Of Mercury, whose time is 87.9692824 days?

45. Of Venus, whose time is 224.7007754 days?

46. The tax needed for school purposes in a certain city was $125000. How much should the board of directors assess upon the property of the inhabitants, valued at $25000000, the taxables being 12000, the poll-tax $1, and allowance being made of 4% for uncollected taxes, 5% for collector's fees, and 1% to the treasurer on all money received by him? What was A's tax, whose property was valued at $65000, and who pays for 1 poll? Ans. Prop. tax, $126445.86,1+; A's tax, $329.75,9+.

47. The sides containing the right angle of a certain triangular field were 30 chains and 40 chains long respectively, and it was inclosed by a "worm fence" made of rails of equal length, and having all its angles right angles except at the corners of the field, where it made an angle with the fence of the other side. I counted 199 panels on the shortest side, and noticed that at each corner of the field the last panel projected half its length beyond the other fence. What was the length of rail, allowing 1ft. of each for waste in lapping, what the number of rails inclosing the field, allowing 7 to a panel, and what the area of the field?

Ans. Length, 15.14+ft.; Number, 5565; Area, 60 acres.

THE END.

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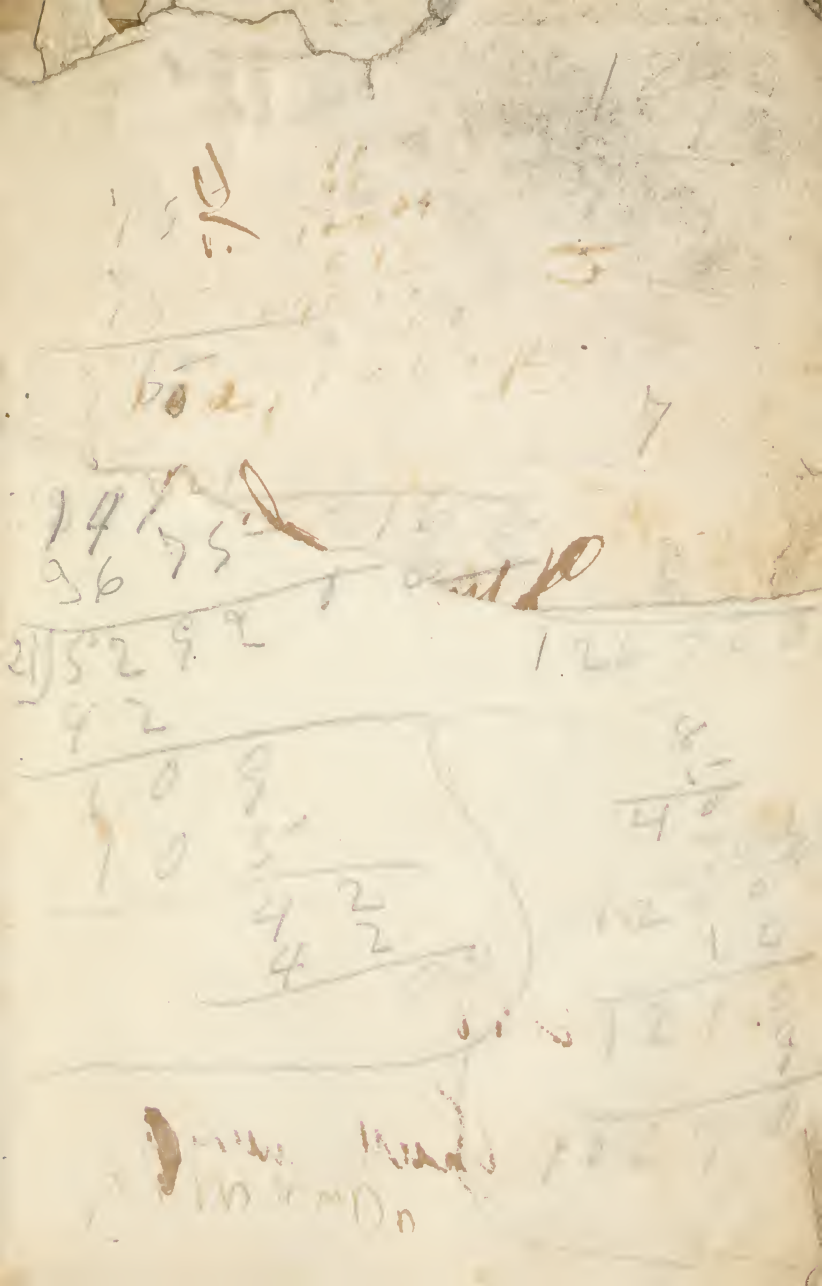

11th. If a body weighing 100lb. on the earth, 4000 miles from its centre, be removed to the distance of the moon, 240000 miles, what would it then weigh? Ans. $7\frac{1}{2}$dr.

43. The square of the time of the revolution of one planet around the sun is to the square of the time of another as the cube of the mean distance of the first is to the cube of the mean distance of the second. The earth being assumed to be 95 millions of miles from the sun, its time of revolution 365.2563744 days, and Jupiter's time being 4332.5848032 days, what is the distance of Jupiter from the sun?

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Dr.

Mr. Stephen Lane
Dan'l Wm. Lane
Lawrence Lane
Mr. Wm. Lane